OCR
Oxford Cambridge and RSA

GCE

Mathematics B (MEI)

Unit **H630/01**: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2018

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for AS Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

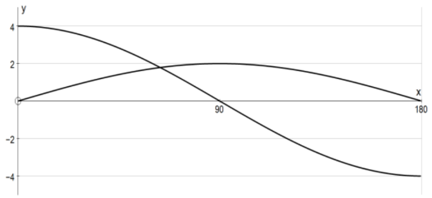
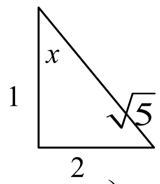
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

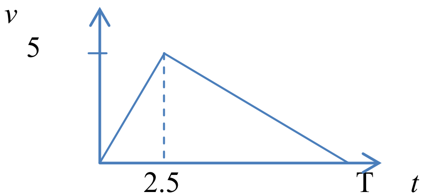
i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1			$\frac{8}{(3-\sqrt{5})}\times\frac{(3+\sqrt{5})}{(3+\sqrt{5})}=6+2\sqrt{5}$	M1 A1 [2]	1.1a 1.1b	Attempt to rationalize the denominator Must be in correct notation	Allow full credit for correct answer
2			EITHER $(3)^3+3(3)^2(-2x)+3(3)(-2x)^2+(-2x)^3$ $=27-54x+36x^2-8x^3$	M1 M1 A1 A1 [4]	1.1a 1.1b 1.1b 1.1b	Use of Binomial coefficients Powers of 3 and (-2x) Condone no brackets or (2x) used. At least 3 simplified terms correct All correct and simplified	
			OR $(3-2x)^2=(9-12x+4x^2)$ $(3-2x)(9-12x+4x^2)$ $=27-54x+36x^2-8x^3$	M1 M1 A1 A1 [4]		Attempting to square Multiplying their answer by third bracket At least 3 simplified terms correct All correct and simplified	

Question			Answer	Marks	AOs		Guidance
3			$\begin{pmatrix}8\\0\end{pmatrix}+\begin{pmatrix}2a\\-3a\end{pmatrix}+\begin{pmatrix}0\\b\end{pmatrix}=0$ $a=-4$, $b=-12$	M1 A1 A1 [3]	2.5 1.1b 1.1b	Setting up a correct vector equilibrium equation or two separate equations.	
4			$T-\frac{1}{2}mg=4ma$ $mg-T=ma$ $a=\dfrac{g}{10}$ m s^{-2} , $T=\dfrac{9mg}{10}$ N	B1 B1 M1dep A1 [4]	3.1b 3.1b 1.1a 1.1b	Award for first of these two equations Signs consistent with first equation Attempting to solve the simultaneous equations leading to either T or a soi Both T and a needed simplified	M1 dependent on at least one B1 earned Allow 0.98 m s^{-2} , and $T=\dfrac{441m}{50}=8.82m$ N
5	i		$y=4\cos x$ $y=2\sin x$ 	B1 B1 [2]	1.2 1.2	Correct shapes and relative sizes Graphs labelled with correct amplitudes clear Ignore any graph beyond the given interval	
	ii		$4\cos x=2\sin x$ $\tan x=2$ $x=\arctan 2$  $4\cos x=4\times\dfrac{1}{\sqrt{5}}=\dfrac{4}{5}\sqrt{5}$ Point of intersection is $\left(\arctan 2,\dfrac{4}{5}\sqrt{5}\right)$	M1 A1 M1 A1 [4]	3.1a 1.1b 1.1a 1.1b	Forming equation leading to $\tan x = k$ oe Award for 63.4^0 for this first A mark Attempting to find exact value for $\cos x$ or $\sin x$ Must be a coordinate in this form	Do not allow for using 63.4^0
	iii		The student's argument is invalid as the period of $\tan x$ is 180^0.	E1 [1]	2.3	Must state or imply that the argument is invalid and give a correct reason	Allow for "roots occur every 180^0" oe

Question			Answer	Marks	AOs		Guidance
6	i		$f(-1)=4\times(-1)^3-3(-1)+1=-4+3+1=0$	M1	2.1	**DR** Use of f(-1) must be seen. Do not allow for algebraic divison.	Allow without conclusion if preceded by "If $f(-1)=0$ then $(x+1)$ will be a factor" or similar
			Therefore $(x+1)$ is a factor	A1 [2]	2.2a	Clear conclusion must be made	
	ii		$f(x)=(x+1)(4x^2-4x+1)=0$	M1	1.1a	**DR** Attempt to divide or to factorise by inspection with $4x^2$	Allow full credit for $(x+1)(4x-2)(x-0.5)$
				A1	1.1b	correct quadratic factor seen or implied by correct linear factors	
			$=(x+1)(2x-1)^2=0$				No marks for solving the cubic on the calculator
			$x=-1,\ \frac{1}{2}$ [repeated]	A1 [3]	1.1b	Both roots seen derived from 3 correct linear factors or use of quadratic formula	
7			EITHER acceleration phase	M1	3.1b	Use of *suvat* equation(s) to find velocity. Do not allow if *s* = 10 used	Must recognise two phases of motion for first 4 marks
			$v=0+2.5\times2=5\ \text{m s}^{-1}$	A1	1.1b		
			slowing phase $v^2=u^2+2as$	M1	3.1b	Use of *suvat* equation(s) with *s* = 10 to find acceleration	
			$0=5^2+2a\times10$ $a=-1.25\ \text{m s}^{-2}$	A1	1.1b	FT their velocity. Must be correct sign.	
			$[-R]=1.5\times(-1.25)=-1.875$	M1	1.1a	Use of Newton's second law.	Consistent sign convention needed for full credit.
			Magnitude of *R* = 1.875 N (1.88 to 3sf)	A1 [6]	1.1b	FT their a $a\neq2$ Must be positive	

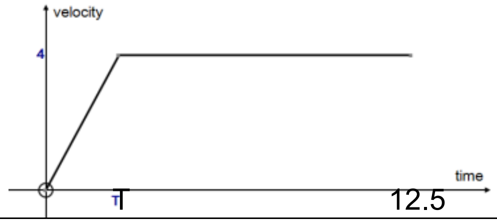
Question	Answer	Marks	AOs		Guidance
	OR acceleration phase $v = 0 + 2.5 \times 2 = 5 \text{ m s}^{-1}$	M1 A1		Use of *suvat* equation(s) to find velocity. Do not allow if s = 10 used	Must recognise two phases of motion for first 4 marks
		M1 A1		Use of area and *suvat* equation(s) to find acceleration Must be correct sign	
	using the distance to find the time it takes to stop *using areas* (second triangle): $\frac{1}{2}(T - 2.5) \times 5 = 10$ $T = 6.5$ so time to stop is 4 s. So $0 = 5 + 4a$ Giving $a = -1.25 \text{ m s}^{-2}$				Consistent sign convention needed for full credit.
	$[-R] = 1.5 \times (-1.25) = -1.875$ Magnitude of R = 1.875 N (1.88 to 3sf)	M1 A1 **[6]**		Use of Newton's second law. FT their a $a \neq 2$ Must be positive	

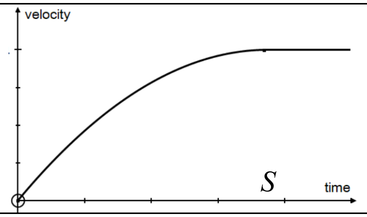
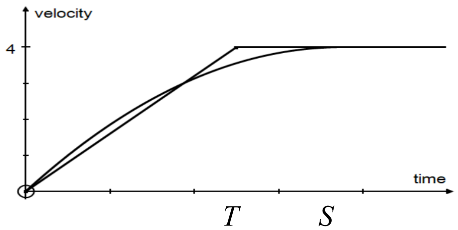
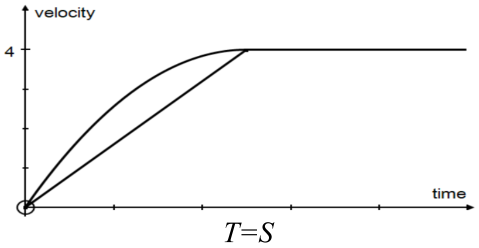
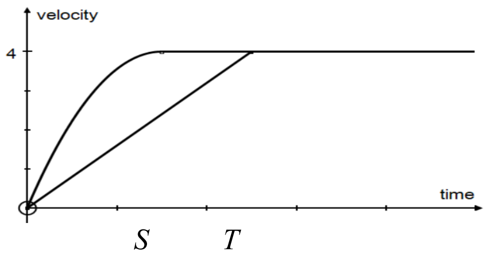
Question			Answer	Marks	AOs		Guidance
8			EITHER Equation of the form $y=k(x+1)(x-2)$ (0, –4) on curve so $k=2$	M1 M1 A1	1.1a 3.1a 1.1b	**DR** Allow with $k = 1$ and without $y =$ Attempt to find $k \neq 1$ All correct	Ignore = 0 if seen
			OR Equation of the form $y=ax^2+bx+c$ (0, -4) on curve $c=-4$ (-1, 0) on the curve $0=a-b-4$ (2, 0) on the curve $0=4a-2b-4$	 (M1) (M1)		 Uses one point to form an equation Uses both other points and attempts to solve simultaneous equations	 Allow for $c = -4$ seen
			Solving simultaneous equations $a=2,\ b=-2$	(A1)		All correct	
			BOTH Area = $\int_{-1}^{2}(2x^2-2x-4)\,\mathrm{d}x$				
			$\left[\frac{2x^3}{3}-x^2-4x\right]_{-1}^{2}$	M1 A1	1.1a 1.1b	Integration – allow without limits – condone one error FT their quadratic	
			$\left(\frac{2\times2^3}{3}-2^2-4\times2\right)-\left(\frac{2\times(-1)^3}{3}-(-1)^2-4\times(-1)\right)$	M1	1.1a	Substitution of limits clearly seen Complete argument leading to exact answer.	
			$=-\frac{20}{3}-\frac{7}{3}=-9$	A1	2.1	Allow for 9 if there is an argument to explain the change of sign even if -9 not seen.	
			Area is 9 below the x-axis.	E1 **[8]**	2.4	Must give modulus and explain the change of sign. FT if their definite integral is negative.	"Area must be positive" is not sufficient explanation.

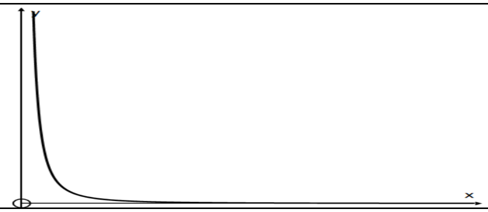
Question			Answer	Marks	AOs		Guidance
9	(i)		Using $y=\mathrm{f}\left(\frac{x}{a}\right)$ $y=\left(\frac{x}{½}-1\right)^2=(2x-1)^2$	M1	1.1a	Allow for 2 instead of ½ used for method mark or attempt to write equation of quadratic that touches axis at (0.5, 0)	$(2x-1)^2$ seen is sufficient for M1
			$=4x^2-4x+1$	A1 [2]	2.1	**AG** Must be a convincing argument that references either stretch or $\mathrm{f}(2x)$ or similar	
	(ii)		EITHER C_2 is $y=4.25x-x^2-3$	B1	3.1a	Finding the equation of C_2. Any form	
			Normal to $y=4x^2-4x+1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=8x-4$	M1	1.1a	Finding the derivative	
			At (0.1) $\frac{\mathrm{d}y}{\mathrm{d}x}=-4$ Gradient of normal is $\frac{1}{4}$	M1	1.1b	Finding negative reciprocal of their gradient	
			(0, 1) on line so equation of normal is $y=\frac{1}{4}x+1$	A1	1.1a	FT their value for derivative	
			Intersection of normal and C_2 $\frac{1}{4}x+1=4.25x-x^2-3$	M1	3.1a	Attempt to solve simultaneous equations	
			$4x^2-16x+16=0$ EITHER $(x-2)^2=0$ OR discriminant $16^2-4\times4\times16=0$	A1	1.1b	Repeated factor or root, or zero discriminant seen.	
			Repeated root so the normal is a tangent to C_2	E1 [7]	3.2a	Must interpret their solution in the context.	

Question			Answer	Marks	AOs		Guidance
			OR C_2 is $y = 4.25x - x^2 - 3$	B1		Finding the equation of C_2. Any form	
			Normal to $y = 4x^2 - 4x + 1$ $\frac{dy}{dx} = 8x - 4$ At (0.1) $\frac{dy}{dx} = -4$	M1		Finding the derivative	
			Gradient of normal is $\frac{1}{4}$	M1		Finding negative reciprocal of their gradient	
			Equation of normal is $y = \frac{1}{4}x + 1$	A1		FT their value for derivative	
			Point on C_1 where gradient is $\frac{1}{4}$ $\frac{dy}{dx} = 4.25 - 2x = \frac{1}{4}$	M1		Attempting to find the point on C_1 where tangent parallel to the normal found.	
			giving $x = 2$ $y = 1.5$	A1		Both coordinates required	
			EITHER So the equation of the tangent is $y - \frac{3}{2} = \frac{1}{4}(x - 2)$ Which is the same equation as the normal to C_1	E1		Correct equation for the tangent in form that makes it clear it is the same line as the normal.	
			OR show that point (2, 1.5) lies on normal So the normal to C_1 is a tangent to C_2	(E1) [7]			

Question			Answer	Marks	AOs		Guidance
			SPECIAL CASE when the candidate tries to show that the normal to C_2 is a tangent to C_1				
			C_2 is $y = 4.25x - x^2 - 3$	B1		Finding the equation of C_2. Any form	
			Normal to $y = 4.25x - x^2 - 3$				
			$\frac{dy}{dx} = 4.25 - 2x$	M1		Finding the derivative	
			At (0, 1) $\frac{dy}{dx} = 4.25$				
			Gradient of normal is $-\frac{4}{17}$	A1		Finding negative reciprocal of their gradient	
			Equation of normal is $y = -\frac{4}{17}x + 1$	A0			(0, 1) does not lie on C_2
			EITHER point of intersection with C_1 $4x^2 - 4x + 1 = -\frac{4}{17}x + 1$	M1		Attempt to solve simultaneous equations	
			OR Attempt to find both coordinates of the point on C_1 with gradient $-\frac{4}{17}$ $\frac{dy}{dx} = 8x - 4 = -\frac{4}{17}$	(M1)		Attempting to find the point on C_1 where tangent parallel to the normal found. **No further marks are available 4/7 maximum**	

Question			Answer	Marks	AOs		Guidance
10	i			B1 B1 [2]	1.1a 1.1a	Two line segments with one horizontal (T, 4) and (12.5, 4) labelled or indicated on scales. Allow their 2.5 marked instead of T. On axes labelled v and t oe	
	ii		$\frac{1}{2}\times 4\times\left(12.5+\left(12.5-T\right)\right)=45$ $T=2.5$	M1 A1 [2]	3.1a 1.1b	Attempt to find area of trapezium or both the the triangle $\left(\frac{1}{2}T\times 4\right)$ and the rectangle $\left(12.5-T\right)\times 4$. cao	*Suvat* equations can be used for two phases of motion.
	iii		EITHER $a=\frac{4}{2.5}=1.6\text{ m s}^{-2}$ $s=\frac{1}{2}\times 1.6t^2=0.8t^2$	M1 A1 [2]	1.1a 3.3	Soi FT their T	
			OR $a=\frac{4}{2.5}=1.6\text{ m s}^{-2}$ $v=\int a\ \mathrm{d}t=1.6t+c$ When $t=0,\ v=0$ so $c=0$ $s=\int v\ \mathrm{d}t=0.8t^2+c$ When $t=0,\ s=0$ so $c=0$ Giving $s=0.8t^2$	M1 A1 [2]		Soi FT their T Must be complete solution – do not award without consideration of $+c$ at least once	
	iv		$0.8t^2=4$ $t=\sqrt{5}=2.24\text{ s}$	B1FT [1]	3.4	FT their quadratic model in (iii)	

Question			Answer	Marks	AOs		Guidance
	v			B1 [1]	1.1a	Must have curved section of the graph decreasing gradient. S must be labelled.	
	vi		Total distance (area under the graph) can only be equal if $S > T$ If $S > T$ If $S = T$ If $S < T$ 	E1 [1]	3.5c	Needs to give reason relating to the refinement of the model. Graphs not required	"It takes longer to reach 4 ms^{-1}" is not sufficient reason

Question			Answer	Marks	AOs		Guidance
11	i		$y = kx^n$	B1 [1]	3.3	Allow any letters used for constant of proportionality and power.	
	ii		$\ln y = \ln\left(kx^n\right)$ $\ln y = \ln k + \ln\left(x^n\right) = \ln k + n\ln x$	M1 E1 [2]	2.1 2.1	Taking natural logs of both sides and one correct use of laws of logs used. Convincing argument. **AG**	
	iii		| | ln$x$ | ln$y$ | | Mercury | -1.179 | 9.575 | | Jupiter | 1.599 | 4.022 |	B1 B1 [2]	1.1b 1.1b	At least 2 correct values All correct and to 4sf	
	iv		EITHER $b = \dfrac{9.575 - 4.022}{-1.179 - 1.599} = -1.999$ (-2.00 to 3sf) $a = 7.218$	M1 A1 A1 [3]	1.1a 3.1a 1.1b	using gradient formula Allow -2 *a* correct to at least 2 sf	These values could be found using the calculator STATS mode, so allow without working
			OR $9.575 = a - 1.179b$ $4.022 = a + 1.599b$ Giving $a = 7.218$ $b = -1.999$ (-2.00 to 3sf)	M1 A1 A1 [3]		Setting up pair of equations by substitution of their values. Allow one slip. *a* correct to at least 2 sf *b* correct to at least 2 sf	Simultaneous equations can be solved using calcuator
	v		$y = 1363x^{-2.00}$	B1 B1 [2]	2.2a 3.3	FT their equation in (i) awrt 1300 or 1400, or $e^{7.2}$ or better. FT their *a* Allow for x^{-2} or better. FT their *b*	
	vi			B1 B1 [2]	1.2 1.1b	Appropriate curve with at least one horizontal asymptote or vertical asymptote shown Both asymptotes correct Ignore $x < 0$ if shown	FT their equation in (v) provided their function is a decreasing function
	vii		Earth $x = 1$, $y = 1363 \times 1^{-2} = 1360$ W m^{-2} (3sf)	B1 [1]	3.4	FT their (v)	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th></th><th>Guidance</th></tr>
<tr><td></td><td></td><td></td><td>SCHEME FOR CANDIDATES USING
LOG BASE 10</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>ii</td><td></td><td></td><td>SC1</td><td></td><td>Correct use of log instead of ln and no other error</td><td></td></tr>
<tr><td></td><td>iii</td><td></td><td><table><tr><th></th><th>ln$x$</th><th>ln$y$</th></tr><tr><td>Mercury</td><td>-0.5122</td><td>4.158</td></tr><tr><td>Jupiter</td><td>0.6946</td><td>1.747</td></tr></table></td><td>B0
B1</td><td></td><td>All correct. Must be 4 sf</td><td></td></tr>
<tr><td></td><td>iv</td><td></td><td>As in main scheme
$a = 3.1336$
$b = -1.999$</td><td>M1

A1
A1</td><td></td><td></td><td></td></tr>
<tr><td></td><td>v</td><td></td><td>$y = 1360x^{-2}$</td><td>B1

B1</td><td></td><td>FT their equation in (i)
awrt 1300 or 1400, or $e^{3.1}$ (= 22.2), $10^{3.1}$ or better.
FT their a
allow for x^{-2} or better. FT their b</td><td></td></tr>
<tr><td></td><td>vii</td><td></td><td>Earth $x = 1$, $y = 1363 \times 1^{-2} = 1360$ W m^{-2} (3sf)</td><td>B1</td><td></td><td>FT their (v)</td><td></td></tr>
</table>

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553