Question	Scheme	Marks	AOs
13 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \pi rh = \frac{6}{r} - \frac{2}{3}\pi r^2,\ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi rh + 2\pi r^2\ \{\Rightarrow A = 3\pi r^2 + 2\pi rh\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2\ \Rightarrow\ A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\}\ \frac{dA}{dr} = -12r^{-2} + \frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{dA}{dr} = 0 \Rightarrow\right\}\ -\frac{12}{r^2} + \frac{10}{3}\pi r = 0\ \Rightarrow\ -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = \ldots\left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736\ldots \Rightarrow r =$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min} = \frac{12}{(1.046\ldots)} + \frac{5}{3}\pi(1.046\ldots)^2$	M1	3.4
	$\{A_{\min} = 17.20\ldots\ \Rightarrow\}\ A = 17\ (\text{m}^2)$ or $A = $ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = \ldots$ or $\pi h = \ldots$ or $\pi rh = \ldots$ or $rh = \ldots$, where $'\ldots' = f(r)$ into an expression for the surface area which is of the form $A = \lambda \pi r^2 + \mu \pi rh;\ \lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\ 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = \ldots$ or $S = \ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{dA}{dr} =\right\}\ -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{dA}{dr} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \to r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r =$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Question Number	Scheme	Marks
14(a)	Area of triangle = $'\frac{1}{2}ab\sin C' = \frac{1}{2}\times 2x\times 2x\times\sin 60 = \sqrt{3}x^2$	M1
	$S = 2\times\sqrt{3}x^2 + 3\times 2xl = 2x^2\sqrt{3} + 6xl$	dM1A1*
		(3)
(b)	$960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \frac{960 - 2x^2\sqrt{3}}{6x}$	M1A1
	$V = x^2\sqrt{3}\ l$	B1
	Substitute $l = \frac{960 - 2x^2\sqrt{3}}{6x}$ into $V = x^2\sqrt{3}\ l$	
	$\Rightarrow V = x^2\sqrt{3}\times\left(\frac{960 - 2x^2\sqrt{3}}{6x}\right) = 160x\sqrt{3} - x^3$	dM1A1*
		(5)
(c)	$\frac{\mathrm{d}V}{\mathrm{d}x} = 160\sqrt{3} - 3x^2 = 0$	M1A1
	$\Rightarrow x = \text{awrt } 9.6$	A1
	$\Rightarrow V = 160\times 9.611\times\sqrt{3} - 9.611^3 = 1776$	dM1 A1
		(5)
(d)	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -6x < 0 \Rightarrow \text{Maximum}$	M1A1
		(2)
		(15 marks)

Question Number	Scheme	Marks
16. (a)	$\pi R^2 H + \frac{2}{3}\pi R^3 = 800\pi$ so $H = \dfrac{800}{R^2} - \dfrac{2}{3}R$ *	M1 A1*
(b)	$A = \pi R^2 + 2\pi RH + 2\pi R^2$	B1
	$A = 3\pi R^2 + 2\pi R\left(\dfrac{800}{R^2} - \dfrac{2}{3}R\right)$ so $A = \dfrac{5\pi R^2}{3} + \dfrac{1600\pi}{R}$ *	M1 A1 *
		[5]
(c)	Find $\dfrac{dA}{dR} = \dfrac{10}{3}\pi R - \dfrac{1600\pi}{R^2}$	M1 A1
	Put derivative equal to zero and obtain $R^3 = 480$	dM1 A1
	So $R = 7.83$	**A1**
		[5]
(d)	Consider $\dfrac{d^2A}{dR^2} = \dfrac{10\pi}{3} + 3200\pi R^{-3} > 0$ so minimum	M1A1
		[2]
(e)	H = awrt 7.83	B1
		[1]
		13 marks

(a)

M1 Sets up volume equation with $800\pi = \pi R^2 H + \dfrac{2}{3}\pi R^3$ and attempts to make H the subject. Condone 800 instead of 800π. Accept for this mark lower case letters $800\pi = \pi r^2 H + \dfrac{2}{3}\pi r^3$ and a lack of consistency in lettering.

A1* This is a show that question and there must be an intermediate line showing (or implying) a division of $\pi r^2 / \pi R^2$. Lettering must be correct and consistent from the point where you see $800\pi = ...$.
Examples of an intermediate line are;

$$800\pi = \pi R^2 H + \frac{2}{3}\pi R^3 \Rightarrow H = \frac{800\pi - \frac{2}{3}\pi R^3}{\pi R^2} \Rightarrow H = \frac{800}{R^2} - \frac{2}{3}R$$

$$800\pi = \pi R^2 H + \frac{2}{3}\pi R^3 \Rightarrow \frac{800}{R^2} = H + \frac{2}{3}R \Rightarrow H = \frac{800}{R^2} - \frac{2}{3}R$$

(b)

B1 A correct expression for the surface area containing three separate correct elements

Allow either $A = \pi R^2 + 2\pi RH + 2\pi R^2$ or $A = \pi R^2 + 2\pi RH + \dfrac{4\pi R^2}{2}$

Allow lower case lettering for this mark

M1 Score for replacing $H = \dfrac{800}{R^2} - \dfrac{2}{3}R$ in their expression for A which must be of the form,

$A = B\pi R^2 + C\pi RH, \quad B, C \in \mathbb{N}$,condoning missing brackets.

A1* This is a show that question and all aspects must be correct. Lettering in (b) must be consistent and correct from the point at which $\dfrac{800}{R^2} - \dfrac{2}{3}R$ is substituted. Do not, however, withhold a second mark for using lower case letters if it has been withheld in part (a) for mixed lettering.

Accept $A = 2\pi R^2 + \pi R^2 + 2\pi R\left(\dfrac{800}{R^2} - \dfrac{2}{3}R\right) \Rightarrow A = \dfrac{5\pi R^2}{3} + \dfrac{1600\pi}{R}$ with little or no evidence

Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $\quad 60000=\pi r^2 h \Rightarrow h=\dfrac{60000}{\pi r^2}$	M1
	Subs in $S=\pi r^2+2\pi rh \Rightarrow S=\pi r^2+2\pi r\times\dfrac{60000}{\pi r^2}$	M1
	$\Rightarrow S=\pi r^2+\dfrac{120000}{r}$	A1*
		(3)
(b)	$\dfrac{dS}{dr}=2\pi r-\dfrac{120000}{r^2}$	M1A1
	$\Rightarrow \dfrac{dS}{dr}=0 \Rightarrow r^3=\dfrac{120000}{2\pi} \Rightarrow r = awrt\ 27\,(\text{cm})$	dM1A1
	$\Rightarrow S=\pi\times"26.7"^2+\dfrac{120000}{"26.7"}=awrt\ 6730\,(\text{cm}^2)$	dM1 A1
		(6)
(c)	$\dfrac{d^2S}{dr^2}=2\pi+\dfrac{240000}{r^3}\Bigg\vert_{r=26.7}=awrt\,19>0\Rightarrow \text{Minimum}$	M1A1
		(2)
		(11 marks)

(a)

M1 Uses $60000=\pi r^2 h \Rightarrow h=..$ Alternatively uses $60000=\pi r^2 h \Rightarrow \pi rh=..$
Condone errors on the number of zeros but the formula must be correct

M1 Score for the attempt to substitute any $h=..$ or $\pi rh=..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000=\dfrac{1}{3}\pi r^2 h \Rightarrow h=..\right)$ into $S=k\pi r^2+2\pi rh$ where $k=1$ or 2 to get S in terms of r
Allow if S is called something else such as A.

A1* Completes proof with no errors (or omissions) $S=\pi r^2+\dfrac{120000}{r}$.
Allow from $S=\pi r^2+\dfrac{2V}{r}$ if quoted. $S=$ must be somewhere in the proof

Question	Scheme	Marks
15 (a)	$200 = \pi r^2 + \pi rh + 2rh$	M1 A1
	$(h =)\dfrac{200 - \pi r^2}{\pi r + 2r}$ or $(rh =)\dfrac{200 - \pi r^2}{\pi + 2}$	dM1
	$V = \dfrac{1}{2}\pi r^2 h =$	M1
	$\Rightarrow V = \dfrac{\pi r^2(200 - \pi r^2)}{2(2r + \pi r)} = \dfrac{\pi r(200 - \pi r^2)}{4 + 2\pi}$ *	A1 cso *
		[5]
(b)	$\dfrac{dV}{dr} = \dfrac{200\pi - 3\pi^2 r^2}{4 + 2\pi}$ Accept awrt $\dfrac{dV}{dr} = 61.1 - 2.9r^2$	M1 A1
	$\dfrac{200\pi - 3\pi^2 r^2}{4 + 2\pi} = 0$ or $200\pi - 3\pi^2 r^2 = 0$ leading to $r^2 =$	dM1
	$r = \sqrt{\dfrac{200}{3\pi}}$ or answers which round to 4.6	dM1 A1
	$V = 188$	B1
		[6]
(c)	$\dfrac{d^2V}{dr^2} = \dfrac{-6\pi^2 r}{4 + 2\pi}$, and sign considered Accept $\dfrac{d^2V}{dr^2} =$ awrt $-5.8r$	M1
	$\left.\dfrac{d^2V}{dr^2}\right\vert_{r=..} = -27 < 0$ and therefore maximum	A1
		[2]
		13 marks

Question Number	Scheme		Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..\text{ (m)}$	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

Question Number	Scheme	Marks
9. (a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if *V* is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ Substitutes expression for *h* into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}\ 12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow *r* = 3, and thus *C* =	ddM1
	Then *C* = awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}\ 12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for *h* in terms of *r* (ft only follows misread of *V*)

M1: Substitutes their expression for *h* into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for *C* and achieves **given** answer in part (a) including "*C* =" or "Cost=" and **no errors seen** such as *C* = area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\dfrac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\dfrac{dC}{dr}$ to 0, and obtains r^k = value where *k* = 2, 3 or 4 (needs correct collection of powers of *r* from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find *r* **or** see *r* = awrt 3 as evidence of cube root and substitutes into correct expression for *C* to obtain value for *C*

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\dfrac{d^2C}{dr^2}$ and deduces value of $\dfrac{d^2C}{dr^2}>0$ so minimum (*r* may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of *C* to left and right of 2.92 and shows that *C* > 483 so deduces minimum (i.e. uses shape of graph) Only ft on misread of *V* for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Question Number	Scheme		Marks
10. (a)	$\frac{1}{2}(9x+6x)4x$ or $2x \times 15x$ or $\left(\frac{1}{2}4x \times (9x-6x) + 6x \times 4x\right)$ or $6x^2 + 24x^2$ or $\left(9x \times 4x - \frac{1}{2}4x \times (9x-6x)\right)$ or $36x^2 - 6x^2$	M1: Correct attempt at the area of a trapezium. Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x \times 6x$ is M0. If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.	M1A1**cso**
	$\Rightarrow 30x^2y = 9600 \Rightarrow y = \frac{9600}{30x^2} \Rightarrow y = \frac{320}{x^2}$ *	A1: Correct proof with at least one intermediate step and no errors seen. **" $y =$" is required.**	
			[2]
(b)	$(S =)\ \frac{1}{2}(9x+6x)4x + \frac{1}{2}(9x+6x)4x + 6xy + 9xy + 5xy + 4xy$		M1A1
	M1: An attempt to find the area of **six** faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct. A1: Correct expression in any form. Allow just $(S =)\ 60x^2 + 24xy$ for M1A1		
	$y = \frac{320}{x^2} \Rightarrow (S =)\ 30x^2 + 30x^2 + 24x\left(\frac{320}{x^2}\right)$		M1
	Substitutes $y = \frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.		
	So, $(S =)\ 60x^2 + \frac{7680}{x}$ *	Correct solution only. "$S =$ " is **not** required here.	A1* **cso**
			[4]

10(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of *x* must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find *S* – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880$ (cm^2)	Substitute candidate's value of $x\,(\neq 0)$ into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]

10(d)	$\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $\left(x^n \to x^{n-1}\right)$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S'' = 0$ is M0 A1: $120 + \frac{15360}{x^3}$ and > 0 **and** conclusion. Requires a **correct** second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) **and** a valid reason (e.g. > 0), **and** conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been evaluated incorrectly.	M1A1ft
	A correct S'' followed by $S''("4") = "360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4") = "360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

Question number	Scheme	Marks
8 (a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso (3)
(c)	$(\dfrac{dA}{dx})=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1
	which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) — Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	A1 (2)
		13 marks
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1:** Accept any equivalent correct form – may be on two or more lines. **M1 :** substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1:** At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1:** Setting $\dfrac{dA}{dx}$=0 and finding a value for x^3 (x^3= may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1:** Using **cube root** to find x **A1 :** For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1 :** Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)	

Question number	Scheme	Marks
8 (a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso (3)
(b)	$P = 2x + cy + k\pi r$ where c = 2 or 4 and k = ¼ or ½	M1
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1 (3)
(c)	$\left(\dfrac{\mathrm{d}P}{\mathrm{d}x} =\right) - \dfrac{8}{x^2} + 2$	M1 A1
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1
	and so $x = 2$ o.e. (ignore extra answer x = -2)	A1
	$P = 4 + 4 = 8$ (m)	B1 (5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) = 21 (cm)	M1, A1 (2) **13**
Notes	(a) **M1**: Putting sum of one or two xy terms and one kr^2 term equal to 4 (*k* and *c* may be wrong) **A1:** For any correct form of this equation with x for radius (may be unsimplified) **B1 :** Making y the subject of their formula to give this printed answer with no errors (b) **M1** : Uses Perimeter formula of the form $2x + cy + k\pi r$ where c = 2 or 4 and k = ¼ or ½ **A1:** Correct unsimplified formula with y substituted as shown, i.e. $c = 4$, k = ½ , $r = x$ and $y = \dfrac{16 - \pi x^2}{8x}$ or $y = \left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ **A1:** obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent (c) **M1:** At least one power of x decreased by 1 (Allow $2x$ becomes 2) **A1:** accept any equivalent correct answer **M1:** Setting $\dfrac{\mathrm{d}P}{\mathrm{d}x}$=0 and finding a value for correct power of x for candidate **A1 :** For x = 2. (This mark may be given for equivalent and may be implied by correct P) **B1:** 8 (cao) N.B. This may be awarded if seen in part (d) (d) **M1 :** Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) **A1** is for 21 or 21cm or 0.21m as this is to nearest cm	