11. (a) Use binomial expansions to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$

(6)

A student substitutes $x = \dfrac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \dfrac{1}{2}$

(1)

(c) Substitute $x = \dfrac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

4. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

Leave blank

6. Given that the binomial expansion, in ascending powers of x, of

$$\frac{6}{\sqrt{(9 + Ax^2)}}, \qquad |x| < \frac{3}{\sqrt{|A|}}$$

is $$B - \frac{2}{3}x^2 + Cx^4 + \ldots$$

(a) find the values of the constants A, B and C.

(7)

(b) Hence find the coefficient of x^6

(2)

Leave blank

5. $$f(x) = (8 + 27x^3)^{\frac{1}{3}}, \quad |x| < \frac{2}{3}$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(5)

Leave blank

5. (a) Use the binomial expansion, in ascending powers of x, of $\frac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2 + 5x + 6x^2, \qquad |x| < 0.5$$

(4)

(b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

Leave blank

2. Given that

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2} \equiv \frac{A}{(1-2x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}$$

(a) find the values of the constants A and C and show that $B = 0$

(4)

(b) Hence, or otherwise, find the series expansion of

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2}, \qquad |x| < \frac{1}{2}$$

in ascending powers of x, up to and including the term in x^2, simplifying each term.

(5)

Leave blank

1. $$\mathrm{f}(x) = (3 - 2x)^{-4}, \qquad |x| < \frac{3}{2}$$

Find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2, giving each coefficient as a simplified fraction.

(4)

Leave blank

3. (a) Find the binomial expansion of

$$(1 + ax)^{-3}, \quad |ax| < 1$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient as simply as possible in terms of the constant a.

(3)

$$f(x) = \frac{2 + 3x}{(1 + ax)^3}, \quad |ax| < 1$$

In the series expansion of $f(x)$, the coefficient of x^2 is 3

Given that $a < 0$

(b) find the value of the constant a,

(4)

(c) find the coefficient of x^3 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

Leave blank

3. (a) Express $\dfrac{9+11x}{(1-x)(3+2x)}$ in partial fractions.

(3)

(b) Hence, or otherwise, find the series expansion of

$$\frac{9+11x}{(1-x)(3+2x)}, \quad |x| < 1$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

Leave blank

4. $$f(x) = \frac{27}{(3 - 5x)^2} \qquad |x| < \frac{3}{5}$$

(a) Find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(5)

Use your answer to part (a) to find the series expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{27}{(3 + 5x)^2} \qquad |x| < \frac{3}{5}$

(1)

(c) $h(x) = \frac{27}{(3 - x)^2} \qquad |x| < 3$

(2)

Leave blank

7. (a) Use the binomial series to expand

$$\frac{1}{(2-3x)^3} \qquad |x| < \frac{2}{3}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

$$\mathrm{f}(x) = \frac{4+kx}{(2-3x)^3} \qquad \text{where } k \text{ is a constant and } |x| < \frac{2}{3}$$

Given that the series expansion of $\mathrm{f}(x)$, in ascending powers of x, is

$$\frac{1}{2} + Ax + \frac{81}{16}x^2 + \cdots$$

where A is a constant,

(b) find the value of k,

(2)

(c) find the value of A.

(2)