



Oxford Cambridge and RSA

## A Level Mathematics B (MEI)

**H640/03** Pure Mathematics and Comprehension  
Insert

**Friday 15 June 2018 – Afternoon**

**Time allowed: 2 hours**



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## Arithmetic and Geometric Means

### Arithmetic and geometric mean of two numbers

For two real numbers  $a$  and  $b$ , the arithmetic mean of the numbers is defined to be  $\frac{a+b}{2}$ . For two non-negative real numbers  $a$  and  $b$ , the geometric mean of the two numbers is defined to be  $\sqrt{ab}$ .

Squares of real numbers cannot be negative, so we know that  $(a-b)^2 \geq 0$ . It follows that  $a^2 + b^2 \geq 2ab$  and so  $(a+b)^2 \geq 4ab$ . Hence the arithmetic mean of two real non-negative numbers is greater than, or equal to, their geometric mean. 5

$$\frac{a+b}{2} \geq \sqrt{ab} \text{ for } a, b \geq 0$$

This result is known as the inequality of the arithmetic and geometric means. If the two numbers  $a$  and  $b$  are equal then the arithmetic mean equals the geometric mean. 10

The three real numbers  $a, \frac{a+b}{2}, b$  form an arithmetic sequence. The three non-negative real numbers  $a, \sqrt{ab}, b$  form a geometric sequence.

### Constructing the arithmetic and geometric mean of two numbers

Lengths representing the arithmetic and geometric mean of two positive numbers can be constructed with a straight edge and compasses. 15

Fig. C1.1 shows a straight line ACB with AC of length  $a$  and CB of length  $b$ .

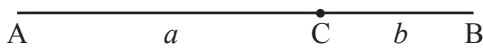


Fig. C1.1

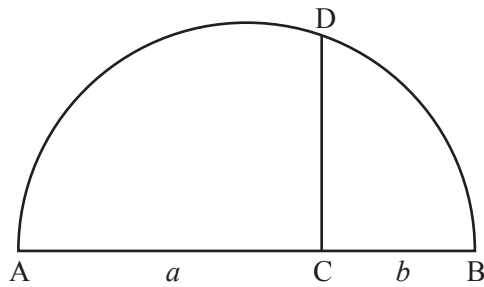


Fig. C1.2

The line AB is first bisected, to locate its midpoint. A semicircle with AB as diameter is then drawn, and a line at C perpendicular to the diameter is constructed. Fig. C1.2 shows this semicircle, with the perpendicular line through C meeting the semicircle at D.

The radius of the semicircle is the arithmetic mean of  $a$  and  $b$ , and the length of CD is the geometric mean of  $a$  and  $b$ . 20

To prove that the length of CD is the geometric mean of  $a$  and  $b$ , consider triangles ACD and BCD, as shown in Fig. C1.3. Letting angle CBD =  $\theta$ , it follows that angle CDA is also  $\theta$ . Finding expressions for  $\tan \theta$  in each of triangles ACD and BCD leads to  $h = \sqrt{ab}$ , where  $h$  is the length of CD.

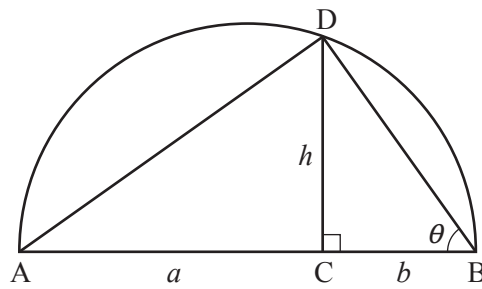


Fig. C1.3

The relationship between  $a$ ,  $b$  and  $h$  in Fig. C1.3 means that a square with side  $CD$  has the same area as a rectangle with sides equal to  $AC$  and  $CB$ . Fig. C2 shows the square and a rectangle  $ACFG$  with  $CF$  equal in length to  $CB$ . This diagram illustrates how a straight edge and compasses can be used to construct a square with area equal to that of a given rectangle. This method appears in Euclid’s books on Geometry (the ‘Elements’) which were published around 2300 years ago. 25

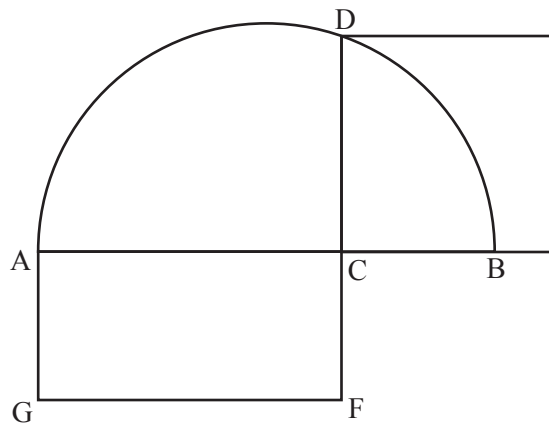


Fig. C2

**Areas of rectangles**

The inequality of arithmetic and geometric means implies that the square has the smallest perimeter of all rectangles with the same area.

Consider a rectangle of given area  $A$  that has sides of lengths  $x$  and  $y$ , so that  $xy = A$ . The perimeter of this rectangle is  $2(x + y)$ . From the inequality of arithmetic and geometric means, we know that  $\frac{x+y}{2} \geq \sqrt{xy}$  so that  $2(x + y) \geq 4\sqrt{xy}$ . But the right-hand side of this last inequality has the fixed value  $4\sqrt{A}$  whatever  $x$  and  $y$  are. For a square of area  $A$ , each side has length  $\sqrt{A}$  and so  $4\sqrt{A}$  is the perimeter of this square. Therefore, the perimeter of any rectangle of area  $A$  is not less than this, so the square has the smallest perimeter of all rectangles with given area. 35

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