Question	Scheme	Marks	AOs
7(a)	$x = \cos\theta + \sin\theta\cos\theta = -y\cos\theta$	M1	2.1
	$\sin\theta = -y - 1$	M1	2.1
	$\left(\frac{x}{-y}\right)^2 = 1 - (-y-1)^2$	M1	2.1
	$x^2 = -\left(y^4 + 2y^3\right)$*	A1*	1.1b
		(4)	
(b)	$V = \pi\int x^2\,\mathrm{d}y = \pi\int -\left(y^4 + 2y^3\right)\,\mathrm{d}y$	M1	3.4
	$= \pi\left[-\left(\frac{y^5}{5} + \frac{y^4}{2}\right)\right]$	A1	1.1b
	$= -\pi\left[\left(\frac{(0)^5}{5} + \frac{(0)^4}{2}\right) - \left(\frac{(-2)^5}{5} + \frac{(-2)^4}{2}\right)\right]$	M1	3.4
	$= 1.6\pi\,\text{cm}^3$ **or** awrt 5.03 cm^3	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)
M1: Obtains x in terms of y and $\cos\theta$
M1: Obtains an equation connecting y and $\sin\theta$
M1: Uses Pythagoras to obtain an equation in x and y only
A1*: Obtains printed answer

(b)
M1: Uses the correct volume of revolution formula with the given expression
A1: Correct integration
M1: Correct use of correct limits
A1: Correct volume

Question	Scheme	Marks	AOs
8(a)	$k = 2.6$	B1	3.4
		(1)	
(b)	$x = 1.18 \Rightarrow \ln\left(3.6\times1.18 - "2.6"\right) = ...$	M1	1.1b
	$h = 0.4995\ldots$ m	A1	2.2b
		(2)	
(c)	$y = \ln\left(3.6x - 2.6\right) \Rightarrow x = \dfrac{e^y + 2.6}{3.6}$ or $\dfrac{5e^y + 13}{18}$	B1ft	1.1a
	$V = \pi\int\left(\dfrac{e^y + 2.6}{3.6}\right)^2 dy = \dfrac{\pi}{3.6^2}\int\left(e^{2y} + 5.2e^y + 6.76\right)dy$ or $\dfrac{\pi}{324}\int\left(25e^{2y} + 130e^y + 169\right)dy$	M1	3.3
	$= \dfrac{\pi}{3.6^2}\left[\dfrac{1}{2}e^{2y} + 5.2e^y + 6.76y\right]$ $\left(\text{or } \dfrac{\pi}{324}\left[\dfrac{25}{2}e^{2y} + 130e^y + 169y\right]\right)$	A1	1.1b
	$= \dfrac{\pi}{3.6^2}\left\{\left(\dfrac{1}{2}e^{2h} + 5.2e^h + 6.76h\right) - \left(\dfrac{1}{2}e^0 + 5.2e^0 + 6.76(0)\right)\right\}$ or e.g. $= \dfrac{\pi}{324}\left\{\left(\dfrac{25}{2}e^{2h} + 130e^h + 169h\right) - \left(\dfrac{25}{2}e^0 + 130e^0 + 6.76(0)\right)\right\}$	M1	2.1
	$= \dfrac{\pi}{3.6^2}\left(\dfrac{1}{2}e^{2h} + 5.2e^h + 6.76h - 5.7\right)$	A1	1.1b
		(5)	
(d)	$\dfrac{dV}{dh} = \dfrac{\pi}{3.6^2}\left(e^{2h} + 5.2e^h + 6.76\right) = \dfrac{\pi}{3.6^2}\left(e^{0.4} + 5.2e^{0.2} + 6.76\right)$	M1	3.1a
	$\dfrac{dh}{dt} = \dfrac{dh}{dV}\dfrac{dV}{dt} = \dfrac{1}{3.539...}\times 0.015\times 60$	M1	1.1b
	$\dfrac{dh}{dt} = 25.4\,\text{cm h}^{-1}$	A1	3.2a
		(3)	
(d) Way 2	$y = 0.2 \Rightarrow x = \dfrac{2.6 + e^{0.2}}{3.6} \Rightarrow A = \pi\left(\dfrac{2.6 + e^{0.2}}{3.6}\right)^2 (= 3.54)$	M1	3.1a
	$\dfrac{dh}{dt} = \dfrac{0.015\times 60}{3.54}$	M1	1.1b
	$\dfrac{dh}{dt} = 25.4\,\text{cm h}^{-1}$	A1	3.2a
			(11 marks)

Notes

(a)
B1: Uses the model to obtain a correct value for *k*. Must be 2.6 not -2.6
(b)

Question	Scheme	Marks	AOs
7(a)	$1=\dfrac{a}{0.5+b},\ 0.5=\dfrac{a}{2.5+b} \Rightarrow a=...,\ b=...$	M1	3.3
	$a = 2,\ b = 1.5$	A1	1.1b
		(2)	
(b)	$V_1=\pi\int x^2\mathrm{d}y=\pi\int\left(\dfrac{"2"}{y+"1.5"}\right)^2\mathrm{d}y$	B1ft	3.4
	$\pi\int_{0.5}^{2.5}\left(\dfrac{"2"}{y+"1.5"}\right)^2\mathrm{d}y$	M1	1.1a
	$=\{4\pi\}\left[-(y+1.5)^{-1}\right]_{0.5}^{2.5}(=\pi)$	M1	1.1b
	$x^2+(y-3)^2=0.5$	B1	2.2a
	$V_2=\pi\int x^2\mathrm{d}y=\pi\int\left(0.5-(y-3)^2\right)\mathrm{d}y$ or $\pi\int\left(-y^2+6y-8.5\right)\mathrm{d}y$	M1	1.1b
	$=\pi\int_{2.5}^{3+\frac{1}{\sqrt{2}}}\left(0.5-(y-3)^2\right)\mathrm{d}y$ or $=\pi\int_{2.5}^{3+\frac{1}{\sqrt{2}}}\left(-y^2+6y-8.5\right)\mathrm{d}y$	M1	3.3
	$=\{\pi\}\left[0.5y-\dfrac{1}{3}(y-3)^3\right]_{2.5}^{3+\frac{1}{\sqrt{2}}}$ or $=\{\pi\}\left[-\dfrac{1}{3}y^3+3y^2-8.5y\right]_{2.5}^{3+\frac{1}{\sqrt{2}}}$	A1	1.1b
	$V_1+V_2+\text{cylinder}=\pi+\pi\left(\dfrac{5}{24}+\dfrac{\sqrt{2}}{6}\right)+\dfrac{1}{2}\pi$	dM1	3.4
	$=\pi\left(\dfrac{41}{24}+\dfrac{\sqrt{2}}{6}\right)\approx 6.11\text{cm}^3$	A1	2.2b
		(9)	
			(11 marks)

Notes

(a)
M1: Uses the given coordinates correctly in the equation modelling the curve to obtain at least one correct equation and attempts to find the values of *a* and *b*
A1: Correct values
(b)
B1ft: Uses the model to obtain $\pi\int\left(\dfrac{\text{their } a}{y+\text{their } b}\right)^2\mathrm{d}y$. Note the p can be recovered if appears later.
M1: Chooses limits appropriate to the model i.e. 0.5 and 2.5
M1: Integrates to obtain an expression of the form $k(y + \text{"}1.5\text{"})^{-1}$
B1: Deduces the correct equation for the circle
M1: Uses their circle equation and $\pi\int x^2\,\mathrm{d}y$ to attempt the top volume. Note the p can be recovered if appears later.

Question	Scheme		Marks	AOs
7(a)	Using $\text{arsinh}\,\alpha = \frac{1}{2}\ln 3$ $\alpha = \dfrac{e^{\frac{1}{2}\ln 3} - e^{-\frac{1}{2}\ln 3}}{2}$	$\ln\left(\alpha + \sqrt{\alpha^2+1}\right) = \frac{1}{2}\ln 3$	B1	1.2
	$\alpha = \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2} \Rightarrow \alpha = ...$	$\alpha + \sqrt{\alpha^2+1} = \sqrt{3}$ $\sqrt{\alpha^2+1} = \sqrt{3} - \alpha$ $\alpha^2 + 1 = 3 - 2\sqrt{3}\alpha + \alpha^2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = \dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$		A1	2.2a
			(3)	
(b)	Volume = $\pi\int_0^{\frac{1}{2}\ln 3} \sinh^2 y\,dy$		B1	2.5
	$\{\pi\}\int\left(\dfrac{e^y - e^{-y}}{2}\right)^2 dy = \{\pi\}\int\left(\dfrac{e^{2y} - 2 + e^{-2y}}{4}\right)dy$ or $\{\pi\}\int \frac{1}{2}\cosh 2y - \frac{1}{2}\,dy$		M1	3.1a
	$\frac{1}{4}\left(\frac{1}{2}e^{2y} - 2y - \frac{1}{2}e^{-2y}\right)$ or $\frac{1}{4}\sinh 2y - \frac{1}{2}y$		dM1 A1	1.1b 1.1b
	Use limits $y = 0$ and $y = \frac{1}{2}\ln 3$ and subtracts the correct way round		M1	1.1b
	$\dfrac{\pi}{4}\left(\dfrac{4}{3} - \ln 3\right)$ or exact equivalent		A1	1.1b
			(6)	
				(9 marks)

Notes:

(a)

B1: Recalls the definition for $\sinh\left(\frac{1}{2}\ln 3\right)$ or forms an equation for arcsinh x

M1: Uses logarithms to find a value for α or forms and solves a correct equation without log

A1: Deduces the correct exact value for α

Note using the result

$$\ln\left(\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right) = \ln\left(\frac{1}{\sqrt{3}} + \sqrt{\frac{4}{3}}\right) = \ln\sqrt{3} = \frac{1}{2}\ln 3 \text{ therefore } \text{arsinh}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln 3$$

Question	Scheme	Marks	AOs
8(a)	$\left(z+\frac{1}{z}\right)^6 = 64\cos^6\theta$	B1	2.1
	$\left(z+\frac{1}{z}\right)^6 = z^6 + 6(z^5)\left(\frac{1}{z}\right) + 15(z^4)\left(\frac{1}{z^2}\right) + 20(z^3)\left(\frac{1}{z^3}\right) + 15(z^2)\left(\frac{1}{z^4}\right) + 6(z)\left(\frac{1}{z^5}\right) + \left(\frac{1}{z^6}\right)$	M1	2.1
	$= \left[z^6 + \frac{1}{z^6}\right] + 6\left[z^4 + \frac{1}{z^4}\right] + 15\left[z^2 + \frac{1}{z^2}\right] + 20$	A1	1.1b
	Uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ $\{64\cos^6\theta\} = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	M1	2.1
	$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$ * cso	A1 *	1.1b
		(5)	
(b)	$\boldsymbol{H = 2}$	B1	3.3
		(1)	
(c)	$\text{vol} = \left\{\frac{1}{2}\right\}\pi\int\left(2\cos^3\left(\frac{x}{4}\right)\right)^2 \mathrm{d}x$	B1ft	3.4
	$\text{vol} = \{2\pi\}\int\cos^6\left(\frac{x}{4}\right)\mathrm{d}x$ $= \{2\pi\}\int\frac{1}{32}\left(\cos\left(\frac{6x}{4}\right) + 6\cos\left(\frac{4x}{4}\right) + 15\cos\left(\frac{2x}{4}\right) + 10\right)\mathrm{d}x = \ldots$	M1	1.1b
	$= \{2\pi\}\left[\frac{1}{32}\underline{\left(\frac{2}{3}\sin\left(\frac{3x}{2}\right) + 6\sin(x) + 30\sin\left(\frac{x}{2}\right) + 10x\right)}\right]$	A1	1.1b
	$= 2\times 2\pi\left[\frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times 4\right) + 6\sin(4) + 30\sin\left(\frac{4}{2}\right) + (10\times 4)\right) - 0\right] = \ldots$ or $=$ $2\pi\left[\frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times 4\right) + 6\sin(4) + 30\sin\left(\frac{4}{2}\right) + (10\times 4)\right) - \frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times -4\right) + 6\sin(-4) + 30\sin\left(-\frac{4}{2}\right) + (10\times -4)\right)\right]$ …	dM1	3.4
	$= 24.56$	A1	1.1b
		(5)	

(d)	The equation of the curve may not be suitable The measurements may not be accurate The paperweight may not be smooth	B1	3.5b
		(1)	
			(12 marks)

Notes:

(a)

B1: Correct identity or equivalent rearrangement. This can appear anywhere in the proof.

M1: Attempts the expansion of $\left(z + \frac{1}{z}\right)^6$ must have at least 3 correct terms. Combining the powers when expanding is fine.

A1: Correct expansion with *z* terms simplified, need not be rearranged. (So a correct expansion will score M1A1.)

M1: Uses $z^n + \frac{1}{z^n} = 2\cos n\,\theta$ to write the expression in terms multiple angles of $\cos 6\,\theta,\ \cos 4\,\theta$ and $\cos 2\,\theta$. Pairing of terms must be seen.

A1*: Achieves the printed answer with no errors or omissions. Cso

For approaches using De Moivre B0M1A1M0A0 may be scored if the binomial expansions is attempted (and correct for the A).

(b)

B1: See scheme

(c)

Note: The question instructs use of algebraic integration and part (a), so answer only can score at most B1 for implied correct formula.

B1ft: Correct expression for the volume of the paperweight **or** the solid formed through 360º rotation, stated or implied, ignore limits. No need to expand, but must be applied, not just a formula in *y,* though allow a correct formula followed by correct integral if the π disappears. Follow through their *H*

M1: Uses the result in part part (a) to express the volume in an integrable form and attempts to integrate. Note use of *θ* instead of *x* is permissible for this mark. Allow if one term is missing or miscopied.

A1: Correct integration in terms of *x*. Ignore π, their H^2 and the $\frac{1}{32}$. Note if *θ* has been used it is A0 unless a correct substitution method has been implied as the coefficients will be incorrect.

dM1: Dependent on previous method mark and must have reached and integral of the correct form - - in terms of *x* with correct arguments allowing for one slip. Finds the required volume using either$\pi\int_0^4 y^2\,dx$ or $\frac{1}{2}\pi\int_{-4}^4 y^2\,dx$ and applies their limits - accept any value following a valid attempt at the integration as an attempt at applying limits.

A1: cao 24.56

(d)

B1: States an appropriate limitation. See scheme for some examples. The limitation should refer to the paperweight, not to paper. Do not accept “it does not take into account thickness of material” as it is a solid, not a shell, being modelled. Award the mark for a correct reason if two reasons are given and one is incorrect.

Question	Scheme	Marks	AOs
7(a)	$\frac{\pi}{16}\int_{-1.545}^{1.257}\left(6-3y^2+y\cos\left(\frac{5}{2}y\right)\right)\{\mathrm{d}y\}$ $\pi\int_{-1.545}^{1.257}\left(\frac{3}{8}-\frac{3}{16}y^2+\frac{1}{16}y\cos\left(\frac{5}{2}y\right)\right)\{\mathrm{d}y\}$	**B1**	1.1a
	$\int x^2\,\mathrm{d}y=\frac{1}{16}\int 6-3y^2+y\cos\left(\frac{5}{2}y\right)\mathrm{d}y\to Ky-Ly^3+\ldots$	**M1**	1.1b
	$\int y\cos\left(\frac{5}{2}y\right)\mathrm{d}y=Ay\sin\left(\frac{5}{2}y\right)+B\cos\left(\frac{5}{2}y\right)(+c)$ $\left\{\int y\cos\left(\frac{5}{2}y\right)\mathrm{d}y=y.\frac{2}{5}\sin\left(\frac{5}{2}y\right)-\int 1.\frac{2}{5}\sin\left(\frac{5}{2}y\right)\mathrm{d}y=\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)(+c)\right\}$	**M1**	3.1a
	$\int x^2\,\mathrm{d}y=\frac{1}{16}\left(6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right)(+c)$ $\int x^2\,\mathrm{d}y=\frac{3}{8}y-\frac{1}{16}y^3+\frac{1}{40}y\sin\left(\frac{5}{2}y\right)+\frac{1}{100}\cos\left(\frac{5}{2}y\right)(+c)$	**A1**	1.1b
	$\int_{-1.545}^{1.257}x^2\,\mathrm{d}y=\frac{1}{16}\left[6y-y^3+\frac{2}{5}y\sin\left(\frac{5}{2}y\right)+\frac{4}{25}\cos\left(\frac{5}{2}y\right)\right]_{-1.545}^{1.257}$ $=\frac{1}{16}(5.3954\ldots-(-6.1101\ldots))=\ldots$ $=(0.3372\ldots)-(-0.3818\ldots)=\ldots$	**M1**	3.4
	Volume = $\pi\times\frac{11.505\ldots}{16}=2.26$ **cm³** (2.2591159…) cso	**A1**	3.2a
		(6)	
(b)	Max volume for 100 berries (as we know volume of the largest) is $100\times 2.26\simeq 226$	**B1ft**	1.1b
	Reason e.g. not all the berries will become juice (e.g. skin, flesh, seeds may not pulp) or not all will be as big as the largest, 150 < 200 or 300 > 200 If their value • is less than 220 –so not likely to produce 200 cm³ of juice. • is between 220 and 250 – they can conclude either way • is greater than 250 – so likely to produce 200 cm³ of juice.	**B1ft**	2.2b

Question	Scheme	Marks	AOs
9(a)	$(4, 14),\ (1, 18) \Rightarrow 14 = a(4)^2 + b,\ 18 = a(1)^2 + b \Rightarrow a = ..., b = ...$	M1	3.3
	$a = -\frac{4}{15}, b = \frac{274}{15}$	A1	1.1b
		(2)	
(b)	$\pi \times 4^2 \times 14$ and $\pi \times 1^2 \times 10$	B1	1.1b
	$\pi \int x^2 \, dy = \frac{\pi}{4} \int (274 - 15y) \, dy$	B1ft	1.1a
	$= \frac{\pi}{4} \int_{14}^{18} (274 - 15y) \, dy$	M1	3.3
	$= \frac{\pi}{4} \left[274y - \frac{15y^2}{2} \right]_{14}^{18}$	M1 A1	1.1b 1.1b
	$V = 234\pi + \frac{\pi}{4} \left[274(18) - \frac{15(18)^2}{2} - \left(274(14) - \frac{15(14)^2}{2} \right) \right]$	ddM1	3.4
	$V = 268\pi \approx 842 \text{ cm}^3$	A1	2.2b
		(7)	
(c)	Any one of e.g. The measurements may not be accurate The equation of the curve may not be a suitable model The bottom of the bottle may not be flat The thickness of the glass may not have been considered The glass may not be smooth This part asks for a limitation of the model so their answer must refer to e.g. : • The measuring of the dimensions • The model used for the curve • The simplified model (the thickness of glass, the simplified shape, smoothness of the glass etc.)	B1	3.5b
		(1)	
(d)	There are 2 criteria for this mark: • A comparison of their value to 750 e.g. larger, smaller, about the same or a difference **demonstrated** e.g. 810 – 750 = … but not **just** a percentage error or **just** a difference with no calculation • A conclusion that is consistent with their values e.g. this is not a good model, this is a good model etc. If they reach an answer that is less than 750, they need to conclude that it is not a good model If they reach an answer that is greater than 750 then look for a sensible comment that is consistent with their value	B1ft	3.5a
		(1)	
			(11 marks)

Question	Scheme	Marks	AOs
9.	A correct overall strategy, an attempt at integrating y^2 with respect to x combine in some way with the volume of revolution formula (use of $\pi\int y^2\mathrm{d}x$ or $\alpha\int y^2\mathrm{d}x$ for any variable α is fine) followed by attempt to find an angle/form an equation in θ	M1	3.1a
	$y^2 = kx^{\frac{2}{3}} + \ldots + \frac{m}{x^{\frac{4}{3}}}$ or $y^2 = kx^{\frac{2}{3}} + \ldots + mx^{-\frac{4}{3}}$ where … is one or two more terms.	M1	1.1b
	$y^2 = 4x^{\frac{2}{3}} + 4x^{-\frac{1}{3}} + x^{-\frac{4}{3}}$ or $y^2 = 4x^{\frac{2}{3}} + 2x^{-\frac{1}{3}} + x^{-\frac{4}{3}} + 2x^{-\frac{1}{3}}$ (oe)	A1	1.1b
	$\int y^2\mathrm{d}x = \int 4x^{\frac{2}{3}} + \frac{4}{x^{\frac{1}{3}}} + \frac{1}{x^{\frac{4}{3}}}\mathrm{d}x = \alpha x^{\frac{5}{3}} + \beta x^{\frac{2}{3}} + \gamma x^{-\frac{1}{3}}$	M1	1.1b
	$= \frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}$ (oe)	A1ft A1	1.1b 1.1b
	$\frac{\theta}{2}\left[\frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \frac{461}{2}$ $\Rightarrow \frac{\theta}{2}\left[\left(\frac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \frac{3}{8^{\frac{1}{3}}}\right) - \left(\frac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\frac{1}{8}\right)^{\frac{2}{3}} - \frac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = \frac{461}{2} \Rightarrow \theta = \ldots$ OR $\pi\left[\frac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \frac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \pi\left[\left(\frac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \frac{3}{8^{\frac{1}{3}}}\right) - \left(\frac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\frac{1}{8}\right)^{\frac{2}{3}} - \frac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = ..$ followed by $\frac{\theta}{2\pi}\times\ldots = \frac{461}{2} \Rightarrow \theta = \ldots$	M1	3.1a
	$\theta = \frac{40}{9}$ (radians)	A1	1.1b
		(8)	
	(8 marks)		

Question	Scheme	Marks	AOs
3	$x^2+y^2=r^2$	B1	1.2
	$\{V\}=\pi\int_{-r}^{r} r^2-x^2 \,\mathrm{d}x$ or $\{V\}=2\pi\int_{0}^{r} r^2-x^2 \,\mathrm{d}x$	B1	2.1
	Integrates to the form $\alpha x \pm \beta x^3$ $\left[\text{note: the correct integration gives } r^2x-\frac{1}{3}x^3\right]$	M1	1.1b
	Substitutes limits of – r and r and subtracts the correct way round $\left(r^2(r)-\frac{1}{3}(r)^3\right)-\left(r^2(-r)-\frac{1}{3}(-r)^3\right)$ or Substitutes limits of 0 and r and subtracts the correct way round with twice the volume. Note the limit of 0 can be implied if gives and answer of 0 $\left(r^2(r)-\frac{1}{3}(r)^3\right)-(0)$	dM1	1.1b
	$V=\frac{4}{3}\pi r^3$ * cso	A1*	1.1b
		(5)	
			(5 marks)

Notes:

B1: Correct equation of the circle, may be implied by correct integral

B1: Correct expression for the volume, including limits, dx may be implied and if using limits r and 0 the 2 could appear later with reasoning

M1: Integrates to the form $\alpha x \pm \beta x^3$. Do not award if $r^2 \to \lambda r^3$

dM1: Dependent on previous method mark. Correct use of limits – r and r or limits of 0 and r with twice the volume.

A1*: $V=\frac{4}{3}\pi r^3$ * cso

Note: rotation about the *y*-axis all marks are available, however for the final accuracy mark must refer to symmetry

Notes
(a) Note ePen B1 M1 M1 A1 A1 A1 B1: Substitutes $n = 1$ into both sides to show that they are both equal to 6. (There is no need to state true for $n = 1$ for this mark) M1: Makes a statement that assumes the result is true for some value of n, say k M1: Adds the $(k + 1)$th term to the assumed result dM1: Dependent on previous M, factorises out $\frac{1}{2}(k+1)(k+2)$ A1: Reaches a correct the required expression no errors and shows that this is the correct sum for $n = k + 1$ A1: Depends on all except **B** mark being scored (must have been some attempt to show true for $n = 1$). Correct conclusion conveying all the points in bold. (b) M1: Realises that $\sum_{r=1}^{2n} r(r+1)(2r+1) - \sum_{r=1}^{n-1} r(r+1)(2r+1)$ is required and uses the result from part (a) to obtain the required sum in terms of n M1: Attempts to factorise by $\frac{1}{2}n(n+1)$ A1: Correct expression or correct values

Question	Scheme	Marks	AOs
9(a)	$(5, 15) \Rightarrow 15 = \frac{\sqrt{225\times5^2 - 2025}}{a} \Rightarrow a = ...$	M1	3.3
	$a = 4$	A1	1.1b
		(2)	
(b)	Evidence of the use of $\pi\int x^2\,dy$ for the curve BC or the curve CD	M1	3.1b
	For BC $V_1 = \frac{\pi}{225}\int(16y^2 + 2025)\,dy$ or $\pi\int\left(\frac{16}{225}y^2 + 9\right)dy$	A1ft	1.1b
	For CD $V_2 = 25\pi\int(16 - y)\,dy$ or $\pi\int(400 - 25y)\,dy$	A1	1.1b
	$V_1 = \frac{\pi}{225}\int_0^{15}(16y^2 + 2025)\,dy$ or $\pi\int_0^{15}\left(\frac{16}{225}y^2 + 9\right)dy$	M1	3.3
	$V_2 = 25\pi\int_{15}^{16}(16 - y)\,dy$ or $\pi\int_{15}^{16}(400 - 25y)\,dy$	M1	3.3
	$V_1 = \frac{\{\pi\}}{225}\left[\frac{16y^3}{3} + 2025y\right]_0^{15}$ or $\{\pi\}\left[\frac{16y^3}{675} + 9y\right]_0^{15}$	A1ft	1.1b
	$V_2 = 25\{\pi\}\left[16y - \frac{y^2}{2}\right]_{15}^{16}$ or $\{\pi\}\left[400y - \frac{25y^2}{2}\right]_{15}^{16}$	A1ft	1.1b
	$V = V_1 + V_2 = \frac{\pi}{225}(18000 + 30375) + 25\pi\left(128 - \frac{255}{2}\right)$ $V = V_1 + V_2 = 215\pi + 12.5\pi$	M1	3.4
	$V = \frac{455\pi}{2}\text{cm}^3$ or $227.5\pi\,\text{cm}^3$	A1	2.2b
		(9)	

(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the candle will not be perfectly curved/smooth • There will be a whole in the middle for the wick	B1	3.5b
		(1)	
(d)	Makes an appropriate comment that is consistent with their value for the volume and 700 cm^3. E.g. a good estimate as 700 cm^3 is only 15 cm^3 less than 715 cm^3	B1ft	3.5a
		(1)	
			(13 marks)

Notes

(a)
M1: Substitutes (5, 15) into the equation modelling the curve in an attempt to find the value of *a*
A1: Infers from the data in the model, the value of *a*
(b)
M1: Uses either model to obtain x^2 in terms of y and applies $\pi \int x^2 \, dy$
A1ft: Correct expression for the volume generated by the curve *BC* (follow through their *a* value)
A1: Correct expression for the volume generated by the curve *CD*
M1: Chooses limits appropriate to their model for the curve *BC*
M1: Chooses limits appropriate to their model for the curve *CD*
A1ft: Correct integration (follow through their *a* value)
A1ft: Correct integration follow through on their volume as long it is of the form $Ay - By^2$
M1: Uses the model to find the sum of volumes
A1: $\frac{455\pi}{2}$
Note: Use of calculator for integration maximum score M1 A1ft A1 M1 M1 A0ft A0ft M1 A1
(c)
B1: States an acceptable limitation of the model
(d)
B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason.

Question	Scheme	Marks	AOs
8(a)	$a = 4$	B1	3.3
		(1)	
(b)	Model A: (i) Widest point will be 4 (cm) from the base	B1	3.4
	(ii) Width at widest point is 12 (cm) $\left(2\times('a'+2)\text{ ft}\right)$	B1ft	3.4
	Model B: (i) $y = 4+\dfrac{x^3-64x}{100} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2-64}{100}$	M1	3.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x = \pm\sqrt{\dfrac{64}{3}} = \pm\dfrac{8\sqrt{3}}{3} = \pm\text{awrt}4.62$	A1	1.1b
	So max width is a distance $8-\dfrac{8}{\sqrt{3}} = 8-\dfrac{8\sqrt{3}}{3} \approx 3.38$ (cm) from base.	A1	3.4
	(ii) $y\big\|_{-4.61..} = 4+\dfrac{(-4.62...)^3-64(-4.62...)}{100} = ...$	dM1	3.4
	$= 5.97...$ so diameter is approximately 11.9 (cm) $\left[2a+3.94...\text{ ft}\right]$	A1ft	3.2a
		(7)	
(c)	Model A and model B both have diameters closed to 12 Model B distance from base is closer to 3 than Model A so is more appropriate.	B1ft	3.5b
		(1)	
(d)	$V_B = \pi\displaystyle\int_{-8}^{8} y^2\,\mathrm{d}x = \pi\int_{-8}^{8}\left(4+\dfrac{x^3-64x}{100}\right)^2\mathrm{d}x = ...$	B1	1.1b
	$= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 400^2 + x^6 + 64^2x^2 + 2\left(400x^3 - 400\times 64x - 64x^4\right)\mathrm{d}x$ $= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 160000 + x^6 + 4096x^2 + 800x^3 - 51200x - 128x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{x^6}{10000} + \dfrac{4096}{10000}x^2 + \dfrac{8}{100}x^3 - \dfrac{512}{100}x - \dfrac{128}{10000}x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{x^6}{1000} + \dfrac{256}{625}x^2 + \dfrac{2}{25}x^3 - \dfrac{128}{25}x - \dfrac{8}{625}x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \dfrac{8x(x-8)(x+8)}{100} + \left(\dfrac{x(x-8)(x+8)}{100}\right)^2\ \mathrm{d}x$	M1	1.1b
	$= \dfrac{\{\pi\}}{10000}\left[160000x + \dfrac{x^7}{7} + 4096\dfrac{x^3}{3} + 800\dfrac{x^4}{4} - 51200\dfrac{x^2}{2} - 128\dfrac{x^5}{5}\right]_{(-8)}^{(8)}$	dM1	1.1b

	$=\{\pi\}\left[16x+\frac{x^7}{70000}+\frac{256}{1875}x^3+\frac{1}{50}x^4-\frac{64}{25}x^2-\frac{8}{3125}x^5\right]_{(-8)}^{(8)}$		
	$=\frac{\{\pi\}}{10000}(620583.00....--2258983.01...)\approx\frac{2879566\pi}{10000}$	M1	3.4
	$=$ awrt $905\left(\text{cm}^3\right)$ cso	A1	1.1b
		(5)	
(e)	Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately.	B1ft	3.5a
		(1)	
			(15 marks)

Notes:

Units not required in this question

(a)

B1: For $a = 4$, ignore any reference to units.

(b)

B1: Correct distance from base for Model A is 4

B1ft: Correct width at widest point. Follow through their '*a*', so $2\times('a'+2)$.

M1: Attempts the derivative for Model B's equation, reduce any power by 1

A1: Sets $\frac{dy}{dx}=0$ and finds correct x coordinate of the stationary point (accept ±)

A1: For $8-\frac{8}{\sqrt{3}}$ or awrt 3.38 cso

dM1: Dependent on previous M mark. Uses their value of x to find the value of y. If no working shown the value of y must come from their x value.

Note using $x = 4.62$ give y = 2.029…

A1: Correct diameter, awrt 11.9 follow through their '*a*', so $[2a+3.94...\text{ ft}]$

Note: Correct answers with no working send to review

Trial and error approach

Candidates could score B1 B1 for model A however if working in integers it is unlikely that they will find the correct value for x (they are using $x = -5$) not a valid method M0A0A0dM0A0

(c)

B1ft: They must have answers for all parts in (b). Accept any well-reasoned comment that follows their answers to (b) If the answers are correct, they must conclude that model B is more appropriate.

- If answers for one model are correct ish but other incorrect, or one value is clearly closer For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**
A	9.4	9.05	4	6
B	3.38	12.06	4.62	4.06
Conclusion	Selects B as distance/diameter closet		Select A as diameter closest	

- If distances and diameters are similar selects the model which has the most appropriate value for distance or diameter
For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**

Question	Scheme	Marks	AOs
5(a)	$\{V=\}\pi\int_0^2\left[(2-y)^{\frac{1}{2}}\right]^2 \mathrm{d}y$ or $\{V=\}\pi\int_0^2(2-y)\,\mathrm{d}y$	B1	3.3
	Integrates to the form $\alpha y \pm \beta y^2$	M1	1.1b
	Correct integration $2y-\frac{1}{2}y^2$	A1	1.1b
	Uses their y limits correctly in a changed expression $\pi\left[2y-\frac{1}{2}y^2\right]_0^2=\pi\left(2(2)-\frac{1}{2}(2^2)\right)-0=\ldots\{2\pi \text{ or } 6.28\ldots\}$	M1	3.4
	mass = 'their volume' $\times 900$	M1	3.1b
	Mass = 5700 (kg) 2 s.f. cao	A1	2.2b
		(6)	
(b)	eg The surface will not be smooth The pile will not follow the shape of the curve The pile will not be solid Equation of the curves may not be a suitable model Concrete is likely to be uneven/may have bumps The pile is unlikely to be symmetrical	B1	3.5b
		(1)	
(c)	Makes a comparison about the difference between their mass and 5500 and draws a conclusion e.g. 200 difference which is a lot of concrete therefore not a good model e.g. the mass of 5700 is very close to 5500 kg and draws a conclusion about the model – e.g. therefore a good model e.g. Finds the percentage error and draw a conclusion about the model e.g. The masses are very close/significantly different and draws an appropriate conclusion Not sufficient to say 5700 > 5500 B0	B1ft	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: Sets up the model to find a correct expression for the volume, including limits, dy may be implied. The limits may be seen later.

M1: Integrates to the form $\alpha y \pm \beta y^2$

A1: Correct integration

M1: Substitutes their y limits the correct way round and subtracts, must be a changed expression

M1: Multiplies their volume by 900 to find the mass

A1: 5700 cao

Note incorrect upper limit of $\sqrt{2}$ leads to 5200kg Scores B0 M1 A1 M1 M1 A0