**5.** The curve $C$ has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

**(3)**

(b) (i) Verify that $C$ has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

**(4)**

**14.** Given that

$$y = \frac{x - 4}{2 + \sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where $A$ is a constant to be found.

**(4)**

**2.** A curve $C$ has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

**(3)**

(b) Verify that $C$ has a stationary point when $x = 4$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

**5.** Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where $A$ is a rational constant to be found.

**(5)**

**11.**

$$\frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants $A$, $B$ and $C$.

**(4)**

$$\mathrm{f}(x) = \frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $\mathrm{f}(x)$ is a decreasing function.

**(3)**

**3.** $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where $A$ and $n$ are constants to be found.

**(4)**

(b) Hence deduce the range of values for $x$ for which $\dfrac{dy}{dx} < 0$

**(1)**

**12.** $$f(x) = 10e^{-0.25x} \sin x, \qquad x \geqslant 0$$

(a) Show that the $x$ coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

**(4)**

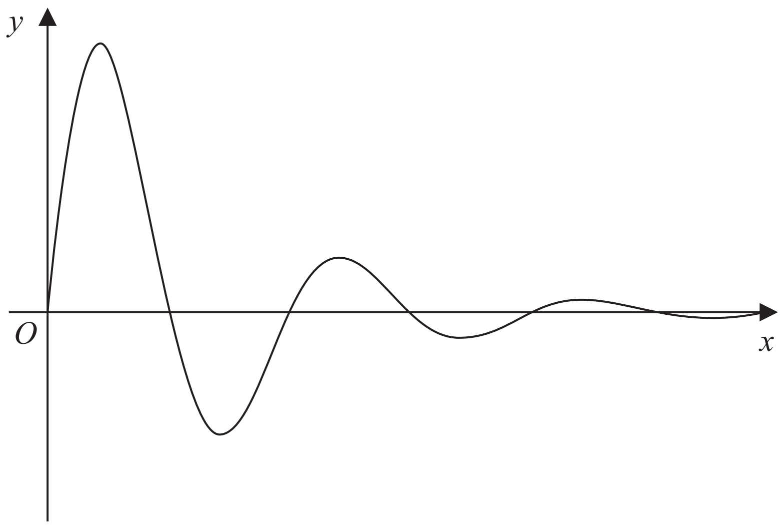


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of $H$ against $t$ where

$$H(t) = \left|10e^{-0.25t} \sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

**(2)**

The function $H(t)$ is used to model the height, in metres, of a ball above the ground $t$ seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

**(3)**

(d) Explain why this model should not be used to predict the time of each bounce.

**(1)**

**9.**

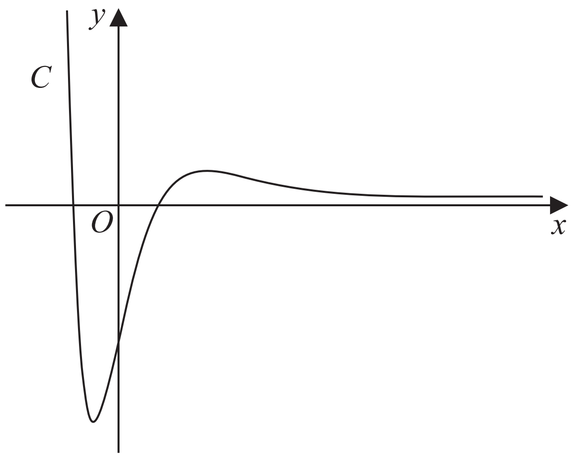


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

**(3)**

(b) Hence find, in simplest form, the exact coordinates of the stationary points of $C$.

**(3)**

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

**(3)**

**13.** The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where $k$ is a constant.

(a) Deduce the value of $k$.

**(1)**

(b) Prove that

$$g'(x) > 0$$

for all values of $x$ in the domain of g.

**(3)**

(c) Find the range of values of $a$ for which

$$g(a) > 0$$

**(2)**

Leave blank

**1.** $$\mathrm{f}(x) = \frac{2x}{x^2 + 3}, \qquad x \in \mathbb{R}$$

Find the set of values of $x$ for which $\mathrm{f}'(x) > 0$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(6)**

Leave blank

**3.** Given that

$$y = \frac{\cos 2\theta}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{a}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where $a$ is a constant to be determined.

**(4)**

Leave blank

**8.** The volume $V$ of a spherical balloon is increasing at a constant rate of 250 $\text{cm}^3\,\text{s}^{-1}$.
Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the volume of the balloon is 12 000 $\text{cm}^3$.
Give your answer to 2 significant figures.

**(5)**

[*You may assume that the volume V of a sphere of radius r is given by the formula* $V = \frac{4}{3}\pi r^3$.]



P 4 4 9 6 9 A 0 2 0 4 8

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**10.** (a) Use the identity for $\sin(A + B)$ to prove that

$$\sin 2A \equiv 2\sin A \cos A$$

**(2)**

(b) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan\left(\tfrac{1}{2}x\right)\right)\right] = \operatorname{cosec} x$$

**(4)**

A curve $C$ has the equation

$$y = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right) - 3\sin x, \qquad 0 < x < \pi$$

(c) Find the $x$ coordinates of the points on $C$ where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Give your answers to 3 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(6)**

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**11.**

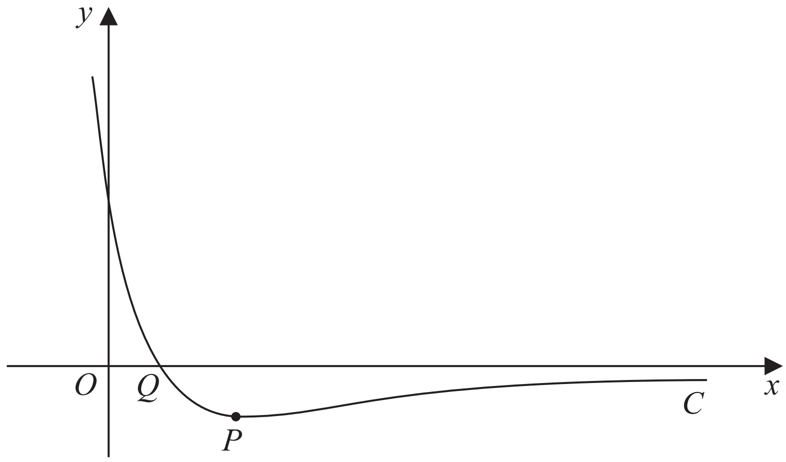


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation

$$y = e^{a-3x} - 3e^{-x}, \quad x \in \mathbb{R}$$

where $a$ is a constant and $a > \ln 4$

The curve $C$ has a turning point $P$ and crosses the $x$-axis at the point $Q$ as shown in Figure 2.

(a) Find, in terms of $a$, the coordinates of the point $P$.
**(6)**

(b) Find, in terms of $a$, the $x$ coordinate of the point $Q$.
**(3)**

(c) Sketch the curve with equation

$$y = \left| e^{a-3x} - 3e^{-x} \right|, \quad x \in \mathbb{R}, \ a > \ln 4$$

Show on your sketch the exact coordinates, in terms of $a$, of the points at which the curve meets or cuts the coordinate axes.
**(3)**

Leave blank

**1.** The curve $C$ has equation

$$y = \frac{3x - 2}{(x - 2)^2}, \quad x \neq 2$$

The point $P$ on $C$ has $x$ coordinate 3

Find an equation of the normal to $C$ at the point $P$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(6)**

Leave blank

**6.** (i) Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{dy}{dx}$ as a function of $x$.

Write your answer in the form $\frac{1}{A(x^p + x^q)}$, where $A$, $p$ and $q$ are constants to be found.

**(5)**

(ii) The volume $V$ of a cube is increasing at a constant rate of 2 $cm^3 s^{-1}$. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 $cm^3$.

**(5)**

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**8.**

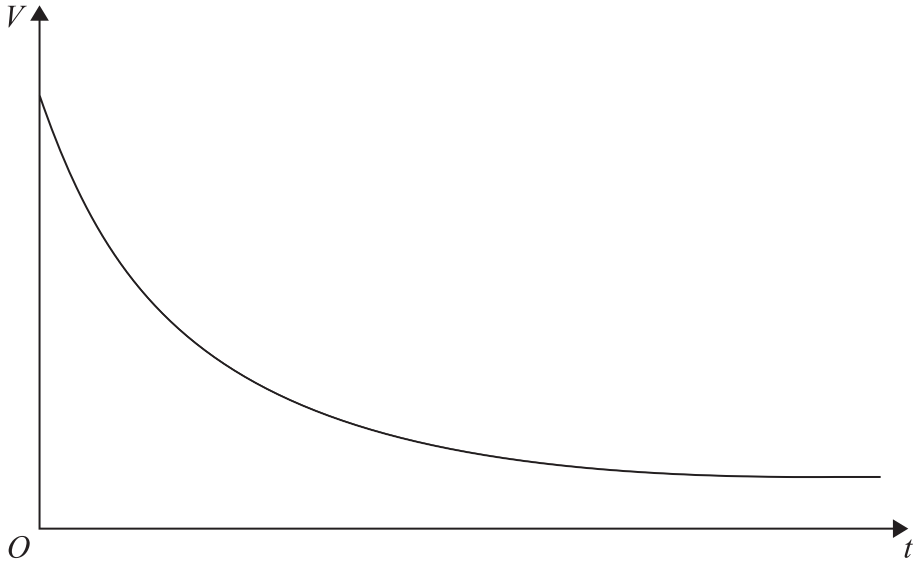


**Figure 1**

The value of Lin's car is modelled by the formula

$$V = 18\,000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is $V$ pounds when the age of the car is $t$ years.

A sketch of $t$ against $V$ is shown in Figure 1.

(a) State the range of $V$.

**(2)**

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

**(3)**

(c) Calculate the exact value of $t$ when $V = 15000$

**(4)**

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**3.**

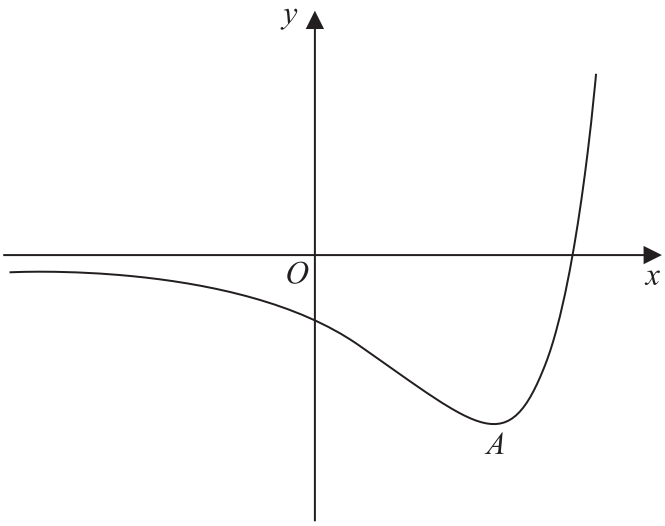


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x - 5)\mathrm{e}^x, \qquad x \in \mathbb{R}$$

The curve has a minimum turning point at $A$.

(a) Use calculus to find the exact coordinates of $A$.

**(5)**

Given that the equation $\mathrm{f}(x) = k$, where $k$ is a constant, has exactly two roots,

(b) state the range of possible values of $k$.

**(2)**

(c) Sketch the curve with equation $y = |\mathrm{f}(x)|$.

Indicate clearly on your sketch the coordinates of the points at which the curve crosses or meets the axes.

**(3)**

Leave blank

**4.** $$g(x) = \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12}, \qquad x > 3, \quad x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + A + \frac{B}{x - 3}$$

find the values of the constants $A$ and $B$.

**(4)**

(b) Hence, or otherwise, find the equation of the tangent to the curve with equation $y = g(x)$ at the point where $x = 4$. Give your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be determined.

**(5)**

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**6.** (i) Differentiate $y = 5x^2 \ln 3x, \quad x > 0$

**(2)**

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

**(4)**