Question	Scheme	Marks	AOs
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	……….and factorise $8k^3 \pm 12k^2 \pm 4k = 4k\left(2k^2 \pm 3k \pm 1\right) =$	dM1	1.1b
	Correct work with statement $4\times..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)

Note: May be in any variable (condone use of n)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = ...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the k term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or 4k from their cubic

A1: Correct work with statement $4\times..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Question	Scheme	Marks	AOs
2(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ **...**

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0, (x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

....

$x^2-8x+17$

$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$ scores M1 A1 A1

Hence $(x-4)^2+1>0$

....

$x^2-8x+17>0$

$(x-4)^2+1>0$ scores M1 A1 A1

This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

....

$x^2-8x+17>0$

$(x-4)^2+1>0$ scores M1 A1 A0

which is true because a squared number is positive incorrect and incomplete

....

$x^2-8x+17=(x-4)^2+1$ scores M1 A1 A0

Minimum is (4,1) so $x^2-8x+17>0$ correct but not explained

....

$x^2-8x+17=(x-4)^2+1$ scores M1 A1 A1

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$ correct and explained

....

Question 15

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions. So $n=5 \Rightarrow n^3+2=127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8}=\frac{1}{8}n^3+\frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8}=\frac{1}{8}n^3+\frac{3}{8}n^2+\frac{3}{8}n+\frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example

"odd number + 2 is still odd and odd numbers are not divisible by 8"

"n^3+2 is odd and cannot be divided by 8 **exactly"**

and for even numbers accept

"a multiple of 8 add 2 is not a multiple of 8, so n^3+2 is not divisible by 8"

"if n^3 is a multiple of 8 then n^3+2 cannot be divisible by 8

Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
15 **Algebraic approach**	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=\underline{\underline{8k^3+12k^2+6k+3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3+12k^2+6k+3}}$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes

Correct expressions are required for the M's. There is no need to state "**If *n* is even**," $n=2k$ and **"If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

:

Question	Scheme	Marks	AOs
13 (a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2 + b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\quad \frac{4a^2}{ab} + \frac{b^2}{ab} \geqslant \frac{4ab}{ab}$	M1	2.2a
	Hence $\quad \frac{4a}{b} + \frac{b}{a} \geqslant 4 \quad$ * $\quad$ CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \frac{4a}{b} + \frac{b}{a} = -20 - \frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2 + b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab} + \frac{b^2}{ab} \geqslant \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Question	Scheme	Marks	AOs
11 (a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes:

(a)

M1: Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging

A1*: Need all three stages making the correct deduction to achieve the printed result

(b)

B1: Chooses two negative values and substitutes, then states conclusion

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2\left(=3\times 3k^2\right)$ is a multiple of 3	A1 **M1 on EPEN**	1.1b
	$(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)