Question	Scheme		Marks	AOs
3(i)(a)	$\lvert\mathbf{M}\rvert = 2(1+2) - a(-1-1) + 4(2-1) = 0 \Rightarrow a = \ldots$		M1	2.3
	The matrix **M** has an inverse when $a \neq -5$		A1	1.1b
			(2)	
(b)	$\text{Minors}: \begin{pmatrix} 3 & -2 & 1 \\ -a-8 & 2 & a+4 \\ 4-a & -6 & -2-a \end{pmatrix}$ or $\text{Cofactors}: \begin{pmatrix} 3 & 2 & 1 \\ a+8 & 2 & -a-4 \\ 4-a & 6 & -2-a \end{pmatrix}$		B1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{\lvert\mathbf{M}\rvert}\text{adj}(\mathbf{M})$		M1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{2a+10}\begin{pmatrix} 3 & a+8 & 4-a \\ 2 & 2 & 6 \\ 1 & -a-4 & -2-a \end{pmatrix}$	2 correct rows or columns. Follow through their det**M**	A1ft	1.1b
		All correct. Follow through their det**M**	A1ft	1.1b
			(4)	
(ii)	When $n = 1$, lhs $= \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$, rhs $= \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ So the statement is true for $n = 1$		B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$		M1	2.4
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$		M1	2.1
	$= \begin{pmatrix} 3\times 3^k & 0 \\ 3\times 3(3^k-1)+6 & 1 \end{pmatrix}$		A1	1.1b
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$		A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n		A1	2.4
			(6)	
				(12 marks)

Question	Scheme	Marks	AOs
7(a)	$\lvert\mathbf{M}\rvert = 2(-k-8)+1(-3-12)+1(6-3k)=0 \Rightarrow k=...$	M1	1.1b
	$k \neq -5$	A1	2.4
		(2)	
(b) Way 1	$\mathbf{M}=\begin{pmatrix}2 & -1 & 1\\ 3 & -6 & 4\\ 3 & 2 & -1\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{M}^{-1}\begin{pmatrix}p\\1\\0\end{pmatrix}$	M1	3.1a
	$\mathbf{M}^{-1}=\frac{1}{5}\begin{pmatrix}-2 & 1 & 2\\ 15 & -5 & -5\\ 24 & -7 & -9\end{pmatrix}$	B1	1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-2 & 1 & 2\\ 15 & -5 & -5\\ 24 & -7 & -9\end{pmatrix}\begin{pmatrix}p\\1\\0\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=...$	M1	2.1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{5}\begin{pmatrix}-2p+1\\15p-5\\24p-7\end{pmatrix}$	A1	1.1b
	$\left(\frac{-2p+1}{5},\ 3p-1, \frac{24p-7}{5}\right)$	A1ft	2.5
		(5)	
(b) Way 2	$\begin{aligned}2x-\ y+\ z&=p\\ 3x-6y+4z&=1\\ 3x+2y-\ z&=0\end{aligned} \Rightarrow \text{e.g.} \begin{aligned}8y-5z&=-1\\ 9y-5z&=3p-2\end{aligned} \Rightarrow y=...$ $\Rightarrow x=...,z=...$	M1	3.1a
	$y=3p-1$(or $x=\frac{-2p+1}{5}$ or $z=\frac{24p-7}{5}$)	B1	1.1b
	$8(3p-1)-5z=-1 \Rightarrow z=... \Rightarrow x=...$	M1	2.1
	$z=\frac{24p-7}{5}$, $x=\frac{-2p+1}{5}$	A1	1.1b
	$\left(\frac{-2p+1}{5},\ 3p-1, \frac{24p-7}{5}\right)$	A1ft	2.5

(c)(i)	For consistency: E.g. $5x+y=4-q$ and $15x+3y=q$	M1	3.1a
	$4-q=\frac{q}{3} \Rightarrow q=...$	M1	2.1
	$q=3$	A1	1.1b
	Alternative for (c)(i): $x=1 \Rightarrow 2-y+z=1,\ 3+2y-z=0 \Rightarrow y=...,z=...$ M1 for allocating a number to one variable and solves for the other 2 $x=1, y=-4, z=-5 \Rightarrow 3+20-20=q$ M1 substitutes into the second equation and solves for q A1: $q=3$		
(ii)	Three **planes** that intersect in a **line** Or Three **planes** that form a **sheaf** allow **sheath**!	B1	2.4
		(4)	
			(11 marks)

Notes

(a)
M1: Attempts determinant, equates to zero and attempts to solve for k in order to establish the restriction for k. For the determinant, at least 2 of the 3 "elements" should be correct.
May see rule of Sarrus used for determinant e.g.

$$|\mathbf{M}|=(2)(k)(-1)+(4)(3)(-1)+(3)(2)(1)-(3)(k)(-1)-(2)(4)(2)-(-1)(3)(-1)=0 \Rightarrow k=...$$

A1: Describes the correct condition for k with no contradictions. Allow e.g. $k < -5$, $k > -5$

(b)**Way 1**
M1: A complete strategy for solving the given equations. Need to see an attempt at the inverse followed by a correct method for finding x, y and z
B1: Correct inverse matrix
M1: Uses their inverse and attempts the multiplication with the correct vector
A1: Correct values for x, y and z in any form
A1ft: Correct values given in coordinate form only. **Follow through their *x*, *y* and *z*.**

Way 2
M1: A complete strategy for solving the given equations. Need to see an attempt at eliminating one variable followed by a correct method for finding x, y and z
B1: One correct value
M1: Uses the equations to find values for the other 2 variables
A1: Correct values for x, y and z in any form
A1ft: Correct values given in coordinate form only. **Follow through their *x*, *y* and *z*.**

(c)(i)
M1: Uses a correct strategy that will lead to establishing a value for q. E.g. eliminating one of x, y or z
M1: Solves a suitable equation to obtain a value for q
A1: Correct value

(ii)
B1: Describes the correct geometrical configuration.
Must include the **two** ideas of **planes** and meeting in a **line** or forming a **sheaf** with no contradictory statements.

Question	Scheme	Marks	AOs
6(a)	$\lvert\mathbf{M}\rvert = k(-1-1)-5(-1-2)+7(1-2)$ $\{=8-2k\}$	M1	1.1b
	Minors: $\begin{pmatrix} -2 & -3 & -1 \\ -12 & -k-14 & k-10 \\ -2 & k-7 & k-5 \end{pmatrix}$ Cofactors: $\begin{pmatrix} -2 & 3 & -1 \\ 12 & -k-14 & 10-k \\ -2 & 7-k & k-5 \end{pmatrix}$	M1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{8-2k}\begin{pmatrix} -2 & 12 & -2 \\ 3 & -k-14 & 7-k \\ -1 & 10-k & k-5 \end{pmatrix}$	M1 A1	2.1 1.1b
		(4)	
(b)	$\mathbf{M}^{-1} = \dfrac{1}{4}\begin{pmatrix} -2 & 12 & -2 \\ 3 & -16 & 5 \\ -1 & 8 & -3 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$ Solve the equations simultaneously to achieve values for x, y and z $y+3z=2p-2$ and $4y+8z=-1 \Rightarrow x=\ldots,\ y=\ldots,\ z=\ldots$	M1	3.1a
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}+3p-1 \\ \frac{3}{4}-4p+\frac{5}{2} \\ -\frac{1}{4}+2p-\frac{3}{2} \end{pmatrix}$	A1ft	1.1b
	$\left(\dfrac{12p-6}{4}, \dfrac{13-16p}{4}, \dfrac{8p-7}{4}\right)$ $\left(3p-\dfrac{3}{2}, \dfrac{13}{4}-4p, 2p-\dfrac{7}{4}\right)$	A1	2.2a
		(3)	
(c)(i)			
	For consistency: E.g. eliminates z to find two equations from $3x+2y=q+2$ $3x+2y=7q-1$ $18x+12y=15$ For consistency: E.g. eliminates x to find two equations from $y+3z=1-4q$ $3y+9z=-3$ $y+3z=2q-2$ For consistency: E.g. eliminates y to find two equations from $x-2z=2-q$ $-x+2z=1-5q$ $-6x+12z=-9$	M1	3.1a
	e.g. $q+2=7q-1 \Rightarrow q=\ldots$ e.g. $-3=3(1-4q) \Rightarrow q=\ldots$ e.g. $-9=6(1-5q) \Rightarrow q=\ldots$	M1	1.1b
	$q=\dfrac{1}{2}$	A1	1.1b

	Alternative Equating coefficients leading to two out of three equations and solves to find values for a and b $4a+b=2, 5a+b=1, 7a+b=-1$ $\{a=-1, b=6\}$	M1	3.1a
	Forms the fourth equation involving q $a+bq=2$ and substitutes in the values of a and b to finds a value for q	M1	1.1b
	$q=\frac{1}{2}$	A1	1.1b
	Finds a coordinate of intersection of the planes $4x+5y+7z=1$ and $2x+y-z=2$ e.g let $z=0 \Rightarrow 4x+5y=1$ and $2x+y=2 \Rightarrow y=-1, x=1.5$	M1	3.1a
	Substitutes the values for x, y and z into $x+y+z=q$ to reach a value for q	M1	1.1b
	$q=\frac{1}{2}$	A1	1.1b
(ii)	For example: $x=\lambda \Rightarrow 3\lambda+2y=\frac{5}{2},\ \lambda-2z=\frac{3}{2} \Rightarrow y=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$ $y=\lambda \Rightarrow 3x+2\lambda=\frac{5}{2},\ \lambda+3z=1 \Rightarrow x=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$ $z=\lambda \Rightarrow 3y+9\lambda=-3,\ -6x+12\lambda=-9 \Rightarrow x=\mathrm{f}(\lambda), y=\mathrm{f}(\lambda)$	M1	3.1a
	Let $x=\lambda,\ \lambda=\frac{y-\frac{5}{4}}{-\frac{3}{2}}=\frac{z+\frac{3}{4}}{\frac{1}{2}}$ or $y=\frac{5}{4}-\frac{3}{2}\lambda, z=-\frac{3}{4}+\frac{1}{2}\lambda$ Let $y=\lambda,\ \lambda=\frac{x-\frac{5}{6}}{-\frac{2}{3}}=\frac{z+\frac{1}{3}}{-\frac{1}{3}}$ or $x=\frac{5}{5}-\frac{2}{3}\lambda, z=-\frac{1}{3}-\frac{1}{3}\lambda$ Let $z=\lambda,\ \lambda=\frac{x-\frac{3}{2}}{2}=\frac{y+1}{-3}$ or $x=\frac{3}{2}+2\lambda, y=-1-3\lambda$	A1	1.1b
	$\mathbf{r}=\frac{5}{4}\mathbf{j}-\frac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\frac{5}{6}\mathbf{i}-\frac{1}{3}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\frac{3}{2}\mathbf{i}-\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	M1 A1	1.1b 2.5
	Alternative (ii) Finds two different coordinates that lie on the line of intersection For example: setting $x=0 \Rightarrow \left(0, \frac{5}{4}, -\frac{3}{4}\right)$ setting $y=0 \Rightarrow \left(\frac{5}{6}, 0, -\frac{1}{3}\right)$ setting $z=0 \Rightarrow \left(\frac{3}{2}, -1, 0\right)$	M1 A1	3.1a 1.1b
	Finds the vector equation of the line passing through their two points	M1	1.1b
	$\mathbf{r}=\frac{5}{4}\mathbf{j}-\frac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	A1	2.5

	$\mathbf{r}=\frac{5}{6}\mathbf{i}-\frac{1}{3}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\frac{3}{2}\mathbf{i}-\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.		
	Alternative (ii): Outside the spec Finds the cross product of two normal vectors and a coordinate that lies on all three planes	M1	3.1a
	Correct cross product $\begin{vmatrix}4&5&7\\1&1&1\end{vmatrix}=\begin{vmatrix}4&5&7\\2&1&-1\end{vmatrix}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$ or $\begin{vmatrix}1&1&1\\2&1&-1\end{vmatrix}=\begin{pmatrix}12\\18\\-3\end{pmatrix}$	A1	1.1b
	Uses the point and the direction vector to find the equation of the line	M1	1.1b
	$\mathbf{r}=\frac{5}{4}\mathbf{j}-\frac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	A1	1.1b
		(7)	
			(14 marks)

Notes

(a)
M1: Correct method to find the determinant. Condone one sign slip
M1: A correct first step in obtaining the inverse. Could be the matrix of minors or cofactors. Condone sign slips as long as the intention is clear.
M1: Fully correct method to obtain the inverse. Attempts matrix of minors, cofactors, transposes and 1/determinant
A1: Correct matrix
(b)
M1: A complete strategy for solving the given equations e.g. multiplies the given coordinates by their inverse or solves simultaneously to achieve values for x, y and z
A1ft: Correct calculation on their inverse matrix (unsimplified) or at least on correct value if solving simultaneously
A1: Correct coordinates
(c)(i)
M1: Uses a correct strategy that will lead to establishing a value for q. E.g. eliminating one of x, y or z
M1: Solves a suitable equation to obtain a value for q
A1: Correct value
(c)(i) **Alternative 1**
M1: Equating coefficients leading to two out of three equations and solves to find values for a and b
M1: Solves a suitable equation to obtain a value for q using their values for a and b
(c)(i) **Alternative 2**
M1: Finds a coordinate of intersection of the planes $4x+5y+7z=1$ and $2x+y-z=2$
M1: Substitutes the values for x, y and z into $x+y+z=q$ to reach a value for q
A1: Correct value
(ii)
M1: Uses a correct strategy to obtain the Cartesian equation of the line or the general coordinates
A1: Correct Cartesian equation or coordinates in terms of a parameter.

Question	Scheme	Marks	AOs
4(i) (a)	It is possible as the number of columns of matrix **A** matches the number of rows of matrix **B**.	B1	2.4
(b)	It is not possible as matrix **A** and matrix **B** have different dimensions o.e. different number of columns	B1	2.4
		(2)	
(ii) (a)	$\lambda = 5$	B1	2.2a
	$a = 1,\ b = 2$	B1	2.2a
(b)	Inverse matrix $= \frac{1}{5}\begin{pmatrix} 0 & 5 & 0 \\ 2 & 12 & -1 \\ -1 & -11 & 3 \end{pmatrix}$	B1 ft	3.1a
		(3)	
(iii)	A complete method to find the determinant of the matrix and set equal to zero.	M1	1.1b
	Determinant = $1(\sin\theta\sin 2\theta - \cos\theta\cos 2\theta) - 1(0) + 1(0) = 0$	A1	1.1b
	Uses compound angle formula to achieve $\cos 3\theta = 0$ leading to $\theta = ...$ or use of $\sin 2q = 2\sin q\cos q$ and $\cos 2q = 1 - 2\sin^2 q$ (e.g. to achieve $\cos q(4\sin^2 q - 1) = 0$) leading to $\theta = ...$ or use of $\sin 2q = 2\sin q\cos q$ and $\cos 2q = 2\cos^2 q - 1$ (e.g. to achieve $4\cos^3 q - 3\cos q = 0$) leading to $\theta = ...$	M1	3.1a
	$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$	A1	1.1b
		(4)	
		(9 marks)	

Notes:

(i)(a)

B1: Comments that the number of columns of matrix **A** (2) equals the number of rows of matrix **B** (2) therefore it is possible. Accept other terminology that is clear in intent e.g. "length of **A**" and "height of **B**"

(b)

B1: Comments that matrix **A** and matrix **B** have different dimensions therefore it is not possible.

(ii)(a)

B1: Deduces the correct value for $\lambda = 5$

B1: Deduces the correct values for a and b

(b)

B1ft: Identifies and applies a correct method find the inverse matrix. May multiply from the given equation, in which case follow through on their value of lambda. Alternatively, award for a correct matrix found by calculator or long hand having found a and b and using these values in the matrix.

Question	Scheme	Marks	AOs
5(a)	$det(\boldsymbol{M}) = a(6) - 2(4) - 3(2a - 12)$	M1	1.1b
	$det(\boldsymbol{M}) = 28 \neq 0$ therefore, non-singular for all values of a	A1	2.4
		(2)	
(b)	Finds the matrix of minors $\begin{pmatrix} 6 & 4 & 2a-12 \\ 4+3a & 2a+12 & a^2-8 \\ 9 & 6 & 3a-4 \end{pmatrix}$	M1	1.1b
	Finds the matrix of cofactors and transposes. $\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$	M1	1.1b
	$\frac{1}{28}\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$	M1 A1	1.1b 2.1
		(4)	
		(6 marks)	

Notes:

(a)

M1: Finds the determinant of the matrix **M**. Must be seen in part (a). Allow one slip if no method shown.

A1: Correct value for determinant, states doesn't equal 0 (accept > 0) and draws the conclusion that the matrix is non-singular. If non-singular meaning determinant is non-zero is given in a preamble then accept a minimal conclusion (e.g. "hence shown"), but there must be a conclusion.

(b)

M1: Finds the matrix of minors, at least 5 correct values.

M1: Finds the matrix of cofactors and transposes (in either order). Note: some will do all these steps in one go, which is fine as long as it is clear what they have done. Allow minor slips if the process is clearly correct.

M1: Completes the process to find the inverse matrix, divides by the determinant.

A1: Correct matrix.

Question	Scheme	Marks	AOs
2(a)(i)	x / C = number of **Construction** students y / D = number of **Design** students z / H = number of **Hospitality** students	B1	3.3
(ii)	The increase in number of students in 2020 $1110 \times 0.0027\{= 2.997 \approx 3\}$ Or The number of students in 2020 $1110 \times 1.0027 = \{1112.997 \approx 1113\}$	M1	1.1b
	$x + y + z = 1110$ $\quad$ $C + D + H = 1110$ $x - z = 370$ o.e. $\quad$ $C - H = 370$ o.e. $0.0125C + 0.025D - 0.02H = 3$or 2.997 o.e $1.0125C + 1.025D + 0.98H = 1113$ or 1112.997o.e. $0.0125x + 0.025y - 0.02z = 3$ or 2.997o.e$1.0125x + 1.025y + 0.98z = 1113$ or 1112.997 o.e.	M1 A1	3.3 1.1b
		(4)	
(b)	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix}$	M1 A1ft	1.1b 1.1b
	$\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}^{-1}\begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$ or $\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}^{-1}\begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$	dM1	1.1b
	So in 2019, **720** students studied **C**onstruction, **40** students studied **D**esign and **350** students studied **H**ospitality	A1	3.2a
		(4)	
			(8 marks)

Notes:

Mark (i) and (ii) together

(a)(i)

B1: Defines 3 variables, minimum e.g. construction = C, Design = D, Hospitality = H. This may be seen in text of the question, abbreviations may be used

(ii)

Question	Scheme	Marks	AOs
4	$\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}^n = \begin{pmatrix}1 & -2n\\ 0 & 1\end{pmatrix}$		
	$\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}^1 = \begin{pmatrix}1 & -2\times 1\\ 0 & 1\end{pmatrix}$ (so true when $n = 1$)	B1	2.2a
	(<u>Assume true for $n = k$</u>, then) $\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}^{k+1} = \begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\ 0 & 1\end{pmatrix}$ or $\begin{pmatrix}1 & -2k\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}$	M1	2.1
	$= \begin{pmatrix}1 & -2k-2\\ 0 & 1\end{pmatrix}$ or $= \begin{pmatrix}1 & -2-2k\\ 0 & 1\end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix}1 & -2(k+1)\\ 0 & 1\end{pmatrix}$	A1	2.2a
	Hence result is true for $n = k + 1$. As <u>true for $n = 1$</u> and have shown <u>if true for $n = k$ then it is true for $n = k + 1$</u>, so it is <u>true for all n</u>.	A1	2.5
		(5)	

(5 marks)

Notes:

B1: Shows true for $n = 1$.

Need to see $n = 1$ **substituted** into rhs. The minimum for B1 would be $\begin{pmatrix}1 & -2\times 1\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}$.

There is no need to state "True for $n = 1$"

M1: (Assumes for $n = k$ and) multiplies original matrix by kth power matrix either way round. Note that the assumption statement is not needed for this mark (but see below) so just look for:

$$\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\ 0 & 1\end{pmatrix} \text{ or } \begin{pmatrix}1 & -2k\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}$$

A1: Achieves a correct unsimplified matrix.

A1: Reaches correct form for the matrix with the $k + 1$ factored out with no errors and the correct unsimplified matrix seen previously. Note that the result may be proved by equivalence (see below).

A1: Correct conclusion **with assumption made** (which may be implied in their conclusion if they say "if true for $n = k$ then…"). This mark is dependent on all the previous marks apart from the B mark and is gained by conveying all the underlined points.

Allow this mark to score as long as all the underlined points are seen as narrative in their solution.

There must be the assumption statement somewhere <u>or</u> the "if…then…" idea in the conclusion. If awarded for the assumption statement condone e.g. true for $n = k$ in the conclusion.

The conclusion must convey the "if true for $n = k$ then true for $n = k + 1$" idea and not e.g. true for k, true for $k + 1$, true for 1 therefore…but see the previous note.

For the "true for all n" part condone e.g. "true for n", "true for all integers after 1", "true for □$^{+}$",
But do **not** allow "true for all values", "true for all real numbers"

Question	Scheme	Marks	AOs
8(a)	Accept E.g. • 1 month is too old for "newborn" • The mammals might not start breeding at exactly 3 months old • The mammals will stop breeding beyond a certain age • Being over 3 months old doesn't necessarily mean the mammal can breed • Some mammals over 3 months may be infertile so will not be breeders • Some juveniles might be breeders But not • The size of the categories is different • There might be overlap • The exact age of mammals might not be known • The numbers in each category will be different • Breeding age is different for different species	**B1**	3.5b
		(1)	

(a)
B1: Any valid limitation – see scheme for some examples. Must refer to a feature of the categories given.

(b)(i)	$\begin{pmatrix}0 & 0 & 2\\ a & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}^2\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}0 & 0 & 2\\ a & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}\begin{pmatrix}2k\\0\\0.96k\end{pmatrix}$ or $\begin{pmatrix}0 & 0.96 & 1.92\\ ab & b^2 & 2a\\ 0.48a & 0.48b+0.96\times0.48 & 0.96^2\end{pmatrix}\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}1.92k\\2ak\\0.9216k\end{pmatrix}$	M1	3.4
	$48=2\times0.96k\Rightarrow k=...$	dM1	1.1b
	$k=25$ so 25 mammals at the start of the study	A1	3.2a
(ii)	$40=2ka\Rightarrow a=0.8$ *	A1*	1.1b
		(4)	

(b)(i)

M1: Attempts to use the given information to set up a matrix equation and find the numbers of mammals after one month e.g.

$$\begin{pmatrix}0 & 0 & 2\\ a & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=\begin{pmatrix}2B_0\\aN_0+bJ_0\\0.48J_0+0.96B_0\end{pmatrix}$$

or attempts to square the matrix to find the number of mammals after two months e.g.

$$\begin{pmatrix}0 & 0.96 & 1.92\\ ab & b^2 & 2a\\ 0.48a & 0.48b+0.96\times0.48 & 0.96^2\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=...$$

dM1: Forms an equation, in their variable for number of breeders at the start, setting their number of newborns after 2 months equal to 48 and solves for their variable to find the initial number of breeders.

A1: For identifying 25 mammals at the start of the study. Allow 25 mammals or just 25 or e.g. $B_0=25$ so ignore how they label it just look for 25

Note that in some cases work may be minimal e.g.

$$\begin{pmatrix}0 & 0 & 2\\ a & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}\begin{pmatrix}0\\0\\B_0\end{pmatrix}=\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}\Rightarrow 0.96B_0=B_1,\quad \begin{pmatrix}0 & 0 & 2\\ a & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow 2B_1=48$$

$$B_1=24=0.96B_0\Rightarrow B_0=25$$

(ii)

A1*: For correctly showing $a = 0.8$. Must see the correct work to establish the correct value or equivalent by verification with a minimal conclusion e.g.

$$\begin{pmatrix}0 & 0 & 2\\ 0.8 & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix}\begin{pmatrix}0\\0\\25\end{pmatrix}=\begin{pmatrix}50\\0\\24\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow 0.8\times50=40 \text{ Hence true *}$$

(c)	$\det\begin{pmatrix}0 & 0 & 2\\ 0.8 & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix} = 0-0+2(0.48\times 0.8-0)=0.768$	**B1**	2.2a
	$\text{adj}\begin{pmatrix}0 & 0 & 2\\ 0.8 & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix} = \begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$	**M1**	1.1b
	$=\begin{pmatrix}1.25b & 1.25 & -\frac{125}{48}b\\ -1 & 0 & \frac{25}{12}\\ 0.5 & 0 & 0\end{pmatrix}$ oe e.g. $\frac{1}{0.768}\begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$ or e.g. $\frac{125}{96}\begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$	**A1**	1.1b
		(3)	

(c)

B1: Deduces correct determinant for the matrix. Allow equivalents e.g. $\frac{96}{125}$ May be implied.

M1: Recognisable attempt at the adjoint matrix. Look for at least 3 non-zero entries correct.

A1: Correct inverse. Accept awrt $-2.6b$ for the upper right entry and awrt 2.08 for middle right entry, or accept with determinant still outside. Apply isw once a correct answer is seen.

(d)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ $\text{Total} = 1.25b\times 596 + 1.25\times 464 - \frac{125}{48}b\times 437 - 596 + \frac{25}{12}\times 437 + 0.5\times 596$	**M1**	3.1b
	$\Rightarrow 1015 = x + y + z = 745b + 580 - 1138b - 596 + 910.4 + 298 \Rightarrow b = ...$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	
	(d) Alternative:		
	$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix} \Rightarrow \begin{matrix} 2z = 596 \\ 0.8x + by = 464 \\ 0.48y + 0.96z = 437 \end{matrix}$ $\Rightarrow z = 298, y = \frac{3773}{12}(314.4...)$ $x + y + z = 1015 \Rightarrow x = ...\frac{4831}{12}(402.5...)$	**M1**	3.1b
	$0.8x + by = 464 \Rightarrow 0.8\times\frac{4831}{12} + b\times\frac{3773}{12} = 464 \Rightarrow b = ...$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	

(d)

M1: Attempts (their inverse matrix) $\times \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ correctly and adds the 3 expressions together to find the total in terms of b.

M1: Sets their total = 1015 and solves for b.

A1: awrt 0.45

Alternative:

M1: Uses the original matrix with $a = 0.8$ and $\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ to form 3 equations in their variables and b and uses these and the 1015 to find the number of Newborns.

M1: Uses their values in the y component and solves for b.

A1: awrt 0.45

(e)	Let NM_n be newborn males and NF_n be newborn females in month n $$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$$ or e.g. $$\begin{pmatrix} NF_{n+1} \\ NM_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1.16 \\ 0 & 0 & 0 & 0.84 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NF_n \\ NM_n \\ J_n \\ B_n \end{pmatrix}$$	**M1** **A1ft**	3.5c 3.3
		(2)	
			(13 marks)

Notes:

(e)

M1: Defines new variables for male and female newborns (accept if a clear notation is used if not defined) and sets up a 4×4 matrix with structure shown, or male and female rows swapped, with the correct 0 entries in at least 4 places.

A1ft: Fully correct matrix system shown, accepting anything (including 0) for the unknown spaces shown – but must have all the 0's **and** upper right entries correct. Accept b or their value of b in place of 0.45

Question	Scheme	Marks	AOs
1(a)	$\mathbf{M}^{-1} = \frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)	$\frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}\begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix} = \ldots$	M1	1.1b
	$x = 2, y = 1, z = 3$ or $(2,1,3)$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$	A1	1.1b
		(2)	
(c)	The **point** where three **planes** meet	B1ft	2.2a
		(1)	
			(5 marks)

Notes

(a)
B1: Evidence that the determinant is ± 69 (may be implied by their matrix e.g. where entries are not in exact form: $\pm\begin{pmatrix} 0.014 & 0.188 & 0.072 \\ -0.159 & -0.072 & 0.203 \\ -0.377 & 0.101 & 0.116 \end{pmatrix}$)(Should be mostly correct)
Must be seen in part (a).
B1: Fully correct inverse with all elements in **<u>exact</u>** form
(b)
M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme)
A1: Correct coordinates
A solution not using the inverse requires a complete method to find values for x, y and z for the method mark.
Correct coordinates only scores both marks.
(c)
B1: Describes the correct geometrical configuration.
Must include the two ideas of **planes** and **meet in a point** with no contradictory statements.
This is dependent on having obtained a unique point in part (b)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>10. (a)</td><td>a represents the proportion of juvenile chimpanzees that (survive and) remain juvenile chimpanzees the next year.</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)(i)</td><td>Determinant = $0.82a - 0.08 \times 0.15$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \ldots \begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \dfrac{1}{0.82a - 0.012} \begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(ii)</td><td>$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} \begin{pmatrix} 15360 \\ 43008 \end{pmatrix} = \dfrac{1}{0.82a - 0.012} \begin{pmatrix} 0.82 \times 15360 - 0.15 \times 43008 \\ (-0.08) \times 15360 + 43008a \end{pmatrix}$
OR forms equations $\begin{matrix} 15360 = aJ_0 + 0.15 \times A_0 \\ 43008 = 0.08 \times J_0 + 0.82 \times A_0 \end{matrix}$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$\dfrac{1}{0.82a - 0.012}\left[6144 + \left(43008a - 1228.8\right)\right] = 64000$
$\Rightarrow 4915.2 + 43008a = 64000(0.82a - 0.012) \Rightarrow a = \ldots$
OR
$A_0 = 64000 - J_0 \Rightarrow 43008 = 0.08 \times J_0 + 0.82 \times \left(64000 - J_0\right) = J_0 = \ldots$
$\Rightarrow a = \dfrac{15360 - \left(64000 - J_0\right)}{J_0} = \ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$a = \dfrac{5683.2}{9472} = 0.60$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(iii)</td><td>Initial juvenile population = $\dfrac{"6144"}{"0.48"} = 12800$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>So change of 2560 juvenile chimpanzees</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>As the number of juveniles has increased, the model is not initially predicting a decline, so is not suitable in the short term.
(Follow through their answer to (b) – but they must have made an attempt at it to find at least a value for J_0)</td><td>B1ft</td><td>3.5a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>Third category needs to be introduced for chimpanzees aged 40 and above, mature chimpanzees M_n, and a matrix multiplication of increased dimension set up. Accept $3 \times 3, 3 \times 2$ or 2×3 matrices including all three categories in the column vector.</td><td>M1</td><td>3.5c</td></tr>
<tr><td></td><td>The corresponding matrix model will have the form
$\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & \underline{\mathbf{0}} \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix} \begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$
(The underlined zero must be correct but do not be concerned about any values used in the other entries.)</td><td>A1</td><td>3.3</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td></td><td colspan="3" align="right">(12 marks)</td></tr>
</table>

Question	Scheme	Marks	AOs
1(a)	$\begin{vmatrix} k & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -k & -k \end{vmatrix} = k(k+k) - 3(-3k+16) - 1(-3k-16)$	M1	2.1
	Solves det = 0 $\Rightarrow 2k^2 + 12k - 32 = 0$ or $k^2 + 6k - 16 = 0$ To achieve $k = 2$ ($k = -8$ must be rejected)	A1	1.1b
		(2)	
	Special case $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -2 & -2 \end{vmatrix} = 2(2+2) - 3(-3\times2+16) - 1(-3\times2-16)$ Shows det = 0, therefore when $k = 2$ there is no unique solution	M1 A0	2.1 1.1b
(b)	Eliminates z to achieve **two** equations in x and y e.g. $5x+2y=1$, $-10x-4y=-2$, $20x+8y=4$ Eliminates x to achieve **two** equations in y and z e.g. $11y-5z=13$, $22y-10z=26$, $-22y-10z=-26$ Eliminates y to achieve **two** equations in x and z e.g. $11x+2z=-3$, $22x+4z=-6$, $-44x-8z=12$	M1 A1	3.1a 1.1b
	Must give a **reason:** e.g. Two equations are a linear multiple of each other e.g. shows they are the same equation therefore the equations are **consistent**.	A1	2.4
	Alternative Eliminates two different variables to form two equations, should be one equation from two of the three sections in the main scheme. e.g $5x+2y=1$ and $11y-5z=13$ rearranges and substitutes in to one of the original equations in three variables. e.g. $2x + 3\left(\frac{1-5x}{2}\right) - \left(\frac{-3-11x}{2}\right) = 3$	M1	3.1a
	Correct equations e.g $5x+2y=1$ and $11y-5z=13$	A1	1.1b
	Shows that the equations are a solution e.g. 3 = 3 therefore **consistent**	A1	2.4
(c)	The three **planes** form a **sheaf.**	B1	2.2a
		(1)	
		(6 marks)	

Question	Scheme	Marks	AOs
4(a)	$$\mathbf{MN}=\begin{pmatrix}2k-24 & 0 & 0\\ k^2-7k+10 & 6k-44 & -10k+50\\ 4k-20 & 0 & -14\end{pmatrix}$$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	$$\mathbf{MN}=\begin{pmatrix}-14 & 0 & 0\\ 0 & -14 & 0\\ 0 & 0 & -14\end{pmatrix}$$	B1ft	1.1b
(ii)	$$\mathbf{M}^{-1}=-\frac{1}{14}\begin{pmatrix}-2 & 6 & -10\\ 2 & -20 & 24\\ -3 & 2 & -1\end{pmatrix}$$	B1	1.1b
		(2)	
(c)	$$\mathbf{M}^{-1}=-\frac{1}{14}\begin{pmatrix}-2 & 6 & -10\\ 2 & -20 & 24\\ -3 & 2 & -1\end{pmatrix}\begin{pmatrix}2\\ 3\\ -1\end{pmatrix}=\ldots$$	M1	1.1b
	$$\left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$$	A1	1.1b
		(2)	
(d)	The coordinates of the only point at which the **planes** represented by the equations in (c) meet.	B1	2.2a
		(1)	
			(7 marks)

Notes

(a)
B1: For 2 correct rows or 2 correct columns (allow unsimplified)
B1: Fully correct simplified matrix
(b)(i)
B1ft: Correct matrix (follow through from part (a)). If an error with part (a) allow the correct matrix stated, restart use of calculator.
(ii)
B1: Deduces the correct inverse matrix, may use calculator
(c)
M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme)
Allow use of a calculator
A1: Correct exact coordinates (allow as a vector or $x = \ldots, y = \ldots, z = \ldots$)
(d)
B1: Describes the correct geometrical configuration of the **planes**

Question	Scheme	Marks	AOs
1(a)	(i) $\mathbf{AB}=\begin{pmatrix}4&-1\\7&2\\-5&8\end{pmatrix}\begin{pmatrix}2&3&2\\-1&6&5\end{pmatrix}=\begin{pmatrix}8+1&12-6&8-5\\14-2&21+12&14+10\\-10-8&-15+48&-10+40\end{pmatrix}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}$	M1	1.1b
	So $\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}-\begin{pmatrix}-15&6&3\\12&9&24\\-18&33&6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$ or $\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}+\begin{pmatrix}15&-6&-3\\-12&-9&-24\\18&-33&-6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$ and states a value for k	M1	1.1b
	Hence $\mathbf{AB}-3\mathbf{C}-24\mathbf{I}=\mathbf{0}$ so $k=-24$	A1	1.1b
	(ii) Need two things One of: • **BA** is a 2×2 matrix • Finds the matrix **BA** (must be a 2×2 matrix) AND One of: • cannot subtract a 3×3 matrix • finds matrix **3C** and comments that they have different dimensions / can't be done • can't subtract matrices of different sizes • 3**C** or **C** is a 3×3 matrix • **BA** needs to be a 3×3 matrix	B1	2.4
		(4)	
(b)(i)	$\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-14\\3\\7\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$ Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{C}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$ Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}$	M1	1.2
(ii)	$=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}=\ldots$	M1	1.1b

	$= \begin{pmatrix} -\frac{41}{180} & \frac{7}{360} & \frac{13}{360} \\ -\frac{7}{45} & -\frac{1}{90} & \frac{11}{90} \\ \frac{31}{180} & \frac{43}{360} & -\frac{23}{360} \end{pmatrix} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$ $\mathbf{C}^{-1} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$		
	So solution is $x = \frac{7}{2}, y = 3, z = -\frac{5}{2}$ or $(3.5,\ 3,\ -2.5)$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) (i)

M1: Attempts to find **AB**. Usually this will be done on calculator so answer implies the method. If answer is incorrect allow for at least 6 correct entries or calculations shown. This mark can be implied by a correct matrix for $\mathbf{AB} - 3\mathbf{C}$ gives the first M1

M1: Uses their **AB** and **3C** matrices to find a multiple **I** and states a value for k

A1: Correct proof with $k = -24$ seen explicitly (may be in equation).

Minimum working required is $\mathbf{AB} - 3\mathbf{C} = \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}$ gets M1 then states a value for k M1 then $k = -24$ gets A1

Special case: If minimum working required is not seen and just $k = -24$ stated then M1 M0 A0 as they have not shown that the value of k works.

(ii)

B1: Correct explanation referring to the dimensions of **BA** and **C** (or 3**C**) and that they do not match in the equation. They can find both these matrices and then comment they cannot be subtracted.

(b) Mark (i) and (ii) altogether

M1: States or implies use of the correct method of using the inverse matrix.

M1: Carries out the process of multiplying after finding the inverse. May find inverse long hand first. Finding the inverse matrix then writes down an answer gains M1.

Note: There is no need to find the inverse matrix. If the inverse matrix is not stated just answers written down then two out of the three correct ordinates imply the M1.

A1: Correct solution. Must be clear that $x = \frac{7}{2}, y = 3, z = -\frac{5}{2}$ allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3.5 \\ 3 \\ -2.5 \end{pmatrix}$

Note: If they solve using simultaneous equations only this is M0 M0 A0

If there is no reference to the inverse matrix and correct answers stated this is M0 M0 A0

Question	Scheme	Marks	AOs
1	$y = 3$	B1	2.2a
	$z = \frac{\text{their } y}{3} = \ldots\{1\}$	B1ft	1.1b
	Uses $z - 3y = k \Rightarrow k = -8$ and $x - 3z = k \Rightarrow x = k + 3z =$ their $k + 3 \times$ their z leading to a value for x **Alternatively** uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x.	M1	3.1a
	$x = -5$	A1	1.1b
		(4)	
			(4 marks)

Notes:

B1: $y = 3$

B1ft: Follow through on the value of z which comes from their y divided by 3

M1: A complete method to find the value of x. Uses $z - 3y = k$ to find a value for k then finds a value for x using $x - 3z = k$ and their values for z and k. Condone a slip with the coefficients if the intention is clear but must have the correct letters.

Alternatively uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x.

A1: $x = -5$

Correct answers only scores full marks.