Question	Scheme	Marks	AOs
6(a)	$\mathrm{f}(x)=\frac{x+2}{x^2+9}=\frac{x}{x^2+9}+\frac{2}{x^2+9}$	B1	3.1a
	$\int\frac{x}{x^2+9}\,\mathrm{d}x=k\ln\left(x^2+9\right)(+c)$	M1	1.1b
	$\int\frac{2}{x^2+9}\,\mathrm{d}x=k\arctan\left(\frac{x}{3}\right)(+c)$	M1	1.1b
	$\int\frac{x+2}{x^2+9}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+c$	A1	1.1b
		(4)	
(b)	$\int_0^3\mathrm{f}(x)\,\mathrm{d}x=\left[\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$ $=\frac{1}{2}\ln 18+\frac{2}{3}\arctan\left(\frac{3}{3}\right)-\left(\frac{1}{2}\ln 9+\frac{2}{3}\arctan(0)\right)$ $=\frac{1}{2}\ln\frac{18}{9}+\frac{2}{3}\arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value $=\frac{1}{3-0}\left(\frac{1}{2}\ln 2+\frac{\pi}{6}\right)$	M1	2.1
	$\frac{1}{6}\ln 2+\frac{1}{18}\pi$ *	A1*	2.2a
		(3)	
(c)	$\frac{1}{6}\ln 2+\frac{1}{18}\pi+\ln k$	M1	2.2a
	$\frac{1}{6}\ln 2k^6+\frac{1}{18}\pi$	A1	1.1b
		(2)	
		(9 marks)	

Notes:

(a)
B1: Splits the fraction into two correct separate expressions
M1: Recognises the required form for the first integration
M1: Recognises the required form for the second integration
A1: Both expressions integrated correctly and added together with constant of integration included

(b)
M1: Uses limits correctly and combines logarithmic terms
M1: Correctly applies the method for the mean value for their integration
A1*: Correct work leading to the given answer

(c)
M1: Realises that the effect of the transformation is to increase the mean value by ln k
A1: Combines ln's correctly to obtain the correct expression

M1: Solve their four equation (using calculator) to find at least one value. This will need checking if incorrect equations used. A1: Correct quartic in terms of z or correct values for a, b, c and d stated. **Note:** Correct answer only will score 5/5

Question	Scheme	Marks	AOs
2	$\frac{8x-12}{(2x^2+3)(x+1)} = \frac{Ax+B}{2x^2+3} + \frac{C}{x+1}$	M1	3.1a
	$8x-12 = (Ax+B)(x+1) + C(2x^2+3)$ E.g. $x=-1 \Rightarrow C=-4, x=0 \Rightarrow B=0, x=1 \Rightarrow A=8$ Or Compares coefficients and solves $(A+2C=0 \quad A+B=8 \quad B+3C=-12)$ $\Rightarrow A=..., B=..., C=...$	dM1	1.1b
	$A=8 \quad B=0 \quad C=-4$	A1	1.1b
	$\int \left(\frac{8x}{2x^2+3} - \frac{4}{x+1} \right) dx = 2\ln(2x^2+3) - 4\ln(x+1)$	A1ft	1.1b
	$2\ln(2x^2+3) - 4\ln(x+1) = \ln\left(\frac{(2x^2+3)^2}{(x+1)^4} \right)$ or $2\ln(2x^2+3) - 4\ln(x+1) = 2\ln\left(\frac{(2x^2+3)}{(x+1)^2} \right)$	M1	2.1
	$\lim_{x\to\infty} \left\{ \ln \frac{(2x^2+3)^2}{(x+1)^4} \right\} = \ln 4$ or $\lim_{x\to\infty} \left\{ 2\ln \frac{(2x^2+3)}{(x+1)^2} \right\} = 2\ln 2$	B1	2.2a
	$\Rightarrow \int_0^\infty \frac{8x-12}{(2x^2+3)(x+1)} dx = \ln\frac{4}{9}$ cao	A1	1.1b
		(7)	
			(7 marks)

Notes

M1: Selects the correct form for partial fractions.
dM1: Full method for finding values for all three constants. Dependent on having the correct form for the partial fractions. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

Question	Scheme	Marks	AOs
3(a) Way 1	$x=\frac{3}{2}\sinh u$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}}=\int \frac{1}{\sqrt{4\left(\frac{9}{4}\right)\sinh^2 u+9}}\times\frac{3}{2}\cosh u\ du$	M1	3.1a
	$=\int \frac{1}{2}\ du$	A1	1.1b
	$=\int \frac{1}{2}\ du=\frac{1}{2}u=\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)+c$	A1	1.1b
		(4)	
(a) Way 2	$x=\frac{3}{2}\tan u$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}}=\int \frac{1}{\sqrt{4\left(\frac{9}{4}\right)\tan^2 u+9}}\times\frac{3}{2}\sec^2 u\ du$	M1	3.1a
	$=\int \frac{1}{2}\sec u\ du$	A1	1.1b
	$=\frac{1}{2}\ln\left(\sec u+\tan u\right)=\frac{1}{2}\ln\left(\frac{2x}{3}+\sqrt{1+\left(\frac{2x}{3}\right)^2}\right)$ $u=\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)+c$	A1	1.1b
(a) Way 3	$x=\frac{1}{2}u$ or $x=ku$ where $k>0$ $k\neq 1$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}}=\int \frac{1}{\sqrt{4\left(\frac{1}{4}\right)u^2+9}}\times\frac{1}{2}du$	M1	3.1a
	$=\frac{1}{2}\int \frac{1}{\sqrt{u^2+9}}du\ \left(\text{or } \frac{1}{2}\int \frac{1}{\sqrt{u^2+\frac{9}{4k^2}}}du \text{ for } x=ku\right)$	A1	1.1b
	$=\frac{1}{2}\sinh^{-1}\frac{u}{3}=\frac{1}{2}\sinh^{-1}\frac{2x}{3}+c$	A1	1.1b
(b)	Mean value = $\frac{1}{3(-0)}\left[\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)\right]_0^3=\frac{1}{3}\times\frac{1}{2}\sinh^{-1}\left(\frac{2\times 3}{3}\right)(-0)$	M1	2.1
	$=\frac{1}{6}\ln\left(2+\sqrt{5}\right)$ **(Brackets are required)**	A1ft	1.1b
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
2(a)	E.g. • Because the interval being integrated over is unbounded • Accept because the upper limit is infinity • Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = ..., B = ...$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)} \, dx = \frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5) = \frac{1}{5}\ln\frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{x}{2x+5}\right\} = \frac{1}{5}\ln\frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^{\infty} \frac{1}{x(2x+5)} \, dx = \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7} = \frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	
			(7 marks)

Notes

(a)

B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity". Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept "because one of the limits is undefined" unless they state they mean ∞. Do not accept "it is undefined when $x = \infty$" without reference to "it" being the upper limit.

(b)

M1: Selects the correct form for partial fractions and proceeds to find values for *A* and *B*

A1: Correct constants or partial fractions

A1ft: $\int \frac{p}{x} + \frac{q}{2x+5} \, dx = p\ln x + \frac{q}{2}\ln(2x+5)$ Note that $\frac{1}{5}\ln 5x - \frac{1}{5}\ln(10x+25)$ is correct.

M1: Combines logs correctly. May see $-\frac{1}{5}\ln\left(\frac{2x+5}{x}\right) = -\frac{1}{5}\ln\left(2 + \frac{5}{x}\right)$

B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing *x* with ∞ scores B0)

A1: Deduces the correct value for the improper integral in the correct form

Question	Scheme	Marks	AOs
5(a)	$y=\tan^{-1}x\Rightarrow \tan y = x \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\sec^2 y$ $y=\tan^{-1}x\Rightarrow \tan y = x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}\sec^2 y=1$	M1	3.1a
	$\frac{\mathrm{d}x}{\mathrm{d}y}=1+\tan^2 y$ or $\frac{\mathrm{d}y}{\mathrm{d}x}\left(1+\tan^2 y\right)=1$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\tan^2 y}=\frac{1}{1+x^2}$ *	A1*	2.1
		(3)	
(b)	$\frac{\mathrm{d}\left(\tan^{-1}4x\right)}{\mathrm{d}x}=\frac{4}{1+16x^2}$	B1	1.1b
	$\int x\tan^{-1}4x\,\mathrm{d}x=\alpha x^2\tan^{-1}4x-\int \alpha x^2\times'\frac{4}{1+16x^2}'\mathrm{d}x$	M1	2.1
	$\int x\tan^{-1}4x\,\mathrm{d}x=\frac{x^2}{2}\tan^{-1}4x-\int\frac{x^2}{2}\times\frac{4}{1+16x^2}\mathrm{d}x$	A1	1.1b
	$=\ldots-\frac{1}{8}\int\frac{16x^2+1-1}{1+16x^2}\mathrm{d}x=\ldots-\frac{1}{8}\int\left(1-\frac{1}{1+16x^2}\right)\mathrm{d}x$ or let $4x=\tan u\Rightarrow \frac{1}{8}\int\frac{\tan^2 u}{1+\tan^2 u}\times\frac{1}{4}\sec^2 u\,\mathrm{d}u$ $\Rightarrow \frac{1}{32}\int\tan^2 u\,\mathrm{d}u=\frac{1}{32}\int\sec^2 u-u\,\mathrm{d}u$	M1	3.1a
	$=\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x+k$	A1	2.1
		(5)	
(c)	Mean value $=\left(\frac{1}{\frac{\sqrt{3}}{4}-0}\right)\left[\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x\right]_0^{\frac{\sqrt{3}}{4}}$ $=\frac{4}{\sqrt{3}}\left(\left(\frac{3}{32}\times\frac{\pi}{3}-\frac{1}{8}\times\frac{\sqrt{3}}{4}+\frac{1}{32}\times\frac{\pi}{3}\right)-0\right)$	M1	2.1
	$=\frac{\sqrt{3}}{72}\left(4\pi-3\sqrt{3}\right)$ or $\frac{\sqrt{3}}{18}\pi-\frac{1}{8}$ oe	A1	1.1b
		(2)	
		(10 marks)	

Notes

(a)
M1: Makes progress in establishing the derivative by taking the tan of both sides and differentiating with respect to *y* or implicitly with respect to *x*
M1: Use of the correct identity
A1*: Fully correct proof

Question	Scheme	Marks	AOs
5(i)	$\int 2e^{-\frac{1}{2}x}\,dx = -4e^{-\frac{1}{2}x}$	B1	1.1b
	$\int_1^{\infty} 2e^{-\frac{1}{2}x}\,dx = \lim_{t\to\infty}\left[\left(-4e^{-\frac{1}{2}t}\right)-\left(-4e^{-\frac{1}{2}}\right)\right]$	M1	2.1
	$= 4e^{-\frac{1}{2}}$	A1	1.1b
		(3)	
(ii)(a)	Mean temperature $= \dfrac{1}{24}\int_0^{24}\left(8-5\sin\left(\dfrac{\pi}{12}t\right)-\cos\left(\dfrac{\pi}{6}t\right)\right)dt$	B1	1.2
	$= \dfrac{1}{24}\left[\left(8t+\dfrac{60}{\pi}\cos\left(\dfrac{\pi}{12}t\right)-\dfrac{6}{\pi}\sin\left(\dfrac{\pi}{6}t\right)\right)\right]_0^{24} = \dfrac{1}{24}[...]$	M1	1.1b
	$= \dfrac{1}{24}\left[\left(8(24)+\dfrac{60}{\pi}-\dfrac{6}{\pi}\times 0\right)-\left(\dfrac{60}{\pi}\right)\right] = 8$ * cso	A1*	2.1
		(3)	
(ii)(b)	E.g. increase the value of the constant 8 / adapt the constant 8 to a function which takes values greater than 8.	B1	3.5c
		(1)	
			(7 marks)

Notes:

(i)
B1: Correct integration.

M1: Attempt to integrate to a form $\lambda e^{-\frac{1}{2}x}$ where $\lambda \neq 2$, and applies correct limits with some consideration of the infinite limit given (e.g. with the limit statement). Only allow with ∞ used as the limit if subsequent work shows the term is zero.
A1: Correct value

(ii)(a)
B1: Recalls the correct formula for finding the mean value of a function. You may see the division by "24" only at the end. No integration is necessary, just a correct statement with an integral.

M1: Integrates to a form $\alpha t+\beta\cos\left(\dfrac{\pi}{12}t\right)+\delta\sin\left(\dfrac{\pi}{6}t\right)$ and uses the limits of 0 and 24 (the correct way around). If no explicit substitution is seen, accept any value following the integral as an attempt. Answers from a calculator with no correct integral seen score M0 as the question requires calculus to be used.
A1*cso: Achieves 8 with no errors seen following a full attempt at the substitution. Must have seen some evidence of the limits used, minimum required for substitution is $\left[\left(8(24)+\dfrac{60}{\pi}\right)-\left(\dfrac{60}{\pi}\right)\right]$.

(ii)(b)
B1: Accept any reasonable adaptation to the equation that will increase the mean value. E.g. as in scheme, or introduce another positive term, or decrease the constant 5 etc. It must be clear which constant they are referring to in their reason, not just "increase the constant".

Question	Scheme	Marks	AOs
9(a)	$\int \frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x \to \int \mathrm{f}(u)\,\mathrm{d}u$ Uses the substitution $x=\cosh u$ fully to achieve an integral in terms of u only, including replacing the dx	M1	3.1a
	$\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u-1}}\sinh u\,(\mathrm{d}u)$	A1	1.1b
	Uses correct identities $\cosh^2 u-1=\sinh^2 u$ and $\cosh 2u=2\cosh^2 u-1$ to achieve an integral of the form $A\int(\cosh 2u\pm 1)\,\mathrm{d}u \qquad A>0$	M1	3.1a
	Integrates to achieve $A\left(\pm\frac{1}{2}\sinh 2u\pm u\right)(+c) \qquad A>0$	M1	1.1b
	Uses the identity $\sinh 2u=2\sinh u\cosh u$ and $\cosh^2 u-1=\sinh^2 u$ $\to \sinh 2u=2x\sqrt{x^2-1}$	M1	2.1
	$\frac{1}{2}\left[x\sqrt{x^2-1}+\mathrm{ar}\cosh x\right]+k$ * cso	A1*	1.1b
		(6)	
(b)	Uses integration by parts the correct way around to achieve $\int \frac{4}{15}x\,\mathrm{arcosh}x\mathrm{d}x = Px^2\mathrm{arcosh}x - Q\int \frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x$	M1	2.1
	$=\frac{4}{15}\left(\underline{\frac{1}{2}x^2\mathrm{arcosh}x-\frac{1}{2}\int\frac{x^2}{\sqrt{x^2-1}}\,\mathrm{d}x}\right)$	A1	1.1b
	$=\frac{4}{15}\left(\frac{1}{2}x^2\mathrm{arcosh}x-\frac{1}{2}\left(\underline{\frac{1}{2}\left[x\sqrt{x^2-1}+\mathrm{arcosh}x\right]}\right)\right)$	B1ft	2.2a
	Uses the limits x =1 and x = 3 the correct way around and subtracts $=\frac{4}{15}\left(\frac{1}{2}(3)^2\,\mathrm{arcosh}3-\frac{1}{2}\left(\frac{1}{2}\left[3\sqrt{(3)^2-1}+\mathrm{arcosh}3\right]\right)\right)-\frac{4}{15}(0)$	dM1	1.1b
	$=\frac{4}{15}\left(\frac{9}{2}\ln\left(3+\sqrt{8}\right)-\frac{3\sqrt{8}}{4}-\frac{1}{4}\ln\left(3+\sqrt{8}\right)\right)$ $=\frac{1}{15}\left[17\ln\left(3+2\sqrt{2}\right)-6\sqrt{2}\right]$*	A1*	1.1b
		(5)	
			(11 marks)

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \frac{-\lambda}{\sqrt{1-\beta x^2}}$ where $\lambda > 0$ and $\beta > 0$ and $\beta \neq 1$ Alternatively $2\cos y = x \Rightarrow \frac{dx}{dy} = \alpha \sin y \Rightarrow \frac{dy}{dx} = \frac{1}{\alpha \sin y}$	M1	1.1b
	$\frac{dy}{dx} = \frac{-\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-\frac{1}{4}x^2}}$ o.e. or $\frac{dy}{dx} = -\frac{1}{2\sin y}$ or	A1	1.1b
	States that $\frac{dy}{dx} \neq 0$ therefore C has no stationary points. Tries to solve $\frac{dy}{dx} = 0$ and ends up with a contradiction e.g. -1 = 0 therefore C has no stationary points. As cosec $y > 1$ therefore C has no stationary points.	A1	2.4
		(3)	
(b)	$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-\frac{1}{4}\times 1^2}} = \left\{-\frac{1}{\sqrt{3}}\right\}$	M1	1.1b
	Normal gradient = $-\frac{1}{m}$ and $y - \frac{\pi}{3} = m_n(x-1)$ Alternatively $\frac{\pi}{3} = m_n(1) + c \Rightarrow c = ...\left\{\frac{\pi}{3} - \sqrt{3}\right\}$ and then $y = m_n x + c$	M1	1.1b
	$y = 0 \Rightarrow 0 - \frac{\pi}{3} = \sqrt{3}(x_A - 1) \Rightarrow x_A = ...\left\{1 - \frac{\pi}{3\sqrt{3}} \text{ or } 1 - \frac{\pi\sqrt{3}}{9}\right\}$ and $x = 0 \Rightarrow y_B - \frac{\pi}{3} = \sqrt{3}(0-1) \Rightarrow y_B = ...\left\{\frac{\pi}{3} - \sqrt{3}\right\}$	M1	3.1a
	Area $= \frac{1}{2} \times x_A \times -y_B = \frac{1}{2}\left(1 - \frac{\pi}{3\sqrt{3}}\right)\left(\sqrt{3} - \frac{\pi}{3}\right)$	M1	1.1b
	Area $\frac{1}{54}\left(27\sqrt{3} - 18\pi + \sqrt{3}\pi^2\right)$ $(p = 27, q = -18, r = 1)$	A1	2.1
		(5)	
		(8 marks)	

Notes:

(a)

M1: Finds the correct form for $\frac{dy}{dx}$

Question	Scheme	Marks	AOs
6(a)	$\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \Rightarrow 2x^2+3x+6$ $= A(x^2+4) + (Bx+C)(x+1)$	M1	1.1b
	e.g. $x=-1 \Rightarrow A = \ldots,\ x=0 \Rightarrow C = \ldots,\ \text{coeff } x^2 \Rightarrow B = \ldots$ or Compares coefficients and solves to find values for A, B and C $2 = A+B,\ 3 = B+C,\ 6 = 4A+C$	dM1	1.1b
	$A = 1, \quad B = 1, \quad C = 2$	A1	1.1b
		(3)	
(b)	$\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4}\,dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\,dx$ $= \left[\alpha \ln(x+1) + \beta \ln(x^2+4) + \lambda \arctan\left(\frac{x}{2}\right)\right]_0^2$	M1	3.1a
	$= \left[ln(x+1) + \frac{1}{2} ln(x^2+4) + arctan\left(\frac{x}{2}\right)\right]_0^2$	A1	2.1
	$= \left[ln(3) + \frac{1}{2} ln(8) + arctan\ 1\right] - \left[ln(1) + \frac{1}{2} ln(4) + arctan(0)\right]$ $=$ $= \left[ln(3) + \frac{1}{2} ln(8) + arctan(1)\right] - \left[\frac{1}{2} ln\ 4\right] = \underline{ln\left(\frac{3\sqrt{8}}{2}\right)} + \frac{\pi}{4}$	dM1	2.1
	$ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
		(4)	
			(7 marks)

Notes:

(a)
M1: Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule).
dM1: Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

(b)
M1: Splits the integral into an integrable form and integrates at least two terms to the correct form. They may use a substitution on the arctan term
A1: Fully correct Integration.
dM1: Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once.
A1: Correct answer

Question	Scheme	Marks	AOs
9(i) (a)	E. g. ● Because the interval being integrated over is unbounded. ● cosh x is undefined at the limit of ∞ ● the upper limit is infinite	B1	1.2
		(1)	
(i) (b)	$\int_0^{\infty} \cosh x \, dx = \lim_{t\to\infty} \int_0^t \cosh x \, dx$ or $\lim_{t\to\infty} \int_0^t \frac{1}{2}(e^x + e^{-x})dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t \, (-0)$ or $\frac{1}{2}\int_0^t e^x + e^{-x} \, dx = \frac{1}{2}[e^x - e^{-x}]_0^t = \frac{1}{2}[e^t - e^{-t}] \left(-\frac{1}{2}[e^0 - e^0]\right)$	M1	1.1b
	When $t \to \infty$ $e^t \to \infty$ and $e^{-t} \to 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4 \sinh x = p \cosh x \Rightarrow \tanh x = \frac{p}{4}$ or $4 \tanh x = p$ **Alternative** $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4-p) = p + 4 \Rightarrow e^{2x} = \frac{p+4}{4-p}$	M1	3.1a
	$\left\{-1 < \frac{p}{4} < 1 \Rightarrow\right\} -4 < p < 4$	A1	2.2a
		(2)	
			(6 marks)

(i)(a)

B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity", but **not** "because it is infinity" without reference to what "it" is. Do not accept "the upper limit tends to infinity" or "the integral is unbounded".

(i)(b)

B1: Writes the integral in terms of a limit as $t \to \infty$ (or other variable) with limits 0 and "t", or implies the integral is a limit by subsequent working by correct language.

M1: Integrates cosh$x$ correctly either as sinh x or in terms of exponentials and applies correctly the limits of 0 and "t". The bottom limit zero may be implied. No need for the $\lim_{t\to\infty}$ for this mark but substitution of ∞ is M0.

A1: cso States that (as $t \to \infty$) $\sinh t \to \infty$ or $e^t \to \infty$ and $e^{-t} \to 0$ therefore divergent (or not convergent), or equivalent working. Accept sinh t is undefined as $t \to \infty$

(ii)

M1: Divides through by cosh$x$ to find an expression involving tanh x

Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x}

A1: Deduces the correct inequality for p. Note $|p| < 4$ is a correct inequality for p.

Question	Scheme		Marks	AOs
5(a)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$	$\sin y = x \Rightarrow \dfrac{dx}{dy} = \cos y$	M1	1.1b
	Uses $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1 - \sin^2 y} \Rightarrow \sqrt{1 - x^2}$		M1	2.1
	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ * cso		A1*	1.1b
			(3)	
(b)	Using the answer to (a) $f'(x) = \dfrac{1}{\sqrt{1 - e^{2x}}} \times \ldots$	Restart $\sin y = e^x \Rightarrow \cos y \dfrac{dy}{dx} = e^x$	M1	3.1a
	$f'(x) = \dfrac{1}{\sqrt{1 - e^{2x}}} \times e^x$	$f'(x) = \dfrac{e^x}{\cos y}$	A1	1.1b
	$e^x \neq 0$ (or $e^x > 0$) therefore, there are no stationary points Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined therefore there are no stationary points.		A1	2.4
			(3)	
				(6 marks)

Notes:

(a)

M1: Finds *x* in terms of *y* and differentiates

M1: Uses the trig identity $\sin^2 y + \cos^2 y = 1$ to express $\cos y$ in terms of *x*. This may be seen in their derivative or stated on the side

A1*: Correctly achieves the printed answer $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. cso

(b)

M1: Differentiates using the chain rule to achieve the correct form, condone $f'(x) = \frac{1}{\sqrt{1-e^{2x}}}$

Note $f'(x) = \frac{1}{\sqrt{1-e^x}}$ is B0 for incorrect form

Alternatively restart, finds *x* in terms of *y* and differentiates

A1: Correct differentiation

A1: Follows correct differentiation. States that as $e^x \neq 0$ (or $e^x > 0$) or no solutions to $e^x = 0$ therefore there are no stationary points.

Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined/error therefore there are no stationary points. Ignore any reference to the denominator = 0

Question	Scheme	Marks	AOs
2(a)	$x^2+4x-5=(x+2)^2-9$	B1	1.1b
		(1)	
(b)	$\int \frac{1}{\sqrt{(x+p)^2-q}}\,dx = \text{arcosh}\left(\frac{x+p}{\sqrt{q}}\right)(+c)$ or $\ln\left(x+p+\sqrt{(x+p)^2-q}\right)(+c)$	M1	1.1a
	$=\text{arcosh}\left(\frac{x+2}{3}\right)$ or $\ln\left(x+2+\sqrt{(x+2)^2-9}\right)$ oe	A1	2.2a
		(2)	
(c)	Mean $=\frac{1}{13-3}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx$	B1	1.2
	$\frac{1}{10}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx=\frac{1}{10}\left(\text{arcosh}\left(\frac{15}{3}\right)-\text{arcosh}\left(\frac{5}{3}\right)\right)$ or $\frac{1}{10}\int_3^{13}\frac{1}{\sqrt{x^2+4x-5}}\,dx=\frac{1}{10}\left(\ln\left(15+\sqrt{216}\right)-\ln\left(5+\sqrt{16}\right)\right)$	M1	1.1b
	$=\frac{1}{10}\ln\left(\frac{5+2\sqrt{6}}{3}\right)$ or $\frac{1}{20}\ln\left(\frac{49+20\sqrt{6}}{9}\right)$	A1	3.2a
		(3)	
		(6 marks)	

Notes:

(a)

B1: Correct completed square form. Allow 3^2 for 9.

(b)

M1: Achieves a correct form for the integration for their p and q from part (a):

$$\text{arcosh}\left(\frac{x+p}{\sqrt{q}}\right)(+c) \textbf{ or } \ln\left(x+p+\sqrt{(x+p)^2-q}\right)(+c) \textbf{ or e.g. } \ln\left(\frac{x+p}{\sqrt{q}}+\sqrt{\left(\frac{x+p}{\sqrt{q}}\right)^2-1}\right)(+c)$$

where $p \neq 0,\ q \neq 1$

Allow $\cosh^{-1}$ for arcosh

Allow attempts that use substitution following an attempt to complete the square but must be an appropriate substitution e.g. $x+p=\sqrt{q}\cosh u$ leading to a correct form as above.

A1: Correct integration. The "+ c" is not required. Apply isw once a correct expression is seen.

Note that $\ln\left(\frac{x+2}{3}+\sqrt{\left(\frac{x+2}{3}\right)^2-1}\right)(+c)$ is also correct