Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx}=20x^3-72x^2+84x-32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2}=60x^2-144x+84$	A1ft	1.1b
		(3)	
(b)(i)	$x=1\Rightarrow\frac{dy}{dx}=20-72+84-32$	M1	1.1b
	$\frac{dy}{dx}=0$ so there is a stationary point at $x=1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3-72x^2+84x-32=4(x-1)^2(5x-8)=0\Rightarrow x=\ldots$	M1	1.1b
	When $x=1$, $\frac{dy}{dx}=0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8}=\ldots$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2}=\ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8}>0$, $\left(\frac{d^2y}{dx^2}\right)_{x=1.2}<0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1}=60x^2-144x+84=0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right)=120x-144\Rightarrow\left(\frac{d^3y}{dx^3}\right)_{x=1}=\ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1}=0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1}\neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8}=\ldots$ $\left(\frac{dy}{dx}\right)_{x=1.2}=\ldots$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8}<0$, $\left(\frac{dy}{dx}\right)_{x=1.2}<0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)
M1: $x^n\to x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx}=20x^3-72x^2+84x-32$
(a)(ii)

Question	Scheme	Marks	AOs
14	$y=\dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
			(4 marks)

Notes

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y=\dfrac{t^2-4}{2+t}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}t}\times\dfrac{\mathrm{d}t}{\mathrm{d}x}=\dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be seen at least once

Question	Scheme	Marks	AOs
14	$y=\dfrac{x-4}{2+\sqrt{x}}\Rightarrow y=\dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}}=\sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
			(4 marks)

Notes

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x}\Rightarrow y=\dfrac{t^2-4}{t+2}\Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be seen at least once

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{dy}{dx}=2\times4-2-12\times4^{-\frac{1}{2}}=\ldots$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{d^2y}{dx^2}=2+6\times4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx}=Ax+B+Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x=4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4}=\ldots$

Alternatively substitutes $x=4$ into an equation resulting from $\frac{dy}{dx}=0$ Eg. $\frac{36}{x}=(x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe

Alt Shows that $x=4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x=4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x=4\Rightarrow\frac{d^2y}{dx^2}>0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x=4$ or calculates the value of y either side of $x=4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x=4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question	Scheme	Marks	AOs
5	$\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \dfrac{dy}{d\theta}=\dfrac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow \dfrac{dy}{d\theta}=\dfrac{3}{2+2\sin 2\theta}=\dfrac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\dfrac{dy}{d\theta}$ (condone it being stated as $\dfrac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\dfrac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\dfrac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\dfrac{dy}{d\theta}=\dfrac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\dfrac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\dfrac{dy}{d\theta}$ condoning it appearing as $\dfrac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \dfrac{dy}{d\theta}=\dfrac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\dfrac{dy}{d\theta}=\dfrac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\dfrac{3}{2}$ stated but allow for example $\dfrac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question	Scheme	Marks	AOs
11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) Way 2	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\mathrm{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1 A1ft	2.1 1.1b
	Correct $\mathrm{f}'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\mathrm{f}'(x)=-(+\text{ ve})-(+\text{ ve})<0$, so $\mathrm{f}(x)$ is a decreasing function	A1	2.4
		(3)	
			(7 marks)

Notes for Question 11

(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for *B* and *C*. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for *B* and *C*
B1:	$A=3$
M1:	Attempts to find the value of either *B* or *C* from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: \ -6=-2A; \quad x: \ 11=7A-2B+C; \quad \text{constant}: \ 1=-3A+B-3C$ **Way 1:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: \ 5=-\frac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: \ -10=-2B+C; \quad \text{constant}: \ 10=B-3C$ **Way 2:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\frac{1}{2}: \ 5=-\frac{5}{2}C \Rightarrow C=-2$

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\dfrac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\dfrac{A}{(x+1)^3}$	M1	2.1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
		(5 marks)	

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\dfrac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

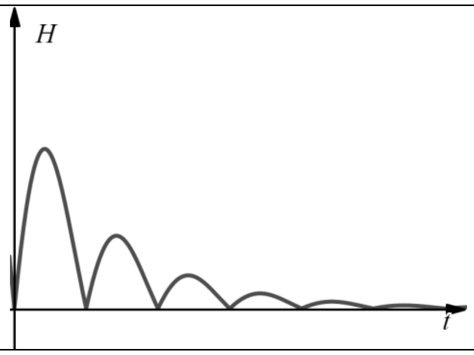
Condone slips but expect $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{\left((x+1)^2\right)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Question	Scheme	Marks	AOs
12 (a)	$f(x)=10e^{-0.25x}\sin x$		
	$\Rightarrow f'(x)=-2.5e^{-0.25x}\sin x+10e^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$f'(x)=0\Rightarrow -2.5\cancel{e^{-0.25x}}\sin x+10\cancel{e^{-0.25x}}\cos x=0$	M1	2.1
	$\frac{\sin x}{\cos x}=\frac{10}{2.5}\Rightarrow \tan x=4*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x=4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47)=\left\|10e^{-0.25\times4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .
So for example score expressions of the form $\pm ...e^{-0.25x}\sin x \pm ...e^{-0.25x}\cos x$ M1
Sight of $vdu-udv$ however is M0

A1: $f'(x)=-2.5e^{-0.25x}\sin x+10e^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $f'(x)=0$, factorising/ cancelling out the $e^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$
Do not allow candidates to substitute $x=\arctan 4$ into $f'(x)$ to score this mark.

A1*: Shows the steps $\frac{\sin x}{\cos x}=\frac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x=4*$. $\frac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.
Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.
The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

Question	Scheme	Marks	AOs
9(a)	$\mathrm{f}(x)=4\left(x^2-2\right)\mathrm{e}^{-2x}$		
	Differentiates to $\mathrm{e}^{-2x}\times 8x+4\left(x^2-2\right)\times -2\mathrm{e}^{-2x}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=8\mathrm{e}^{-2x}\left\{x-\left(x^2-2\right)\right\}=8\left(2+x-x^2\right)\mathrm{e}^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $\mathrm{f}'(x)=0$ $\quad x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $\left(-1,-4\mathrm{e}^2\right)$ and $\left(2,8\mathrm{e}^{-4}\right)$	A1	1.1b
		(3)	
(c)	(i) Range $\left[-8\mathrm{e}^2,\infty\right)$ o.e. such as $\mathrm{g}(x)\geqslant -8\mathrm{e}^2$	B1ft	2.5
	(ii) For • Either attempting to find $2\mathrm{f}(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times "8\mathrm{e}^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $\left[-19,16\mathrm{e}^{-4}-3\right]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $\mathrm{e}^{-2x}\to k\mathrm{e}^{-2x},\quad k\neq 0$

If candidate states $u=4(x^2-2), v=\mathrm{e}^{-2x}$ with $u'=....,v'=...\mathrm{e}^{-2x}$ it can be implied by their $vu'+uv'$

If they just write down an answer without working award for $\mathrm{f}'(x)=px\mathrm{e}^{-2x}\pm q\left(x^2-2\right)\mathrm{e}^{-2x}$

They may multiply out first $\mathrm{f}(x)=4x^2\mathrm{e}^{-2x}-8\mathrm{e}^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $\mathrm{f}(x)=\dfrac{u}{v}=\dfrac{4\left(x^2-2\right)}{\mathrm{e}^{2x}}$ with $v'=k\mathrm{e}^{2x}$ and $\mathrm{f}'(x)=\dfrac{vu'-uv'}{v^2}$

A1: A correct $\mathrm{f}'(x)$ which may be unsimplified.

Via the quotient rule you can award for $\mathrm{f}'(x)=\dfrac{8x\mathrm{e}^{2x}-8\left(x^2-2\right)\mathrm{e}^{2x}}{\mathrm{e}^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $\mathrm{f}'(x)$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $\mathrm{f}'(x)=8x\mathrm{e}^{-2x}-8\left(x^2-2\right)\mathrm{e}^{-2x}$

Question	Scheme	Marks	AOs
13(a)	$k = \mathrm{e}^2$ **or** $x \neq \mathrm{e}^2$	B1	2.2a
		(1)	
(b)	$\mathrm{g}'(x) = \dfrac{(\ln x - 2)\times\dfrac{3}{x} - (3\ln x - 7)\times\dfrac{1}{x}}{(\ln x - 2)^2} = \dfrac{1}{x(\ln x - 2)^2}$ or $\mathrm{g}'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(3 - (\ln(x) - 2)^{-1}\right) = (\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$ or $\mathrm{g}'(x) = (\ln x - 2)^{-1}\times\dfrac{3}{x} - (3\ln x - 7)(\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $\mathrm{g}'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < \mathrm{e}^2, \quad a > \mathrm{e}^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

Question Number	Scheme	Marks
1.	$\mathrm{f}(x)=\dfrac{2x}{x^2+3} \quad \Rightarrow \mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}=\left(\dfrac{6-2x^2}{(x^2+3)^2}\right)$	M1A1
	$\mathrm{f}'(x)>0 \Rightarrow \dfrac{6-2x^2}{(x^2+3)^2}>0$	
	Critical values $\quad 6-2x^2=0 \Rightarrow x=\pm\sqrt{3}$	M1A1
	Inside region chosen $\quad -\sqrt{3}<x<\sqrt{3}$	dM1A1
		(6 marks)

Notes

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\dfrac{2x}{x^2+3}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=2x, v=x^2+3, u'=.., v'=...$ followed by their $\dfrac{vu'-uv'}{v^2}$, then only accept answers of the form

$\dfrac{(x^2+3)A-2x\times Bx}{(x^2+3)^2}$ $A,B>0$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=2x, v=\left(x^2+3\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=2x, v=\left(x^2+3\right)^{-1}, u'=.., v'=...$ followed by their $vu'+uv'$, then only accept answers of the form

$\left(x^2+3\right)^{-1}\times A\pm 2x\times\left(x^2+3\right)^{-2}\times Bx$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $\mathrm{f}'(x)$

Accept versions of $\mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}$ for the quotient rule or

Versions of $\mathrm{f}'(x)=\left(x^2+3\right)^{-1}\times 2-2x\times\left(x^2+3\right)^{-2}\times 2x$ for use of the product rule.

M1 Setting their numerator of $\mathrm{f}'(x)=0$ or >0, and proceeding to find two critical values.

A1 Both critical values $\pm\sqrt{3}$ are found. Accept for this mark expressions like $x>\pm\sqrt{3}$ and ± 1.73

dM1 For choosing the inside region of their critical values.

The inequality (if seen) must have been of the correct form. Either $Ax^2....-B<0$, $C......-Dx^2>0$ or $x^2<C$ .It is dependent upon having set the numerator >0 or $=0$.

A1 Correct solution only. $-\sqrt{3}<x<\sqrt{3}$. Accept $\left(-\sqrt{3},\sqrt{3}\right)$ $x<\sqrt{3}$ and $x>-\sqrt{3}$

Do not accept $x<\sqrt{3}$ or $x>-\sqrt{3}$ or $-1.73<x<1.73$.

Do not accept a correct answer coming from an incorrect inequality. This would be dM0A0.

Condone a solution $x^2<3 \Rightarrow x<\pm\sqrt{3} \Rightarrow -\sqrt{3}<x<\sqrt{3}$

Do not accept a solution without seeing a correct $\mathrm{f}'(x)$ first. Note that this is a demand of the question.

Question Number	Scheme		Marks
3.	Apply quotient rule : $\left\{\begin{array}{ll} u = \cos 2\theta & v = 1+\sin 2\theta \\ \frac{du}{d\theta} = -2\sin 2\theta & \frac{dv}{d\theta} = 2\cos 2\theta \end{array}\right\}$	Or apply product rule to $y = \cos 2\theta(1+\sin 2\theta)^{-1}$ $\left\{\begin{array}{ll} u = \cos 2\theta & v = (1+\sin 2\theta)^{-1} \\ \frac{du}{d\theta} = -2\sin 2\theta & \frac{dv}{d\theta} = -2\cos 2\theta(1+\sin 2\theta)^{-2} \end{array}\right\}$	
	$\frac{dy}{d\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$-2(1+\sin 2\theta)^{-1}\sin 2\theta - 2\cos^2 2\theta(1+\sin 2\theta)^{-2}$	M1 A1
	$= \frac{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta$	
	$= \frac{-2\sin 2\theta - 2}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\}$	M1
	$= \frac{-2(1+\sin 2\theta)}{(1+\sin 2\theta)^2} = \frac{-2}{1+\sin 2\theta}$		A1 cso
			[4] **4**

Notes

M1: **Applies the Quotient rule**, a form of which appears in the formula book, to $\frac{\cos 2\theta}{1+\sin 2\theta}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that terms are written out

$u = \cos 2\theta, v = 1+\sin 2\theta, u' = .., v' = ...$ followed by their $\frac{vu'-uv'}{v^2}$, then only accept answers of the form

$\frac{(1+\sin 2\theta)A\sin 2\theta - \cos 2\theta \times (B\cos 2\theta)}{(1+\sin 2\theta)^2}$ where A and B are constant (could be 1) Condone "invisible" brackets for the M mark. If double angle formulae are used give marks for correct work.

Alternatively applies the product rule with $u = \cos 2\theta, v = (1+\sin 2\theta)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that terms are written out
$u = \cos 2\theta, v = (1+\sin 2\theta)^{-1}, u' = .., v' = ...$ followed by their $vu'+uv'$,
then only accept answers of the form $(1+\sin 2\theta)^{-1} \times A\sin 2\theta \pm \cos 2\theta \times (1+\sin 2\theta)^{-2} \times B\cos 2\theta$.
Condone "invisible brackets" for the M. If double angle formulae are used give marks for correct work.

A1: Any fully correct (unsimplified) form of $\frac{dy}{d\theta}$ If double angle formulae are used give marks for correct work.

Accept versions of $\frac{dy}{d\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$ for use of the quotient rule or versions of

$\frac{dy}{d\theta} = (1+\sin 2\theta)^{-1} \times -2\sin 2\theta + \cos 2\theta \times (-1) \times (1+\sin 2\theta)^{-2} \times 2\cos 2\theta$ for use of the product rule.

M1: **Applies** $\sin^2 2\theta + \cos^2 2\theta \equiv 1$ or $-2\sin^2 2\theta - 2\cos^2 2\theta \to -2$ **correctly to eliminate squared trig. terms from the numerator** to obtain an expression of the form $k\sin 2\theta + \lambda$ where k and λ are constants (including 1) If double angle formulae have been used give marks only if correct work leads to answer in correct form. (If in doubt, send to review)

A1: Need to see **factorisation of numerator then** answer, which is cso
so $\frac{-2}{1+\sin 2\theta}$ or $\frac{a}{1+\sin 2\theta}$ and $a = -2$, with no previous errors

Question Number	Scheme	Marks
8.	$\frac{dV}{dt} = 250$	
	$\left\{V = \frac{4}{3}\pi r^3 \Rightarrow\right\} \frac{dV}{dr} = 4\pi r^2$	B1
	$V = 12\,000 \Rightarrow 12\,000 = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{9000}{\pi}} \; (= 14.202480...)$	B1
	$\frac{dr}{dt} \left\{= \frac{dr}{dV} \times \frac{dV}{dt}\right\} = \frac{1}{4\pi r^2} \times 250$	M1
	When $r = \sqrt[3]{\frac{9000}{\pi}}$, $\frac{dr}{dt} = \frac{250}{4\pi\left(\sqrt[3]{\frac{9000}{\pi}}\right)^2}$	dM1
	So, $\frac{dr}{dt} = 0.0986283... \left(\text{cm s}^{-1}\right)$ awrt 0.099	A1
		[5] **5**

Notes

B1: $\frac{dV}{dr} = 4\pi r^2$. This may be stated or used and need not be simplified

Applies $12\,000 = \frac{4}{3}\pi r^3$ and rearranges to find r using division then cube root with accurate algebra

May state $r = \sqrt[3]{\frac{3V}{4\pi}}$ then substitute $V = 12000$ later which is equivalent. r does not need to be evaluated.

M1: **Uses** chain rule correctly so $\frac{1}{\left(\text{their } \frac{dV}{dr}\right)} \times 250$

dM1: Substitutes their r **correctly** into their equation for $\frac{dr}{dt}$ This depends on the previous method mark

A1: awrt 0.099 (Units may be ignored) If this answer is seen, then award A1 and isw.
Premature approximation usually results in all marks being earned prior to this one.

Question Number	Scheme	Marks
10. (a)	$A=B \Rightarrow \sin 2A = \underline{\underline{\sin(A+A)}} = \underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$	M1
	Hence, $\underline{\underline{\sin 2A = 2\sin A\cos A}}$ (as required) *	A1 * **[2]**
(b)	Way 1A: $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] \Rightarrow\right\} \dfrac{dy}{dx} = \dfrac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$	M1 A1
	$= \dfrac{1}{2\tan\left(\frac{1}{2}x\right)\cos^2\left(\frac{1}{2}x\right)} = \dfrac{1}{\dfrac{2\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}\cdot\dfrac{\cos^2\left(\frac{1}{2}x\right)}{1}}$	dM1
	$= \dfrac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)} = \dfrac{1}{\sin x} = \operatorname{cosec} x$ *	A1 *
	Way 1B $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$	M1 A1
	$= \dfrac{1+\tan^2\left(\frac{1}{2}x\right)}{2\tan\left(\frac{1}{2}x\right)} = \; = \dfrac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$	dM1
	$= \dfrac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)} = \dfrac{1}{\sin x} = \operatorname{cosec} x$ *	A1 * **[4]**
	Way 2: $\left\{y = \ln\left[\sin\left(\tfrac{1}{2}x\right)\right] - \ln\left[\cos\left(\tfrac{1}{2}x\right)\right] \Rightarrow\right\} \dfrac{dy}{dx} = \dfrac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \dfrac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$	M1 A1
	$= \dfrac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}; = \dfrac{1}{\sin x} = \operatorname{cosec} x$	M1;A1 [4]
	Way3: quotes $\int \operatorname{cosec} x\, dx = \ln(\tan\left(\tfrac{1}{2}x\right))$	M1 A1
	(As differentiation is reverse of integration) $\dfrac{d}{dx}\left[\tan\left(\tfrac{1}{2}x\right)\right] = \operatorname{cosec} x$	M1 A1 [4]
(c)	$\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\} \dfrac{dy}{dx} = \operatorname{cosec} x - 3\cos x$	B1
	$\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}$ $\operatorname{cosec} x - 3\cos x = 0 \Rightarrow \dfrac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow 1 = 3\sin x\cos x \Rightarrow 1 = \tfrac{3}{2}(2\sin x\cos x)$ so $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
	So $\sin 2x = \dfrac{2}{3}$	A1
	$\{\, 2x = \{0.729727..., 2.411864...\} \,\}$ So $x = \{0.364863..., 1.205932...\}$	A1 A1 **[6]** **12**
Way2 10 (c)	Method (Squaring Method) $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\} \dfrac{dy}{dx} = \operatorname{cosec} x - 3\cos x$	**B1**
	$\left\{\dfrac{dy}{dx} = 0 \Rightarrow\right\}$ $\operatorname{cosec} x - 3\cos x = 0 \Rightarrow \dfrac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow \dfrac{1}{1-\cos^2 x} = 9\cos^2 x$ so $9\cos^4 x - 9\cos^2 x + 1 = 0$ or $9\sin^4 x - 9\sin^2 x + 1 = 0$	M1
	So $\cos^2 x = 0.873$ *or* 0.127 or $\sin^2 x = 0.873$ *or* 0.127	A1
	So $x = \{0.364863..., 1.205932...\}$	A1 A1 **[6]**

Way 3 10c)	"t" method $\left\{ y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow \right\} \dfrac{dy}{dx} = \text{cosec}\, x - 3\cos x$	B1
	$\left\{ \dfrac{dy}{dx} = 0 \Rightarrow \right\}$ $\text{cosec}\, x - 3\cos x = 0 \Rightarrow \dfrac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow \dfrac{1+t^2}{2t} - 3\dfrac{1-t^2}{1+t^2} = 0$ so $t^4 + 6t^3 + 2t^2 - 6t + 1 = 0$	M1
	$t = 0.1845$ or 0.6885	A1
	So $x = \{0.364863..., 1.205932...\}$	A1 A1 **[6]**

Notes

(a) M1: This mark is for the underlined equation in either form
$\underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$

A1: For this mark need to see :
sin2*A* at the start of the proof , or as part of a conclusion
sin(*A* + *A*) = at the start
= $\underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$
= 2sin*A*cos*A* at the end

(b)M1: For expression of the form $\dfrac{\pm k\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$, where k is constant (could even be 1)

A1: Correct differentiation so $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$

Way 1A:

dM1: Use both $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and $\sec^2\left(\frac{1}{2}x\right) = \dfrac{1}{\cos^2\left(\frac{1}{2}x\right)}$ in their differentiated expression. This may be implied.

This depends on **the** previous Method mark.

A1*: Simplify the fraction, use double angle formula, see $\dfrac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 1B

dM1: Use both $\sec^2\left(\frac{1}{2}x\right) = 1 + \tan^2\left(\frac{1}{2}x\right)$ and $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

A1*: Simplify the fraction, use double angle formula, see $\dfrac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 2:

M1:Split into $\left\{ y = \ln\left[\sin\left(\frac{1}{2}x\right)\right] - \ln\left[\cos\left(\frac{1}{2}x\right)\right] \Rightarrow \right\}$ then differentiate to give $\dfrac{dy}{dx} = \dfrac{k\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \dfrac{c\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

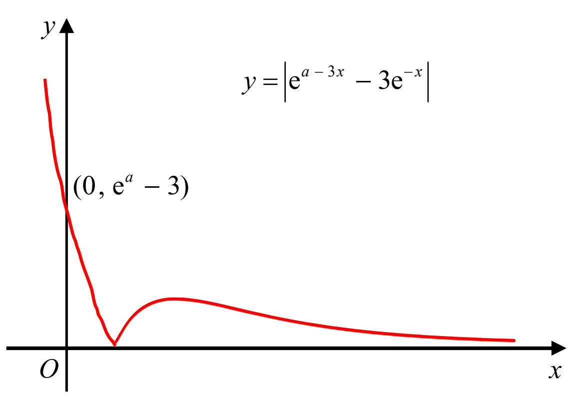
A1: Correct answer $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \dfrac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

M1: Obtain $= \dfrac{\cos^2\left(\frac{1}{2}x\right) + \sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$ A1*: As before

Way 3:

Alternative method: This is rare, but is acceptable. Must be completely correct.

Quotes $\int \text{cos}\,\text{ec}x\,dx = \ln(\tan\left(\frac{1}{2}x\right))$ and follows this by $\dfrac{d}{dx}\left[\tan\left(\frac{1}{2}x\right)\right] = \text{cosec}x$ gets 4/4

Question Number	Scheme	Marks
11.		
(a)	$\frac{dy}{dx} = -3e^{a-3x} + 3e^{-x}$	M1 A1
	$-3e^{a-3x} + 3e^{-x} = 0 \Rightarrow e^{-x} = e^{a-3x} \Rightarrow -x = a - 3x \Rightarrow x =$	M1
	$x = \frac{1}{2}a$	A1
	So, $y_P = e^{a-3\left(\frac{a}{2}\right)} - 3e^{-\left(\frac{a}{2}\right)}; = -2e^{-\frac{a}{2}}$	ddM1; A1
		[6]
	Mark parts (b) and (c) together.	
(b)	Method 1: $0 = e^{a-3x} - 3e^{-x} \Rightarrow e^{a-"2"x} = 3$; Method 2: $0 = e^{a-3x} - 3e^{-x} \Rightarrow e^{"2"x} = \frac{e^{a}}{3}$; Method 3: $0 = e^{a-3x} - 3e^{-x} \Rightarrow 3e^{"2"x} = e^{a}$	M1
	Method 1: $\Rightarrow a - "2"x = \ln 3$; Method 2: $"2"x = a - \ln 3$; Method 3: $\ln 3 + "2"x = a$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ or equivalent e.g. $\frac{1}{2}\ln\left(\frac{e^{a}}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^{a}}\right)}$ etc	A1
		[3]
	Method 4	
	$0 = e^{a-3x} - 3e^{-x} \Rightarrow e^{a-3x} = 3e^{-x}$ and so $a - 3x = \ln 3 - x$	M1
	$"2"x = a - \ln 3$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ o.e. e.g. $\frac{1}{2}\ln\left(\frac{e^{a}}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^{a}}\right)}$ etc	A1 **[3]**
(c)		
	Shape	B1
	Cusp and behaviour for large x	B1
	$(0, e^{a} - 3)$	B1
		[3]
		12

Question Number	Scheme	Marks
1	$y=7$ at point P	B1
	$y=\dfrac{3x-2}{(x-2)^2}\Rightarrow\dfrac{dy}{dx}=\dfrac{(x-2)^2\times3-(3x-2)\times2(x-2)}{(x-2)^4}$	M1A1
	Sub $x=3$ into $\dfrac{dy}{dx}=(-11)$	M1
	$\dfrac{1}{11}=\dfrac{y-7}{x-3}\Rightarrow x-11y+74=0$ cso	M1A1 cso
		(6 marks)

B1 For seeing $y=7$ when $x=3$. This may be awarded if embedded within an equation.

M1 Application of Quotient rule. If the rule is quoted it must be correct.

It may be implied by their $u=3x-2, u'=.., v=(x-2)^2, v'=..$ followed by their $\dfrac{vu'-uv'}{v^2}$

If the rule is neither stated nor implied only accept expressions of the form

$\dfrac{(x-2)^2\times A-(3x-2)\times B(x-2)}{\left((x-2)^2\right)^2}$ $A,B>0$ condoning missing brackets

Alternatively applies the Product rule to $(3x-2)(x-2)^{-2}$ If the rule is quoted it must be correct.

It may be implied by their u or $v=3x-2, u', v$ or $u=(x-2)^{-2}, v'$ followed by their $vu'+uv'$

If the rule is neither stated nor implied only accept expressions of the form $A(x-2)^{-2}\pm B(3x-2)(x-2)^{-3}$

If they use partial fractions expect to see $y=\dfrac{3x-2}{(x-2)^2}\Rightarrow y=\dfrac{P}{(x-2)}+\dfrac{Q}{(x-2)^2}(P=3,Q=4)\Rightarrow\dfrac{dy}{dx}=\pm\dfrac{R}{(x-2)^2}\pm\dfrac{S}{(x-2)^3}$

You may also see implicit differentiation etc where the scheme is easily applied.

A1 A correct (unsimplified) form of the derivative.

Accept from the quotient rule versions equivalent to $\dfrac{dy}{dx}=\dfrac{(x-2)^2\times3-(3x-2)\times2(x-2)}{\left((x-2)^2\right)^2}$

Accept from the product rule versions equivalent to $\dfrac{dy}{dx}=3(x-2)^{-2}-2(3x-2)(x-2)^{-3}$

Accept from partial fractions $\dfrac{dy}{dx}=-3(x-2)^{-2}-8(x-2)^{-3}$

or $(x-2)^2\dfrac{dy}{dx}+y\times2(x-2)=3$ from implicit differentiation

FYI: Correct simplified expressions are $\dfrac{dy}{dx}=\dfrac{-3x^2+4x+4}{(x-2)^4}$ or $\dfrac{-3x-2}{(x-2)^3}$

M1 Sub $x=3$ into what they believe is their derivative to find a numerical value of $\dfrac{dy}{dx}$.

M1 Uses $x=3$ and their numerical value of y with their numerical $-\dfrac{dx}{dy}$ at $x=3$ to form an equation of a normal. If the form $y=mx+c$ is used then it must be a full method reaching a value for c.

A1 Correct solution only Accept $\pm A(x-11y+74)=0$ where $A\in\mathbb{N}$. from correct working.

Watch for correct answers coming from incorrect versions of $\dfrac{dy}{dx}$ with eg. $(x-2)^2$ on the denominator

Question Number	Scheme	Marks
6(i)	$x=\tan^2 4y \Rightarrow \frac{dx}{dy}=8\tan 4y\sec^2 4y$ oe	M1A1
	$\frac{dy}{dx}=\frac{1}{8\tan 4y\sec^2 4y}, =\frac{1}{8\tan 4y(1+\tan^2 4y)}=\frac{1}{8\sqrt{x}(1+x)}=\frac{1}{8\left(x^{0.5}+x^{1.5}\right)}$	M1,M1A1
		(5)
(ii)	$\frac{dV}{dt}=2,\quad V=x^3 \Rightarrow \frac{dV}{dx}=3x^2$	B1,B1
	Uses $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$	M1
	$\left.\frac{dx}{dt}\right\rvert_{x=4}=\frac{2}{3x^2}=\frac{1}{24}\left(\text{cm s}^{-1}\right)$	M1A1
		(5)
		(10 marks)

(i)

M1 Differentiates $\tan^2 4y$ to get an expression equivalent to the form $C\tan 4y\sec^2 4y$

You may see $\tan 4y\times A\sec^2 4y+\tan 4y\times B\sec^2 4y$ from the product rule or versions appearing from $\sqrt{x}=\tan 4y \Rightarrow Ax^{-0.5}\times ...=B\sec^2 4y$ or

$Ax^{-0.5}=B\sec^2 4y\times ...$ $x=\frac{\sin^2 4y}{\cos^2 4y}\Rightarrow\frac{dx}{dy}=\frac{\cos^2 4y\times A\sin 4y\cos 4y-\sin^2 4y\times B\cos 4y\sin 4y}{\left(\cos^2 4y\right)^2}$

from the quotient rule

A1 Any fully correct answer, or equivalent, including the left hand side. $\frac{dx}{dy}=2\tan 4y\times 4\sec^2 4y$

Also accept the equivalent by implicit differentiation $1=8\tan 4y\sec^2 4y\frac{dy}{dx}$

M1 Uses $\frac{dy}{dx}=1\Big/\frac{dx}{dy}$ Follow through on their $\frac{dx}{dy}$.

Condone issues with reciprocating the '8' but not the trigonometrical terms.

If implicit differentiation is used it is scored for writing $\frac{dy}{dx}$ as the subject.

M1 Uses $\sec^2 4y=1+\tan^2 4y$ where $x=\tan^2 4y$ to get their expression for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of just x.

If they use other functions it is for using $\sin^2 4y=\frac{x}{1+x}$ and $\cos^2 4y=\frac{1}{1+x}$ where $x=\tan^2 4y$ to get their expression for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of just x.

A1 Correct answer and solution. Accept $\frac{1}{8\left(x^{0.5}+x^{1.5}\right)},\frac{1}{8\left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)}$ or A=8, p =0.5and q =1.5

Candidates do not have to explicitly state the values of A, p and q. Remember to isw after the sight of an acceptable answer.

Question Number	Scheme	Marks
8 (a)	$1000 < V \le 23000$	B1,B1
		(2)
(b)	$\dfrac{dV}{dt} = 18000 \times -0.2e^{-0.2t} + 4000 \times -0.1e^{-0.1t}$	M1
	$\left.\dfrac{dV}{dt}\right\|_{t=10} = 18000 \times -0.2e^{-2} + 4000 \times -0.1e^{-1} = \text{awrt}(-)634$	M1A1
		(3)
(c)	$15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$	
	$0 = 9e^{-0.2t} + 2e^{-0.1t} - 7$	
	$0 = (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$	M1A1
	$9e^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\dfrac{9}{7}\right)$ oe	dM1A1
		(4)
		(9 marks)

(a)

B1 Accept either boundary: $V < 23000$ or $V \le 23000$ or $V_{\max}$ 23000 for the upper boundary and $V > 1000$ or $V \ge 1000$ or $V_{\min}$ 1000 for the lower boundary. Answers like $V \ge 23000$ are B0

B1 Completely correct solution.
Accept $1000 < V \le 23000$, $1000 < \mathit{Range}$ or $y \le 23000$, $(1000, 23000]$, $V > 1000$ *and* $V \le 23000$

(b)

M1 Score for a $\dfrac{dV}{dt} = Ae^{-0.2t} + Be^{-0.1t}$, where $A \ne 18000,\ B \ne 4000$

M1 Sub $t = 10$ into a $\dfrac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t}$ where $A \ne 18000,\ B \ne 4000$

Condone substitution of $t = 10$ into a $\dfrac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t} + 1000 \quad A \ne 18000,\ B \ne 4000$

A1 Correct solution and answer only. Accept ± 634 following correct $\dfrac{dV}{dt} = -3600e^{-0.2t} - 400e^{-0.1t}$

Watch for students who sub $t = 10$ into their V first and then differentiate. This is 0,0,0.

Watch for students who achieve +634 following $\dfrac{dV}{dt} = 3600e^{-0.2t} + 400e^{-0.1t}$. This is 1,1,0

A correct answer with no working can score all marks.

(c)

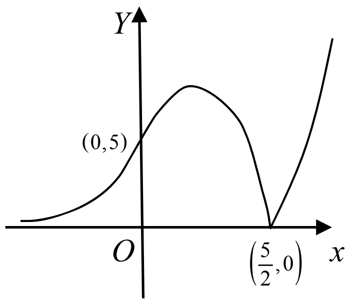
M1 Setting up 3TQ in $e^{\pm 0.1t}$ AND correct attempt to factorise or solve by the formula.
For this to be scored the $e^{\pm 0.2t}$ term must be the x^2 term.

A1 Correct factors $(9e^{-0.1t} - 7)(e^{-0.1t} + 1)$ or $(7e^{0.1t} - 9)(e^{0.1t} + 1)$ or a root $e^{-0.1t} = \dfrac{7}{9}$

dM1 Dependent upon the previous M1.
This is scored for setting the $ae^{\pm 0.1t} - b = 0$ and proceeding using correct ln work to $t = \ldots$

A1 $t = 10\ln\left(\dfrac{9}{7}\right)$. Accept alternatives such as $t = \dfrac{1}{0.1}\ln\left(\dfrac{9}{7}\right), \dfrac{1}{-0.1}\ln\left(\dfrac{7}{9}\right), -10\ln\left(\dfrac{7}{9}\right)$

If any extra solutions are given withhold this mark.

Question Number	Scheme	Marks
3 (a)	$\mathrm{f}'(x)=\mathrm{e}^{x}\times 2+(2x-5)\mathrm{e}^{x}$	M1A1
	$\mathrm{f}'(x)=0\Rightarrow(2x-3)\mathrm{e}^{x}=0\Rightarrow x=\frac{3}{2}$	M1A1
	{Coordinates of $A=\left(\frac{3}{2},-2\mathrm{e}^{\frac{3}{2}}\right)$} obtains $y=-2\mathrm{e}^{\frac{3}{2}}$	A1ft
		(5)
(b)	$-2\mathrm{e}^{\frac{3}{2}}<k<0$	M1A1
		(2)
(c)	 Shape including cusp	B1
	$\left(\frac{5}{2},0\right)$ only	B1
	$(0,5)$ only	B1
		(3)
		(10 marks)

Question Number	Scheme	Notes	Marks
4 (a)	$\begin{array}{r} x^2+5 \\ x^2+x-12\overline{\big)\,x^4+x^3-7x^2+8x-48} \\ \underline{x^4+x^3-12x^2\quad\quad\quad\quad} \\ 5x^2+8x-48 \\ \underline{5x^2+5x-60} \\ 3x+12 \end{array}$ M1: Divides $x^4+x^3-7x^2+8x-48$ by x^2+x-12 to get a quadratic quotient and a remainder of the form $\alpha x+\beta$ where α and β are not both zero A1: Correct quotient and remainder		M1A1
	$\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+5+\dfrac{3(x+4) \text{ or } 3x+12}{(x+4)(x-3)}$ Writes their answer as $\dfrac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv \text{Their Quotient}+\dfrac{\text{Their Remainder}}{(x+4)(x-3)}$		M1
	$\equiv x^2+5+\dfrac{3}{(x-3)}$ or states $A=5$, $B=3$		A1
			(4)

(b)	$g'(x)=2x-\dfrac{3}{(x-3)^2}$	M1: $x^2+A+\dfrac{B}{x-3}\to 2x\pm\dfrac{B}{(x-3)^2}$ A1: $x^2+A+\dfrac{B}{x-3}\to 2x-\dfrac{B}{(x-3)^2}$ Follow through their B or the letter B or a made up B.	M1A1ft
	Special Case: If they write g(x) as $x^2+5+\dfrac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x$ + the quotient rule on $\dfrac{3x+12}{(x-3)}$ then the M mark is available but **not** the A1ft. It must be the correct quotient rule and the numerator must be a linear expression.		
	$g'(4)=2\times4-\dfrac{3}{(4-3)^2}(=5)$	Substitutes $x=4$ into their derivative	M1
	Uses $m=g'(4)=(5)$ with $(4, g(4))=(4,24)$ to form eqn of tangent		
	$y-24=5(x-4)$	Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4, g(4))$	M1
	$y=5x+4$	Cso. This mark may be withheld for an incorrect "A" earlier or any incorrect work leading to a correct gradient.	A1
			(5)
			(9 marks)
	Alternative to part (b) for first 3 marks		
	$g'(x)=\dfrac{(x^2+x-12)(4x^3+3x^2-14x+8)-(x^4+x^3-7x^2+8x-48)(2x+1)}{(x^2+x-12)^2}$ M1: Correct use of the quotient rule – there must be evidence of the application of $\dfrac{vu'-uv'}{v^2}$ or this formula quoted and attempted. A1: Correct derivative		M1A1
	$g'(4)=\dfrac{8\times256-192\times9}{8^2}(=5)$	Substitutes $x=4$ into their derivative	M1

Qu	Scheme	Marks
6.(i)	$\frac{dy}{dx} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1 **(2)**
(ii)	$\frac{dy}{dx} = \frac{(\sin x+\cos x)1 - x(\cos x - \sin x)}{(\sin x+\cos x)^2}$	M1
	$\frac{dy}{dx} = \frac{(\sin x+\cos x)1 - x(\cos x - \sin x)}{(\sin^2 x+\cos^2 x)+(2\sin x\cos x)} = \frac{(\sin x+\cos x)1 - x(\cos x - \sin x)}{1+\sin 2x}$	B1, B1
	$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1+\sin 2x}$ *	A1 * **(4)**
		(6 marks)

(i)

M1: Applies the Product rule to $y = 5x^2 \ln 3x$

Expect $\frac{dy}{dx} = Ax + Bx\ln(3x)$ for this mark (A, B positive constant)

A1: cao- need not be simplified

(ii)

M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$

Expect $\frac{dy}{dx} = \frac{(\sin x+\cos x)1 - x(\pm\cos x \pm \sin x)}{(\sin x+\cos x)^2}$ for M1

Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator

Eg $\sin^2 x + \cos^2 x$

B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + ...$ and $(\sin^2 x + \cos^2 x) \to 1$

B1: Denominator should be expanded to $... + k\sin x\cos x$ and $(k\sin x\cos x) \to \frac{k}{2}\sin 2x$.

For example sight of $(\sin x+\cos x)^2 = 1 + 2\sin x\cos x = 1+\sin 2x$ without the intermediate line on the Denominator is B0 B1

A1: cso – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos$ $\sin^2 x \leftrightarrow \sin x^2$

If the only error is the omission of $(\sin^2 x + \cos^2 x) \to 1$ then this final A1* can be awarded.

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Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct

Product rule $\frac{dy}{dx} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2}(\pm\cos x \pm \sin x)$

Implicit differentiation $(\sin x + \cos x)y = x \Rightarrow (\sin x + \cos x)\frac{dy}{dx} + y(\pm\cos x \pm \sin x) = 1$

To score the B's under this method there must have been an attempt to write $\frac{dy}{dx}$ as a single fraction