15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

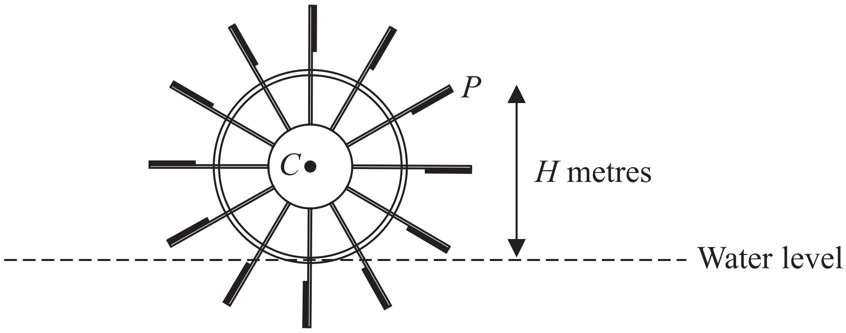


Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

Leave blank

13. (a) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give your value of α to 2 decimal places. **(3)**

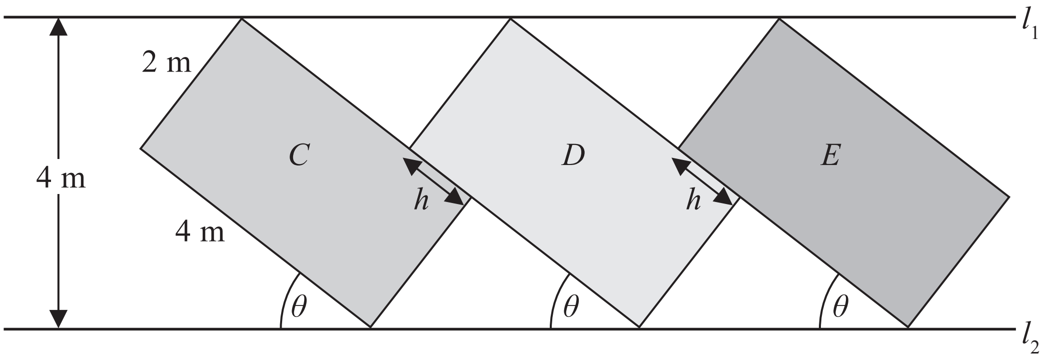


Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, C, D and E, each of which is in contact with two horizontal parallel lines l_1 and l_2. Rectangle D touches rectangles C and E as shown in Figure 4.

Rectangles C, D and E each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

(b) show that

$$2\sin\theta + \cos\theta = 2$$

(2)

Given also that $0 < \theta < 45°$,

(c) solve the equation

$$2\sin\theta + \cos\theta = 2$$

giving the value of θ to 1 decimal place. **(3)**

Rectangles C and D and rectangles D and E touch for a distance h m as shown in Figure 4.

Using your answer to part (c), or otherwise,

(d) find the value of h, giving your answer to 2 significant figures. **(3)**

Leave blank

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

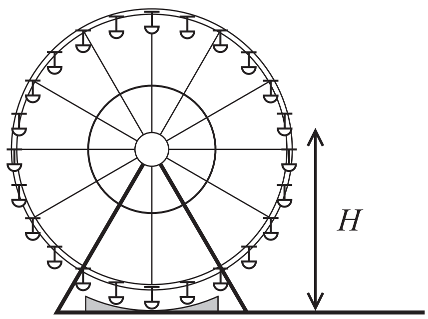
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)° + 3\sin(30t)°$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



(b) Calculate

(i) the maximum value of H predicted by this model,

(ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

(c) Calculate the value of t when the passenger is 18 m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

(d) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

Leave blank

11. (a) Express $1.5\sin\theta - 1.2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of R and the value of α to 3 decimal places.

(3)

The height, H metres, of sea water at the entrance to a harbour on a particular day, is modelled by the equation

$$H = 3 + 1.5\sin\left(\frac{\pi t}{6}\right) - 1.2\cos\left(\frac{\pi t}{6}\right), \quad 0 \leqslant t < 12$$

where t is the number of hours after midday.

(b) Using your answer to part (a), calculate the minimum value of H predicted by this model and the value of t, to 2 decimal places, when this minimum occurs.

(4)

(c) Find, to the nearest minute, the times when the height of sea water at the entrance to the harbour is predicted by this model to be 4 metres.

(6)

Leave blank

10. (a) Express $3\sin 2x + 5\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

(b) Solve, for $0 < x < \pi$,

$$3\sin 2x + 5\cos 2x = 4$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

$$g(x) = 4(3\sin 2x + 5\cos 2x)^2 + 3$$

(c) Using your answer to part (a) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

Leave blank

1. (a) Express $3\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$3\cos\theta + 5\sin\theta = 2$$

Give your answers to one decimal place.

(4)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$3\cos\theta - 5\sin\theta = 2$$

(2)

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11. (a) Express $35 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.
(3)

(b) Hence solve, for $0 \leqslant x < 2\pi$,

$$70 \sin x - 24 \cos x = 37$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(4)

$$y = \frac{7000}{31 + (35\sin x - 12\cos x)^2}, \quad x > 0$$

(c) Use your answer to part (a) to calculate

(i) the minimum value of y,

(ii) the smallest value of x, $x > 0$, at which this minimum value occurs.
(4)

Leave blank

10. (a) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to one decimal place.
(3)

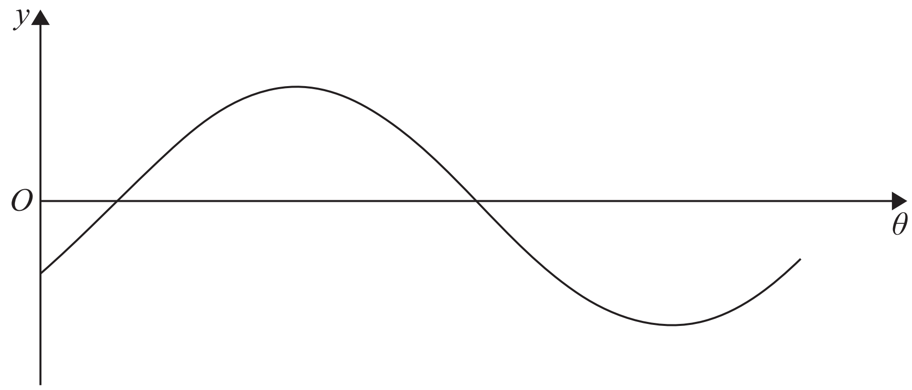

Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta, \quad 0 \leqslant \theta < 360°$

(b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \qquad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.
(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |2\sin(15t)° - \cos(15t)°|, \qquad 0 \leqslant t < 24$$

where f(t) is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

(c) (i) the maximum value of f(t),

(ii) the largest value of t, for $0 \leqslant t < 24$, at which this maximum value occurs. Give your answer to one decimal place.
(3)

Leave blank

4. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Leave blank

3. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leqslant \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15$$

Give your answer to one decimal place.

(2)

Leave blank

3. $$g(\theta) = 4\cos 2\theta + 2\sin 2\theta$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to 2 decimal places.

(3)

(b) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k.

(2)

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7.

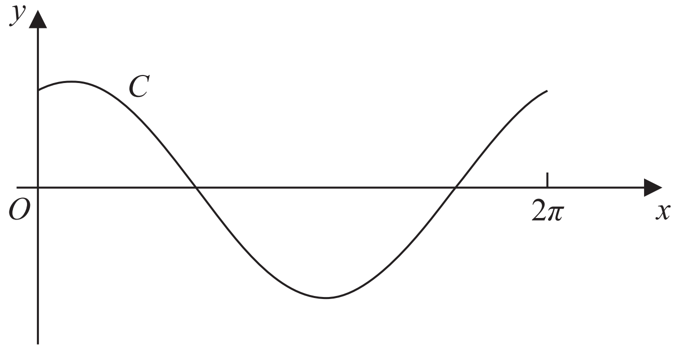

Figure 1

Figure 1 shows the curve C, with equation $y = 6\cos x + 2.5\sin x$ for $0 \leqslant x \leqslant 2\pi$

(a) Express $6\cos x + 2.5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places.

(3)

(b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes.

(3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right), \quad 0 \leqslant t \leqslant 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(c) the maximum and minimum values of H predicted by the model,

(3)

(d) the values for t when $H = 16$, giving your answers to the nearest whole number.

[*You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.*]

(6)

Leave blank

9. (a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leqslant \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leqslant \theta < \pi$, at which this minimum value occurs.

(3)

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8.

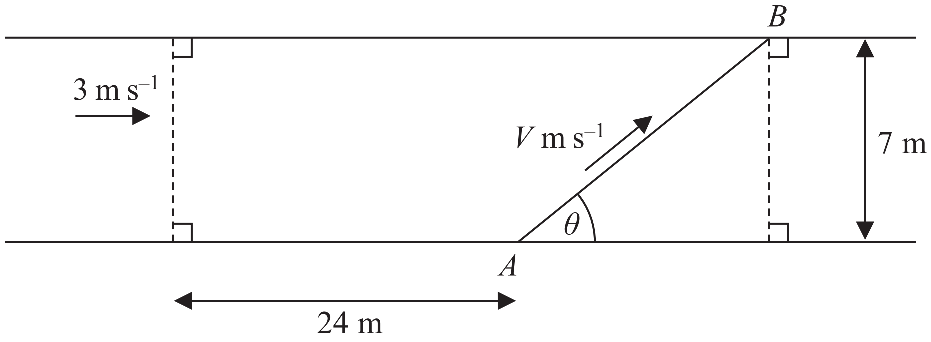

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.
Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.
Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.
(3)

Given that θ varies,

(b) find the minimum value of V.
(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance AB.
(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.
(6)

Leave blank

4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b) $$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)