10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

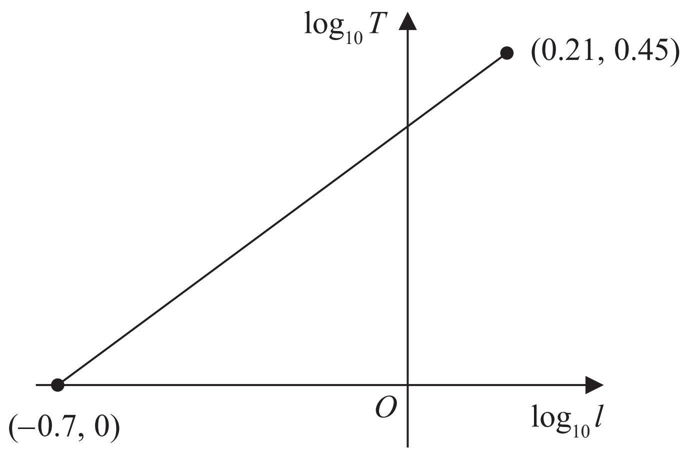

Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

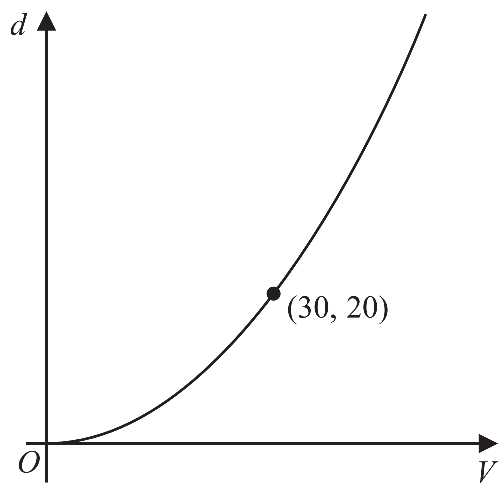

Figure 5

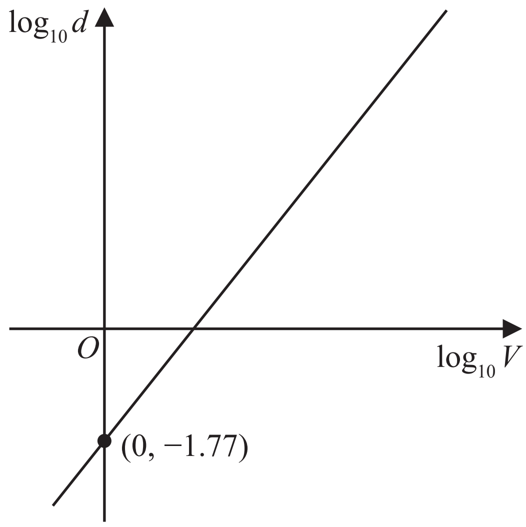

Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

Leave blank

6. The mass, m grams, of a radioactive substance t years after first being observed, is modelled by the equation

$$m = 25e^{1-kt}$$

where k is a positive constant.

(a) State the value of m when the radioactive substance was first observed.

(1)

Given that the mass is 50 grams, 10 years after first being observed,

(b) show that $k = \frac{1}{10}\ln\left(\frac{1}{2}e\right)$

(4)

(c) Find the value of t when $m = 20$, giving your answer to the nearest year.

(3)

Leave blank

6. A hot piece of metal is dropped into a cool liquid. As the metal cools, its temperature T degrees Celsius, t minutes after it enters the liquid, is modelled by

$$T = 300e^{-0.04t} + 20, \quad t \geqslant 0$$

(a) Find the temperature of the piece of metal as it enters the liquid.

(1)

(b) Find the value of t for which $T = 180$, giving your answer to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(c) Show, by differentiation, that the rate, in degrees Celsius per minute, at which the temperature of the metal is changing, is given by the expression

$$\frac{20 - T}{25}$$

(3)

Leave blank

9.

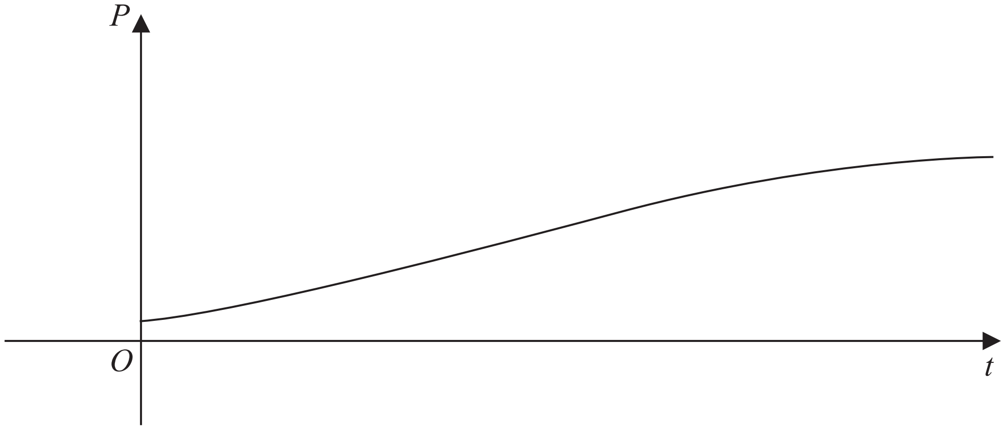

Figure 2

The population of a species of animal is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{9000e^{kt}}{3e^{kt} + 7}, \qquad t \geqslant 0$$

where k is a positive constant.

A sketch of the graph of P against t is shown in Figure 2.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find the value for the upper limit of the population.

(1)

Given that $P = 2500$ when $t = 4$

(c) calculate the value of k, giving your answer to 3 decimal places.

(5)

Using this value for k,

(d) find, using $\frac{dP}{dt}$, the rate at which the population is increasing when $t = 10$

Give your answer to the nearest integer.

(3)

Leave blank

10. A population of insects is being studied. The number of insects, N, in the population, is modelled by the equation

$$N = \frac{300}{3 + 17e^{-0.2t}} \qquad t \in \mathbb{R},\ t \geqslant 0$$

where t is the time, in weeks, from the start of the study.

Using the model,

(a) find the number of insects at the start of the study,

(1)

(b) find the number of insects when $t = 10$,

(2)

(c) find the time from the start of the study when there are 82 insects.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(d) Find, by differentiating, the rate, measured in insects per week, at which the number of insects is increasing when $t = 5$. Give your answer to the nearest whole number.

(3)

Leave blank

13.

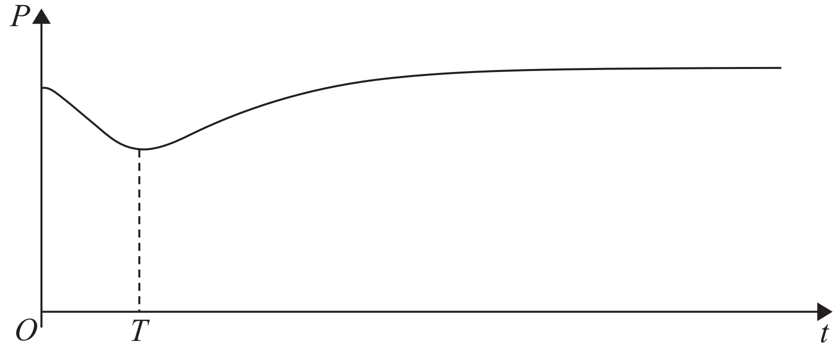

Figure 5

A colony of ants is being studied. The number of ants in the colony is modelled by the equation

$$P = 200 - \frac{160e^{0.6t}}{15 + e^{0.8t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of ants, measured in thousands, t years after the study started. A sketch of the graph of P against t is shown in Figure 5

(a) Calculate the number of ants in the colony at the start of the study.
(2)

(b) Find $\frac{dP}{dt}$
(3)

The population of ants initially decreases, reaching a minimum value after T years, as shown in Figure 5

(c) Using your answer to part (b), calculate the value of T to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(4)

Leave blank

3. The number of bacteria in a liquid culture is modelled by the formula

$$N = 3500(1.035)^t, \quad t \geqslant 0$$

where N is the number of bacteria t hours after the start of a scientific study.

(a) State the number of bacteria at the start of the scientific study.

(1)

(b) Find the time taken from the start of the study for the number of bacteria to reach 10 000
Give your answer in hours and minutes, to the nearest minute.

(4)

(c) Use calculus to find the rate of increase in the number of bacteria when $t = 8$
Give your answer, in bacteria per hour, to the nearest whole number.

(3)

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8.

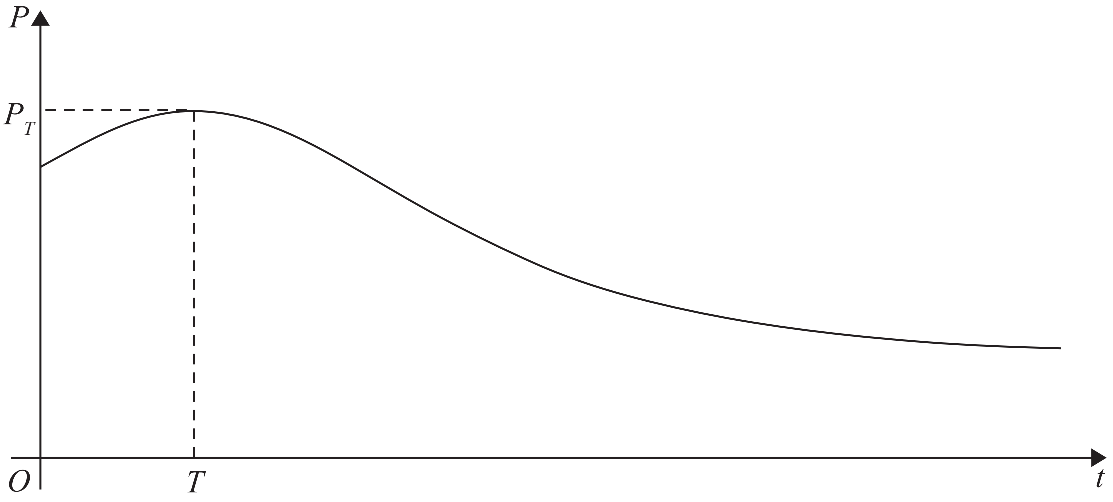

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1 + 3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

(a) Calculate the number of rabbits that were introduced onto the island.

(1)

(b) Find $\frac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(c) Using your answer from part (b), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(d) Use the model to state the maximum value of k.

(1)

Leave blank

9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a\ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

Leave blank

4. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(c) Calculate the value of T to the nearest whole number.

(2)

Leave blank

8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270

(1)

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8.

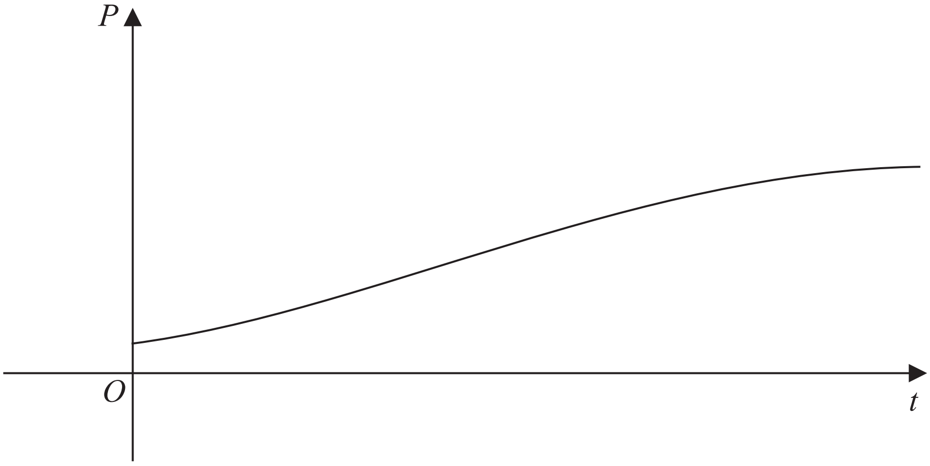

Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \qquad t \geqslant 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.

(2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study.

(3)