OCR
Oxford Cambridge and RSA

GCE

Mathematics B (MEI)

Unit **H640/02**: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

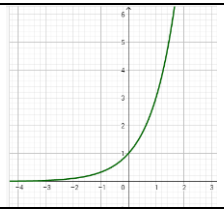
f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$3\sqrt{3}$ or $8\sqrt{3}$ seen $[3\sqrt{3}+8\sqrt{3}]=11\sqrt{3}$	**M1** **A1** **[2]**	**1.1** **2.1**		
2			$-5 < 2x+1 < 5$ $-6 < 2x < 4$ $-3 < x < 2$	**M1** **A1** **A1** **[3]**	**2.1** **1.1** **1.1**	$-(2x+1) < 5$ oe and $2x+1 < 5$ -3 and 2 identified $-3 < x < 2$ allow $x > -3$ **and** $x < 2$	or $(2x+1)^2 < 25$ if **M0** allow **B1** for either condition identified
3	**(i)**		0.36	**B1** **[1]**	**1.1**		
3	**(ii)**		P(draw) $\neq 0$ oe	**B1** **[1]**	**2.4**	allow any comment which identifies that other outcomes are possible	eg winning and losing are not exhaustive
4	**(i)**		2.031578947….rounded to two or more sf isw BC	**B1** **[1]**	**1.1**	NB 2.0, 2.03 or 2.032	
4	**(ii)**		1.076367330…rounded to two or more sf isw BC	**B1** **[1]**	**1.1**	NB 1.1, 1.08 or 1.076	
5	**(i)**	*A*		**B1** **[1]**	**1.2**	correct shape in both quadrants	condone touching the *x*-axis, but not cutting it
5	**(i)**	*B*	(0, 1)	**B1** **[1]**	**1.1**	do not allow just $y = 1$	

Question			Answer	Marks	AOs	Guidance	
5	(ii)		$(f(x)=)\log_3 x$	B1 [1]	1.1	allow eg $\frac{\log x}{\log 3}$	
6	(i)		$R = 25$ $\tan^{-1}\left(\frac{24}{7}\right)$ or $\sin^{-1}\left(\frac{24}{25}\right)$ or $\cos^{-1}\left(\frac{7}{25}\right)$ $25\cos(x + 1.29)$	B1 M1 A1 [3]	1.1 1.1 1.1	 α = 1.28700221759 rounded to 2 or more sf	 73.739795° rounded to 2 or more sf may imply **M1A0** allow **A1** for α found to 2 or more sf
6	(ii)		$12 \pm$ *their* 25 $-13 \le f(x) \le 37$	M1 A1 [2]	3.1a 1.1	or one of – 13 and 37 identified allow eg from – 13 to 37 inclusive	 **A0** if inequality is strict
7			$kx^{\frac{3}{2}}$ kx^{-2} $\frac{8}{3}x^{\frac{3}{2}}$ or $+ 3x^{-2}$ seen $\frac{8}{3}x^{\frac{3}{2}} + 3x^{-2} + c$ isw	M1 M1 A1 A1 [4]	1.1 1.1 1.1 1.1	 	
8	(i)		use of $B \sim (20, 0.7)$ soi 0.191638982753…rounded to 2 or more dp isw BC	M1 A1 [2]	3.1b 1.1	 NB 0.1916 or 0.192 or 0.19	

Question			Answer	Marks	AOs	Guidance	
8	(ii)		P($X \leq 13$) found soi 0.6080098118 13…rounded to 2 or more dp isw BC	M1 A1 [2]	3.1b 1.1	NB 0.391990188187 NB 0.6080 or 0.608 or 0.61	**M0** if P($X = 13$) used NB 0.1643… if **M0** allow **SC1** for $1 – P(X \leq 14) = 1 – 0.58362..$ =0.41637083 rounded to 2 or more dp
9	(i)		negative skew	B1 [1]	1.2		
9	(ii)		(used) the mode	B1 [1]	1.1		
9	(iii)		(used) the median	B1 [1]	1.1		
9	(iv)		61– 1.5 × (88 – 61) 20.5 < 35 [so 35 is not an outlier] so he does not move to set 2	M1 A1 [2]	2.1 2.2b	*Alternatively,* 73.61 – 2 × 17.03 39.6 > 35 [so 35 is an outlier] so he moves to set 2	allow eg only marks below 20.5 (or 39.6) would lead to a move down plus correct conclusion
10	(i)		[μ =]19	B1 [1]	1.1		

Question			Answer	Marks	AOs	Guidance	
10	(ii)		$1.96 = \frac{21.548 - 19}{\sigma}$ $[\sigma =]$ awrt 1.3 $[\sigma^2 =]$ awrt 1.69	**M1** **A1** **A1** **[3]**	**3.1a** **1.1** **1.1**	or $-1.96 = \frac{16.452 - 19}{\sigma}$ may be implied by final answer allow **B3** for awrt 1.69 unsupported	NB 1.959963985…rounded to 3 or more sf **M0** if $z = 2$
10	(iii)	***A***	$[\mu =]$ 4 × *their* 19 + 5 $[\sigma^2 =] 4^2 \times$ *their* 1.69 or $\sigma = 4\times$ *their* 1.3 $[Y \sim] \text{N}(81, 5.2^2)$ oe	**M1** **M1** **A1** **[3]**	**2.1** **1.1** **1.1**	NB 27.04	
10	(iii)	***B***	0.04175 or 0.0417 or 0.042 BC	**B1** **[1]**	**1.1**		NB 0.0417462427103
11	(i)		$k(1\times 6 + 2\times 5 + 3\times 4 + 4\times 3 + 5\times 2 + 6\times 1) = 1$ oe $[k =] \frac{1}{56}$ isw	**M1** **A1** **[2]**	**3.1a** **1.1**	allow one slip in arithmetic **B2** if unsupported	
11	(ii)		$(6\times k)^3 \times (12\times k)^2$ oe seen $\frac{243}{4302592}$ or 0.000056477584 rounded to 2 or more sf	**M1** **A1** **[2]**	**2.1** **1.1**	FT their *k*	
11	(iii)		$40 \times 6k$ 4.286 or 4.29 or 4.3	**M1** **A1** **[2]**	**3.1b** **3.2b**	FT their *k* mark the final answer	

Question			Answer	Marks	AOs	Guidance	
12			use of contingency table or Venn diagram or P(A or B) = P(A) + P(B) – P(A and B)	M1	3.1b	0.56, 0.8 and 0.14 must be correctly placed; eg $1 - 0.14 = 0.56 + 0.8 - P(A \text{ and } B)$	where A denotes "passing" maths and B denotes "passing" English
			P(A and B) = 0.5	A1	2.1		
			P(A) × P(B) = 0.56 × 0.80	M1	1.1	or $P(A/B) = \frac{0.5}{0.80}$	the first **M1A1** may be awarded for working with percentages
			= 0.448 seen	A1	1.1	= 0.625 or ⅝ seen	
			0.448 ≠ 0.5 or 0.56 × 0.80 ≠ 0.5 so not independent	A1 [5]	3.2a	0.625 or ⅝ ≠ 0.56	allow equivalent argument based on showing A' and B' not independent
13	(i)		calculation of P(X < 14) and P(X > 18)	M1	3.4	or solves $-1.476 = \frac{14-\mu}{\sigma}$ and $0.496 = \frac{18-\mu}{\sigma}$ simultaneously	or solves $-1.476 = \frac{x-15}{2}$ and $0.496 = \frac{x-15}{2}$
			0.3085 and 0.0668 to 1 sf or better	A1	1.1	$\mu \approx 17$ and $\sigma \approx 2.02$	x = 12.048 and 15.992 to nearest whole number or better
			these figures do not support the model	A1 [3]	3.5a	17 is (relatively) far from 15 so not a good fit the second **A1** is only available if the first **A1** is awarded allow **SC2** for showing the model is not a good fit for either value with all working correct or for a complete argument based on symmetry which refers to both tails	which are not close to 14 and 18 or $\frac{14-15}{2}$ and $\frac{18-15}{2}$ evaluated – 0.5 and 1.5 obtained which are not close to –1.476 and 0.496 respectively

Question			Answer	Marks	AOs	Guidance	
13	(ii)		$\Phi^{-1}(0.07) = -1.476 = \frac{14-\mu}{2}$ [$\mu = 16.95$] **OR** $\Phi^{-1}(0.69) = 0.496 = \frac{18-\mu}{2}$ [$\mu = 17.008$] [$\mu =$]17	**M1** **A1** **[2]**	**3.5c** **2.4**	*alternatively* since the variance is assumed to be correct, the mean must be as far above the midpoint as it was previously below it. 16 + 1 = 17	if **M0** allow **B2** for 17 unsupported
13	(iii)		$z = \pm 1.96$ used $\frac{16-\mu}{\frac{2}{\sqrt{n}}} < -1.96$ or $\frac{\mu-16}{\frac{2}{\sqrt{n}}} > 1.96$ $\sqrt{n}$ isolated from their $\frac{16-\mu}{\frac{\sigma}{\sqrt{n}}} < -1.96$ oe [$n >$] 15.3664 – 15.4 $n = 16$ cao	**B1** **M1** **M1** **A1** **A1** **[5]**	**1.1a** **3.1b** **2.1** **3.4** **2.2b**	 allow method marks only if other z – value, eg – 1.645 used; FT μ eg $\sqrt{n} > 2 \times 1.96$ previous **A1** must be awarded for the award of final **A1**	NB 1.959963985…rounded to 3 or more sf **M0** if other value for σ used all marks are available if works with = instead of < or > throughout, but withhold final **A1** if works with < instead of > or > instead of < throughout
14	(i)	***A***	the cumulative frequencies have been plotted against the mid-points of the class intervals, mis-plotting [at centre of each class] reduces estimate (by 2.5) oe	**B1** **B1** **[2]**	**2.4** **2.4**		

Question			Answer	Marks	AOs	Guidance	
		B	grouped data has been used grouping has slightly reduced the error introduced by misplotting (because the error is less than 2.5)	**B1** **B1** **[2]**	**2.4** **2.4**	or eg Hodge used the graph (instead of the raw data)	
14	**(ii)**		percentage unemployment is often estimated oe	**E1** **[1]**	**2.4**	allow data (on percentage unemployment) is not available for all countries **in Europe** oe	
14	**(iii)**		there are many other countries in the pre-release material; it is very unlikely that a random sample would only include European countries.	**E1** **[1]**	**2.4**		
14	**(iv)**		negative correlation / association (may be embedded) comparison of *p*-value with 0.05 or 0.01 or other appropriate significance level and supporting comment	**B1** **B1** **[2]**	**2.2b** **2.2b**	if **B0B0** allow **SC2** for eg comment on no significant association justified by comparison of *p*-value with appropriate significance level (eg 0.025)	
14	**(v)**		(even though this is interpolation), the scatter / weak correlation / presence of an outlier would suggest that the use of of a line of best fit is inappropriate	**E1** **[1]**	**2.2b**	allow explanation based on the value for Kosovo being an outlier or on it lying in the (large) gap in the scatter	

Question			Answer	Marks	AOs	Guidance	
15			substitution of $y = 1$	M1	1.1a		
			$x - 4\sqrt{x} + 3 = 0$ or $4\sqrt{x} = x + 3$	A1	2.1		
			$x = 1$ or 9	A1	1.1		
			$3y^2 \frac{dy}{dx}$	B1	3.1a		
			$-x \times \frac{dy}{dx} - y$ or $x \times \frac{dy}{dx} + y$	M1	2.1	allow one sign error	
			$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} - y + \frac{2}{\sqrt{x}} [= 0]$	A1	1.1	dependent on at least two terms correct on LHS following differentiation	allow following wrong rearrangement after differentiating
			substitution of $y = 1$ and *their* $x = 1$ or *their* $x = 9$	M1	1.1		
			$m = -\frac{1}{2}$ [at (1, 1)]	A1	1.1	allow – 0.05555…to 2 sf or better	
			$m = -\frac{1}{18}$ [at (1, 9)]	A1	1.1		
				[9]			
16	(i)		$C = 2$	B1	3.3	since max when $t = 2$	
			$A = 62$	B1	3.3	since max when $(t - 2)^2 = 0$	
			$B = 10$	B1	1.1	from substitution of 22, 62 and 2	
				[3]			
16	(ii)		substitution of 0.75 in $p = 62 - 10(t - 2)^2$	M1	3.4	FT *their* 2, 62, 10	
			46	A1	1.1	allow 46.375 rounded to 2 or more sf	
				[2]			

Question			Answer	Marks	AOs	Guidance	
16	(iii)		*their* $62 - 10(t-2)^2 = 0$	M1	3.4	or ≥ 0 or > 0 for **M1** NB $t = 2 + \sqrt{6.2}$	
			[t =] 4 hours 29 minutes or 4 hours 30 minutes	A1 [2]	2.4	allow 4.49 or 4.5 [hours]	
16	(iv)		substitution of t = 1, 3 and 5 awrt $59.4 \approx 59$ awrt $83.8 \approx 84$ awrt $88.8 \approx 89$	M1 A1 [2]	3.4 3.5a	 or awrt 59.4, 83.8 and 88.8 found and supporting comment made eg they are approximately the same as the values in the table	if **M0** allow **SC1** for two values correctly found and shown to be consistent or supporting comment made
16	(v)		$p \rightarrow 90$ as $t \rightarrow$ large or when $t = 12$ $p = 89.99539\ldots$rounded to 2 or more sf comparison with value of p for $t = 5$ eg model predicts $p = 89$ for $t = 5$ and $p = 90$ for $t = 12$ so not good advice	B1 B1 [2]	3.5a 3.5a	 or model predicts $p = 90$ for (any) $t \geq 7$ so not good advice	 allow equivalent comment on 7 hours work for one extra mark
17	(i)		$\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ $x^2 - 8x + 9 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$ $A = 2$ $B = -1$ $C = -1$	B1 M1 A1 A1 A1 [5]	3.1a 2.1 1.1 1.1 1.1	may be seen later $\frac{2}{(x+1)} - \frac{1}{(x-2)} - \frac{1}{(x-2)^2}$	

Question			Answer	Marks	AOs	Guidance	
17	(ii)		$\int \frac{dy}{y} = \int \frac{x^2 - 8x + 9}{(x+1)(x-2)^2} dx$ soi	**M1***	**3.1a**	allow omission of integral signs and/or omission of dy and/or dx	
			use of their partial fractions in integration	**M1***	**2.1**	allow one sign error and/or one coefficient error	condone use of brackets instead of modulus signs; these two **A** marks are only available following the award of **both M** marks
			$\ln\lvert y\rvert = 2\ln\lvert x+1\rvert - \ln\lvert x-2\rvert + \frac{1}{x-2} + c$	**A1**	**1.1**	**A1** for any correct natural log integral on RHS FT *their* $\frac{2}{x+1}$ or *their* $\frac{-1}{x-2}$	
				A1	**1.1**	**A1** for $\frac{1}{x-2}$ FT *their* $\frac{k}{(x-2)^2}$	
			substitution of $y = 16$ and $x = 3$	**M1dep***	**1.1**	expression must include + c and must include at least one natural log term; may be awarded after exponentiating	may be awarded following collection of like terms, which may contain errors NB $c = -1$
			correctly exponentiate both sides of their equation	**M1**	**1.1**		
			$y = \frac{(x+1)^2}{x-2} e^{\frac{3-x}{x-2}}$ oe	**A1**	**2.1**	eg $\frac{(x+1)^2}{x-2} e^{\frac{1}{x-2}} e^{-1}$	
				[7]			

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

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