

Leave
blank

1. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

Handwritten answer area containing 25 horizontal lines for writing.



Question 1 continued

Leave
blank

(This area contains horizontal lines for writing the answer.)

(Total 7 marks)

Q1



Leave
blank

2.

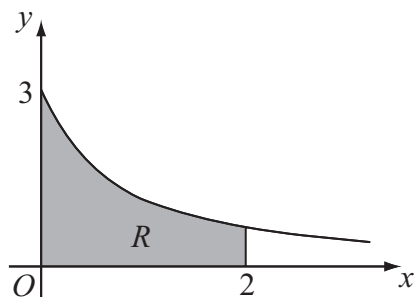


Figure 1

Figure 1 shows part of the curve $y = \frac{3}{\sqrt{1+4x}}$. The region R is bounded by the curve, the x -axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R . **(4)**

The region R is rotated 360° about the x -axis.

(b) Use integration to find the exact value of the volume of the solid formed. **(5)**



Leave blank

3.

$$f(x) = \frac{27x^2 + 32x + 16}{(3x + 2)^2(1 - x)}, \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x + 2)} + \frac{B}{(3x + 2)^2} + \frac{C}{(1 - x)},$$

- (a) find the values of B and C and show that $A = 0$. **(4)**

- (b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Simplify each term. **(6)**

- (c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures. **(4)**



Leave blank

4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$. (2)

Given further that l_1 and l_2 intersect, find

(b) the value of p , (6)

(c) the coordinates of the point of intersection. (2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2 .

Given that a circle, with centre C , cuts the line l_1 at the points A and B ,

(d) find the position vector of B . (3)



Leave blank

5.

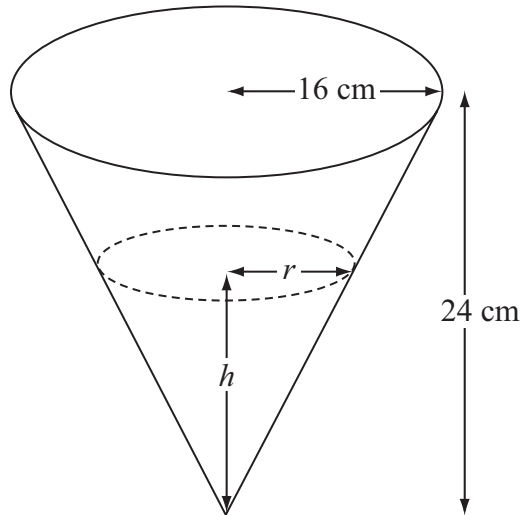


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm³.

(a) Show that $V = \frac{4\pi h^3}{27}$. (2)

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$.

(b) Find, in terms of π , the rate of change of h when $h = 12$. (5)





Question 5 continued

Leave blank

Lined area for writing the answer to Question 5.

Q5

(Total 7 marks)



N 3 1 0 1 3 A 0 1 9 2 8



Leave
blank

6. (a) Find $\int \tan^2 x \, dx$. (2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$. (4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant. (7)



Question 6 continued

Lined writing area for the question.

Leave blank

(Total 13 marks)

Q6

23



Turn over

Leave blank

7.

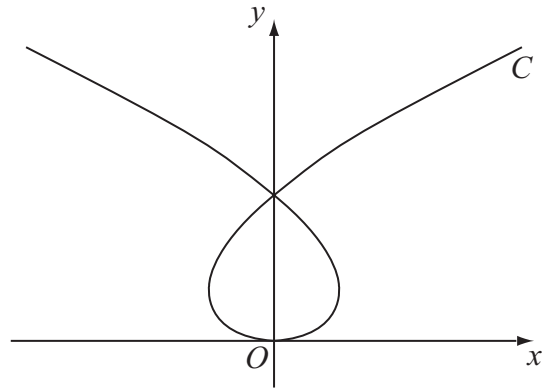


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

- (a) find the coordinates of A . (1)

The line l is the tangent to C at A .

- (b) Show that an equation for l is $2x - 5y - 9 = 0$. (5)

The line l also intersects the curve at the point B .

- (c) Find the coordinates of B . (6)





Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.



BLANK PAGE

