June 2017
WST02 STATISTICS 2
Mark Scheme

Question	Scheme	Marks
1.(a)	$X \sim \text{Po}(\frac{1}{4})$	B1
	$\text{P}(X = 0) = e^{-\frac{1}{4}} = 0.778800\ldots$ awrt **0.779**	B1
		(2)
(b)	$[(\text{P}(X \geqslant 1))^3] = (1 - \text{'0.7788'})^3 = 0.010823\ldots$ awrt **0.0108**	M1 A1
		(2)
(c)	$Y \sim \text{B}(7, 0.7788\ldots)$	B1ft
	$\text{P}(Y = 5) = 7\text{C}5\ (0.7788)^5(1 - 0.7788)^2 = 0.294386\ldots$ awrt **0.294**	M1 A1
		(3)
(d)	H_0: $\mu = 8$ or $\lambda = 0.25$ $\quad$ H_1: $\mu < 8$ or $\lambda < 0.25$	B1
		(1)
(e)	$W \sim \text{Po}(8)$	B1
	$\text{P}(W \leqslant 3) = 0.0424\ (< 0.05)$	M1
	$\text{P}(W \leqslant 4) = 0.0996\ (> 0.05)$	
	Largest possible value of f is 3	A1
		(3)
		Total 11

	Notes
(a)	1st B1 for writing or using $\text{Po}(\frac{1}{4})$. May be implied by a correct answer or by awrt 0.78
(b)	M1 for $(1-0.779)^3$ or (1 – 'their (a)')^3
(c)	B1ft for writing or using B(7, 'their a'). May be implied by M1 scored.
	M1 for a correct binomial expression for $\text{P}(Y = 5)$ (ft their value of p). Allow $\binom{7}{2}$ etc or 21
	May be implied by the correct answer but if $p \neq 0.779$ or better we must see expression
ALT	They may use $W \sim \text{B}(7, \text{" 1 – their (a)"})$ for B1ft then $\text{P}(W = 2)$ for the M1
(d)	B1 for both hypotheses correct. Must use λ or μ for either 8 or 0.25 [Use of $\leqslant$ is B0]
	If (d) is blank but correct hypotheses are seen in (e) can award retrospectively BUT if hypotheses are given in (d) and (e) award this mark for answer in (d).
(e)	B1 for writing Po(8) can be awarded if seen in (d) (may be implied e.g. by scoring M1)
	M1 for using Po(8) to find a lower-tail critical region
	Need to see one of the given probability statements or implied by Po(8) and $f = 3$ seen.
	A1 for $[f] = 3$ but allow $f \leqslant 3$ Correct answer only scores 3/3

Question Number	Scheme	Marks
3. (a)	$P(X \leqslant 7) = 0.8883$ or $P(X \leqslant 8) = 0.9644$ or $P(X \geqslant 8) = 0.1117$ or $P(X \geqslant 9) = 0.0356$	**M1**
	Critical Region is $X \geqslant 9$ (o.e.)	**A1**
		(2)
(b)	(1 – 0.9644=) 0.0356 [NB Calculator gives: 0.03557486...]	**B1cao**
		(1)
(c)	Reject H_0/Significant or value of p is > 0.45	**B1ft**
		(1)
(d)(i)	Conclusion would not change as H_0 would still be rejected	**B1**
(ii)	Conclusion would change as H_0 would not be rejected	**B1**
		(2)
		[6]

Notes

(a)
M1 for one of these 4 probabilities - may be implied by a correct critical region
A1 for $X \geqslant 9$ (allow $X > 8$) (o.e.) e.g. [9, 12], {9, 10, 11, 12} etc Ans. only 2/2
NB Must be $X \geqslant 9$ for A1, do not award for just seeing $P(X \geqslant 9)$

(b)
B1 for 0.0356 or better

(c)
B1f ft their critical region in (a) Must say "reject" **and** "H_0" No contradictory statements
Just saying "9 is not in the critical region" is not enough
Allow a restart i.e. calculating $P(X \geqslant 9) = 0.0356 < 0.05$ so significant

If they score B0 in (c) then score B0B0 in (d)

(d)	In (c) they reject H_0	In (c) they accept H_0
(i)	B1 for "No", "no change", "significant" etc	B0 whatever they say
(ii)	B1 for "Yes" , "do not reject H_0" etc	B1 for "no change" or "do not reject H_0" etc

CR
(i) NB new CR is $X \geqslant 9$ but can treat any incorrect mention of CR as ISW
(ii) NB new CR is $X \geqslant 10$ but can treat any incorrect mention of CR as ISW

Question Number	Scheme	Marks
5. (a)(i)	$H_0 : p = 0.35$ $H_1 : p \neq 0.35$	**B1**
(ii)	B(15,0.35)	**M1**
	CR $X \leq 1 \cup X \geq 10$ (Allow any letter)	**A1A1**
		(4)
(b)	8 is not in CR	**M1**
	There is evidence that the Company's claim is true	**A1ft**
		(2)
(c)	0.0142 + 0.0124 = 0.0266	**B1**
		(1)
		[7]

	Notes
(a) (i)	B1 both hypotheses correct. Must mention p (or π). Words only is B0
(ii)	M1 Writing B(15,0.35) May be implied by e.g. P($X \leq 1$) =0.0142 or P($X \leq 9$) = 0.9876
	1st A1 $X \leq 1$ (accept $X < 2$) Allow $0 \leq X \leq 1$ but P($X \leq 1$) is A0
	2nd A1 $X \geq 10$ (accept $X > 9$) Allow $10 \leq X \leq 15$ but P($X \geq 10$) is A0
	Either correct answer will imply M1
(b)	M1 for a reason that matches their CR. "Interpret" their CR of P($X \geq 10$) as $X \geq 10$ etc
	Allow calculation of P($X \geq 8$) = 1 – 0.8868 = 0.1132 and "not sig" comment
	Do not allow contradictory remarks e.g. 8 is not in CR so significant (this gets M0)
	A1ft for a conclusion correct for their CR in context
	Must mention "claim" or "peas" and "germinating"
	NB A correct contextual claim on its own scores M1A1
(c)	B1 for 0.0266 or awrt 0.0266 (calc gives 0.02662196...)

Question Number	Scheme	Marks
6.	Let X = the number of seeds that germinate	
	Let Y = the number of seeds that don't germinate. $x_{obs} = 66,\ y_{obs} = 9$	
	$H_0 : p = 0.96$, $H_1 : p < 0.96$ **or** $H_0 : p = 0.04$, $H_1 : p > 0.04$ **or** $H_0 : \lambda = 3, H_1 : \lambda > 3$	B1 B1
	$\{Y \sim \text{Bin}(75, 0.04)$ approximates to $\}$ $Y \sim \text{Po}(3)$	B1
	$P(Y \geqslant 9) = 1 - P(Y \leqslant 8)$ or $P(Y \leqslant 7) = 0.9881 \Rightarrow P(Y \geqslant 8) = 0.0119$ $P(Y \leqslant 8) = 0.9962$	M1
	$= 1 - 0.9962$	
	$= 0.0038$ CR: $Y \geqslant 9$	A1
	$\{0.0038 < 0.01\}$	
	Reject H_0 or significant or 9 lies in the CR	dM1
	Either • There is evidence that the producer has overstated the probability/percentage/proportion/number of bean seeds that germinate. • Producer's claim is not true. • There is evidence that the producer has understated the probability/percentage/proportion/number/ of bean seeds that don't germinate.	A1 cso
		[7]
		7

Notes

1st B1 for $H_0 : p = 0.96$ or $H_0 : p = 0.04$ or $H_0 : \lambda = 3$

2nd B1 for $H_0 : p = 0.96$ and $H_1 : p < 0.96$
or $H_0 : p = 0.04$ and $H_1 : p > 0.04$
or $H_0 : \lambda = 3$ and $H_1 : \lambda > 3$

3rd B1 Po(3) seen or implied

1st M1 for writing or using $1 - P(Y \leqslant 8)$ **or** giving $P(Y \leqslant 7) = 0.9881$ or $P(Y \geqslant 8) = 0.0119$ for a CR method
(may be implied by probability = 0.0038 or correct CR)

1st A1 for 0.0038 or CR: $Y \geqslant 9$

2nd M1 Dependent on the 1st M1. For a correct statement i.e. significant/reject H_0/9 is in CR
Follow through their probability/CR and their H_1
May be implied by a correct contextual statement.
Ignore comparison of probability with the significance level.
Do not allow non-contextual conflicting statements.

2nd A1cso fully correct solution and correct contextual statement

B1 B1 Correct hypotheses (same mark scheme as above)

B0 N(72, 2.88)

M1 $\dfrac{\pm(66.5 - 72)}{\sqrt{2.88}} (= \pm 3.24)$

A0 awrt 0.0006

dM1A0cso (same mark scheme as above)

Question Number	Scheme	Marks
2(a)	**List** of all the **customers** (who eat in the restaurant)	B1 (1)
(b)	**Customer(s)** (who ate in the restaurant)	B1 (1)
(c)	Advantage: more/total accuracy, unbiased	B1
	Disadvantage: time consuming to obtain data and analyse it, expensive, difficult to ensure entire population is included	B1 (2)
(d)	Let X = the number of customers who would like more choice on the menu.	
	H_0: $p = 0.3$ H_1: $p > 0.3$	B1
	$X \sim B(50,0.3)$	M1
	$P(X \geqslant 20) = 1 - P(X \leqslant 19)$ or CR $P(X \leqslant 20) = 0.9522$ $= 1 - 0.9152$ $P(X \geqslant 21) = 0.0478$ $= 0.0848$ $X \geqslant 21$	M1 A1
	Do not reject H_0/ not significant/20 is not in critical region	M1
	The percentage of customers who would like more choice on the menu is not more than Bill believes. or There is no evidence to reject Bill's belief.	A1cso (6)
		Total (10)

Notes

(a) B1 Need the idea of list/register/database and 'customer(s)'
Do not allow customer's opinions.
'All' may be implied. Do not allow a partial list e.g. 'A list of 50 customers'

(b) B1 customer(s)

(c) If not labelled, assume the response refers to a census.
1st B1 is for the advantage and 2nd B1 is for the disadvantage.

(d) B1 need both hypotheses with p
M1 using B(50,0.3)
M1 for $1 - P(X \leqslant 19)$ or
$P(X \leqslant 20) = 0.9522$ or $P(X \geqslant 21) = 0.0478$ leading to a **critical region** $X > k$ or $X \geqslant k$
A1 awrt 0.0848 or critical region $X \geqslant 21$ or $X > 20$
M1 a correct conclusion for their probability. May be implied by a correct contextual conclusion.
A1 a correct contextual conclusion for their hypotheses and a fully correct solution with no errors seen. Must mention 'customers' and 'choice' or 'Bill' and 'belief'.

NB P(X=20) can score B1M1M0A0M0A0
NB normal approximation gives 0.082(457…) and loses all A marks

Question Number	Scheme	Marks
2(a)	Only 2 outcomes **Heads** and **Tails oe**	
	Constant probability of **spinning** a **Head/Tail oe**	
	Coin is **spun** a fixed number of times **oe**	
	Each **spin** of the **coin** is independent **oe**	B1 B1
		(2)
(b)	$T \sim B(6, 0.5)$	
	$P(T \le 5) - P(T \le 4) = 0.9844 - 0.8906$ or $6\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)$ **oe**	M1
	$= 0.09375$ or $\frac{3}{32}$ oe awrt 0.0938	A1
		(2)
(c)	$P(T = 4,5,6) = 1 - P(T \le 3)$	M1
	$= 1 - 0.6563$	
	$= 0.3437$ or $\frac{11}{32}$ awrt 0.344	A1
		(2)
(d)	$P(H = 3,4,5,6) = 1 - P(H \le 2)$	B1M1d
	$= 1 - 0.8306$	
	$= 0.1694$ or $\frac{347}{2048}$ awrt 0.169	A1
		(3)

	Notes		Total 9
(a)	B1 A correct statement – does not need to be in context B1 A second correct statement in context include coin or heads or tails(do not allow H and T) or spins/flip oe.		
(b)	M1 [writing or using B(6, 0.5) **and** writing or using $P(T \le 5) - P(T \le 4)$] **or** [$6\left(\frac{1}{2}\right)^6$ **oe**]		
(c)	M1 for realising they need find $P(T = 4, 5 \text{ or } 6)$ eg $1 - P(T \le 3)$ or $P(T \ge 4)$		
(d)	B1	writing/using B(6, 0.25) **and** $P(H \ge 3)$ **oe**	writing/using B(6, 0.75) **and** $P(T \le 3)$
	M1d	dep on B1 for $1 - P(H \le 2)$	dep on B1 $(0.25)^6 + 6(0.75)(0.25)^5 + 15(0.75)^2(0.25)^4 + 20(0.75)^3(0.25)^3$
	A1	awrt 0.169	awrt 0.169
	NB	Only accept correct use of H and T in the probability statement unless their variable is correctly defined	
	NB	awrt 0.169 with no incorrect working gains B1M1A1	

Question Number	Scheme		Marks
	Allow any letter instead of X or c for this question		
1 (a)	$X \sim \mathrm{B}(25, 0.2)$	**M1** Writing or using B(25,0.2)or B(25,1/5) [allow Po(5)] May be written in full or implied by a correct CR (allow written as a probability statement)	M1
	$[\mathrm{P}(X \geq 9) =] 0.0468$ $[\mathrm{P}(X \leq 1) =] 0.0274$	**1st A1** both awrt 0.0468 and awrt 0.0274 seen.	A1
	$X = [0 \leq] \ X \leq 1$	**2nd A1** $X \leq 1$ or $X < 2$ or $0 \leq X \leq 1$ or [0,1] or 0,1 or equivalent statements. $X \leq c$ and $c = 1$	A1
	$9 \leq X \ [\leq 25]$	**3rd A1d** dependent on seeing a probability from the B(25, 0.2) and $X \geq 9$ or $X > 8$ or $9 \leq X \leq 25$ or 9,10,11,12,13,14,15,16,17,18,19,20,21,22, 23,24,25 or [9,25] or equivalent statements. $X \geq c$ and $c = 9$	A1d
	NB These two final 2 A marks must be for statements with "X" only(or list) – not in probability statements **SC If** a probability from the B(25, 0.2) is seen **and** they either have both CR correct but written as probability statements **or** the CR is written as $1 \geq X \geq 9$ they get A1 A0 for final 2 marks		**(4)**
(b)	H_0: $p = 0.2$ H_1: $p < 0.2$	**B1** both hypotheses with p or π and clear which is H_0 and which is H_1	**B1**
	$\mathrm{P}(X \leq 6) = 0.1034$ or CR $X \leq 5$	**1st M1** writing or using B(50, 0.2) and writing or using P($X \leq 6$) or P($X \geq 7$) on its own. May be implied by a correct CR	**M1**
		1st A1 awrt 0.103. Allow CR $X \leq 5$ or $X < 6$. or if not using CR allow awrt 0.897.	**A1**
	Insufficient evidence to reject H_0, Accept H_0, Not significant. 6 does not lie in the Critical region.	**2nd M1** dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non-contextual statements). ft their Prob/CR compared with 0.05/6/(0.95 if using 0.8979). Do not follow through their hypotheses	**M1d**
	No evidence that increasing the batch size has **reduced** the **percentage** of broken **pots (oe)** **or** evidence that there is **no change** in the **percentage** of broken **pots (oe)**	**2nd A1cso** Conclusion must contain the words **reduced/ no change/not affect oe number/percentage/proportion/ probability oe**, and **pots**. All previous marks must be awarded for this mark to be awarded. Do **not** allow the potters claim /belief is wrong/true **NB** Correct contextual statement on its own scores M1A1	**A1cso** **(5)**
			(Total 9)

Question Number	Scheme		Marks
2(a)		**notes**	
	$X \sim$ B(30, 0.25)	B1: using B(30, 0.25)	B1
	P($X \leq 10$) – P($X \leq 4$) = 0.8943 – 0.0979	M1: using P($X \leq 10$) – P($X \leq 4$) or P($X \geq 5$) – P($X \geq 11$) oe	M1 A1
	= 0.7964	A1: awrt 0.796	
	NB a correct answer gains full marks		

(b)	$H_0 : p = 0.25$ $H_1 : p < 0.25$	B1: Both hypotheses correct, labelled H_0 or NH or H_n and H_1or AH or H_a , must use p or $p(x)$ or π	B1
	B(15, 0.25)	M1: for using B(15, 0.25)	M1 A1
	P($X \leq 1$) = 0.0802	A1: awrt 0.0802 or CR $X \leq 1$ (allow P($X \geq 2$) = 0.9198)	
	NB: Allow M1 A1 for a correct CR with no incorrect working		
	Reject H_0 or Significant or 1 1ies in the critical region	M1: A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR (for 1 or 2 tail test). If no H_1 given then M0. Ignore their comparison. For a probabillity < 0.5, statement must be correct compared to 0.1 for 1 tail test and 0.05 for 2 tailed test or if the probability > 0.5, statement must be correct compared to 0.9 for 1 tail test and 0.95 for 2 tailed test.	dM1 A1cso
	There is evidence that the radio **company's** claim is true. **Or** The new transmitter will reduce the proportion of houses unable to receive **radio**	A1: cso (all previous marks awarded) and a correct statement containing the word **company** if writing about the claim or **radio** if full context**.**	