Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$ *	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$ *	A1*	2.1
			(5 marks)
	Notes		

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\dfrac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

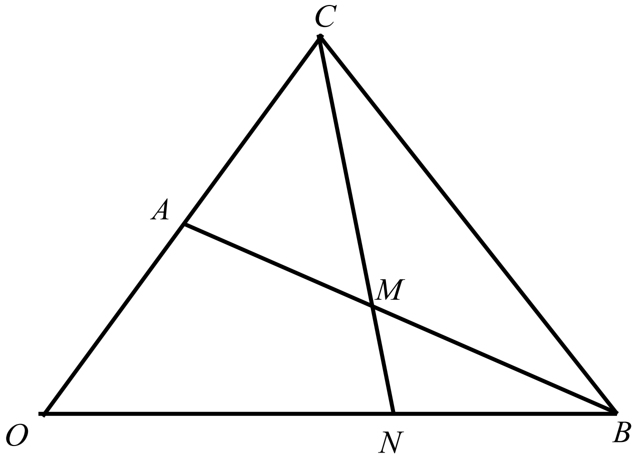
M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

Question	Scheme	Marks	AOs
2	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $\overrightarrow{OC} = a\mathbf{i}+5\mathbf{j}-2\mathbf{k}$, $a<0$ $\overrightarrow{AB} = \overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OB} + \overrightarrow{OB} - \overrightarrow{OA} = 2\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OB} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix} \left\{ = \begin{pmatrix}4\\-2\\3\end{pmatrix} + \begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$ **or** $= \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix} - \begin{pmatrix}2\\3\\-4\end{pmatrix}\right) \left\{ = \begin{pmatrix}2\\3\\-4\end{pmatrix} + 2\begin{pmatrix}2\\-5\\7\end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2+(5-3)^2+(-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = ...$ **or** $\Rightarrow a^2-4a-4=0 \Rightarrow a = ...$	dM1	2.1
	(as $a<0 \Rightarrow$) $a = 2-2\sqrt{2}$ (or $a = 2-\sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

Notes for Question 2

(a)	
M1:	Complete ***applied*** strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})-(4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-7\mathbf{k}) = (2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB} - \overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of ***correctly*** applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = ...$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2-2\sqrt{2}$
Note:	Writing $a = 2\pm2\sqrt{2}$, without evidence of rejecting $a = 2+2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2-\sqrt{8}$ or $\frac{4-\sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828...$, without reference to a correct exact value is A0

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (***needs to be simplified and seen in (a) only***)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON} = \mu\mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} = \mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda = \mu\ \&\ \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$ $\left\{ \mathbf{b}: 1 = \frac{1}{2}\lambda + K \ \ \& \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}$ & $\overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}$, $\mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ $\left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**<u>Special Case</u>**
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $\left\{ = \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b} \right\}$
Note:	**<u>Alternative 1:</u>** Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as *OC* is parallel to *AB*, so *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why *OABC* is a trapezium.
Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$
It requires a reason and minimal conclusion.
Example 1:
$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium
Example 2:
A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.
Example 3
As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, *OC* and *AB* are parallel, so proven
Example 4
Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

Question Number	Scheme	Marks	AO's
2	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=2(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=3(\mathbf{q}-\mathbf{p})$ $\frac{2}{3}(\mathbf{q}-\mathbf{p})=\frac{1}{3}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{3}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{2}{3}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=2\mathbf{q}-2\mathbf{p}\Rightarrow 2\mathbf{p}+\mathbf{r}=3\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{3}(\mathbf{r}+2\mathbf{p})$ *	A1*	2.1
		(3)	
			(3 marks)

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{p}),\ \pm(\mathbf{r}-\mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow OQ = … as long as OQ has been defined as **q** earlier.

In the working allow use of P instead of **p** and Q instead of **q** as long as the intention is clear.

Question	Scheme	Marks	AOs

Qu	Scheme	Marks
14 (a)	Attempts $\overrightarrow{BA}=\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ or $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$ either way around	M1
	Finds $\overrightarrow{OD}=\mathbf{a}-\mathbf{b}+\mathbf{c}=(-2\mathbf{i}+2\mathbf{j}-8\mathbf{k})+(-\mathbf{i}+3\mathbf{j}+6\mathbf{k})=-3\mathbf{i}+5\mathbf{j}-2\mathbf{k}$	M1 A1
		(3)
(b)	$\overrightarrow{BA}=\mathbf{a}-\mathbf{b}=-2\mathbf{i}+2\mathbf{j}-8\mathbf{k}$ and $\overrightarrow{BC}=\mathbf{c}-\mathbf{b}=-4\mathbf{i}+4\mathbf{j}$	M1
	$\cos\theta=\dfrac{\begin{pmatrix}-2\\2\\-8\end{pmatrix}.\begin{pmatrix}-4\\4\\0\end{pmatrix}}{\sqrt{(-2)^2+2^2+(-8)^2}\sqrt{(-4)^2+4^2}}=\dfrac{16}{\sqrt{72}\sqrt{32}}=\dfrac{1}{3}$	dM1 A1
	So angle is 1.23 radians or 70.5 degrees	A1
		(4)
(c)	Area = $\sqrt{72}\sqrt{32}\sin\theta=45.3$ **or** $32\sqrt{2}$ **oe**	M1A1
		(2)
(d)	Area = $\dfrac{3}{2}\times"45.3"=67.9$ or $48\sqrt{2}$ **oe**	M1 A1
		(2)
		(11 marks)

(a)

M1: For attempting one of **b – a** or **a – b or c – b** or **b – c.** It must be correct for at least one of the components. Condone coordinate notation for the first two M marks

M1: For attempting $\mathbf{d}=\mathbf{a}-\mathbf{b}+\mathbf{c}$ = It must be correct for at least one of the components.

A1: cao. Correct answer no working scores all 3 marks. It must be the vector (either form) and not a coordinate

Note this can be attempted by finding the mid point E of AC and then using **d** =**b**+2$\overrightarrow{BE}$ but it must be a full method M1 Attempts $mp_{AC}=(0,2,2)$ **and uses** .... M1 Attempts $(3,-1,6)+2\times(-3,3,-4)$ A1

(b)

M1: Uses correct pair of vectors , so $\pm k\overrightarrow{BA}$ and $\pm k\overrightarrow{BC}$. Each must be correct for at least one of the components

dM1: A clear attempt to use the dot product formula to find $\cos\theta=k, -1<k<1$. It is dependent upon having chosen the correct pair of vectors. Allow for arithmetical slips in both their dot product calculation and the moduli, but the process must be correct.

It could also be found using the cosine rule. $\dfrac{72+32-72}{2\sqrt{72}\sqrt{32}}=$

(M1 is for attempt at all three lengths, so $\pm\overrightarrow{BA}, \pm\overrightarrow{BC}, \pm\overrightarrow{AC}$ and dM1 correct angle attempted using the correct formula)

A1: For 1/3 or – 1/3 or equivalent - may be implied by 70.5 or 109.5 or 1.23 radians or 1.91 radians

A1: cso for awrt 70.5 degrees or 1.23 radians. (Note that invcos(– 1/3)=109.5 followed by 70.5 is A0 .... unless accompanied by a convincing argument that the angle 109.5 is the exterior angle, and therefore the interior angle is 70.5. It is not awarded for simply finding the acute angle. A diagram with correct angles would be ok)

(c)

M1: Uses correct area formula for parallelogram.

You may see the area of the triangle ABC doubled which is fine.

A1: Obtains awrt 45.3. Allow this from an angle of 109.5

(d)

M1: Realises connection with part (c) and uses 1.5 times answer to the area of $ABCD$ (It can be implied by 67.9)

A1: awrt 67.9

Question Number	Scheme	Marks
9(a)	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}.\begin{pmatrix}5\\-6\\1\end{pmatrix}=\sqrt{(2^2+3^2+(-2)^2)}\sqrt{(5^2+(-6)^2+1^2)}\cos(\angle CAB)$	M1
	$\angle CAB=\arccos\left(-\dfrac{10}{\sqrt{17}\sqrt{62}}\right)=107.94°$	dM1A1
		(3)
(b)	Area $=\dfrac{1}{2}\sqrt{17}\sqrt{62}\sin(107.94°)=15.44$	M1A1
		(2)
(c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses Area $=\dfrac{1}{2}\lvert BC\rvert\times\lvert AD\rvert \Rightarrow 15.44=\dfrac{1}{2}\times\sqrt{99}\times\lvert AD\rvert\Rightarrow\lvert AD\rvert=3.10$	M1A1
		(3)
		(8 marks)
ALT (a)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses cosine rule $\cos\angle CAB=\dfrac{17+62-99}{2\times\sqrt{17}\times\sqrt{62}}$	dM1
	$\Rightarrow\angle CAB=\arccos\left(-\dfrac{20}{2\sqrt{17}\sqrt{62}}\right)=107.94°$	A1
		(3)
ALT (b)	Area = $\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\left(\dfrac{\sqrt{17}+\sqrt{62}+\sqrt{99}}{2}\right)\left(\dfrac{\sqrt{17}-\sqrt{62}+\sqrt{99}}{2}\right)\left(\dfrac{\sqrt{17}+\sqrt{62}-\sqrt{99}}{2}\right)\left(\dfrac{\sqrt{62}+\sqrt{99}-\sqrt{17}}{2}\right)}$	M1A1
Alt (b) ii	Area = $\dfrac{1}{2}\sqrt{\lvert AB\rvert^2\lvert AC\rvert^2-\left(\overrightarrow{AB}.\overrightarrow{AC}\right)^2}=\dfrac{1}{2}\sqrt{17\times62-100}=\dfrac{3}{2}\sqrt{106}$	M1A1
ALT (b)	**Area =** $\dfrac{1}{2}\lvert a\times b\rvert=\dfrac{1}{2}\begin{vmatrix}i&j&k\\2&3&-2\\5&-6&1\end{vmatrix}=\dfrac{1}{2}\lvert-9i-12j-27k\rvert=\dfrac{1}{2}\sqrt{9^2+12^2+27^2}=15.44$	M1A1
ALT (c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	$\dfrac{\sin(\text{"}107.94\text{"})}{BC}=\dfrac{\sin B}{\text{"}\sqrt{62}\text{"}}\Rightarrow B=48.84...\Rightarrow AD=\text{"}\sqrt{17}\text{"}\sin B=3.10$	M1A1

(a)

M1: Attempts to use $\mathbf{a}.\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ where $\mathbf{a}=\pm(2\mathbf{i}+3\mathbf{j}-2\mathbf{k})$, $\mathbf{b}=\pm(5\mathbf{i}-6\mathbf{j}+\mathbf{k})$ I.e. correct use of Pythagoras to find and multiply the lengths together and multiplies and adds components (allow arithmetic slips) for dot product.

dM1: Dependent upon previous M1. For continuing to find $\angle CAB$ using invcos. Allow $\arccos\left(\dfrac{\pm10}{\sqrt{17}\sqrt{62}}\right)$

Implied by previous M1 + angle rounding to $108°$ or $72.1°$ (NB $\cos\theta$= -0.308…)