

Mark Scheme (Results)

January 2007

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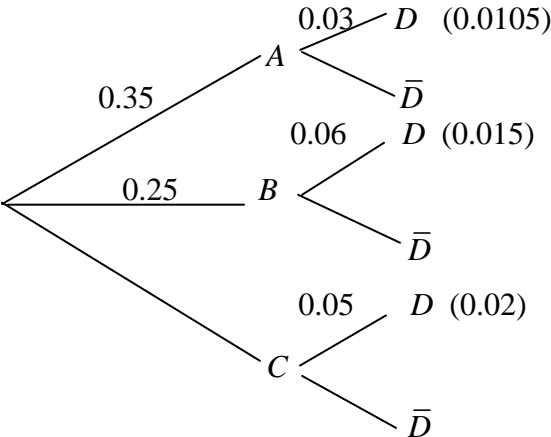
GCE

GCE Mathematics

Statistics (6683)

January 2007
6683 Statistics S1
Mark Scheme

| Question number | Scheme | Marks |
|-----------------|--|---|
| 1. (a) | (£) 17 | Just 17 |
| (b) | $\sum t = 212$ and $\sum m = 61$ (Accept as totals under each column in qu.) | B1, B1 |
| | $S_m = 2485 - \frac{61 \times 212}{10}, = 1191.8$ | awrt 1190 or 119 (3sf) |
| | $S_n = 983.6$ (awrt 984) and $S_{mm} = 1728.9$ (awrt 1730) | (or 98.4 and 173) |
| (c) | $r = \frac{1191.8}{\sqrt{983.6 \times 1728.9}}$ | M1, A1 |
| | $= 0.913922\dots$ | awrt 0.914 |
| (d) | 0.914 | (Must be the same as (c) or awrt 0.914) |
| | e.g. linear transformation, coding does not affect coefficient (or recalculate) | B1f.t. ($ r < 1$) |
| (e) | 0.914 suggests longer spent shopping the more spent. (Idea more time, more spent) | dB1 |
| | 0.178 different amounts spent for same time. | B1 |
| (f) | e.g. might spend short time buying 1 expensive item <u>OR</u> might spend a long time checking for bargains, talking, buying lots of cheap items. | B1g |
| | | 15 marks |
| (b) | M1 for one correct formula seen, f.t. their $\sum t, \sum m$ [Use 1 st A1 for 1 correct, 2 nd A1 for 2 etc] | |
| (c) | M1 for attempt at correct formula, $\frac{2485}{\sqrt{2101 \times 5478}}$ scores M1A0A0 | |
| | A1ft f.t. their values for S_n etc from (b) but don't give for $S_n = 5478$ etc (see above) | |
| | Answer only (awrt 0.914) scores 3/3, 0.913 (i.e. truncation) can score M1A1ft by implication. | |
| (d) | 2 nd B1 dependent on 1 st B1 Accept $\sum m = 261, \sum m^2 = 8541, \sum tm = 6725 \rightarrow 0.914$ | |
| (e) | One mark for a sensible comment relating to each coefficient | |
| | For 0.178 allow "little or no link between time and amount spent". Must be in context. | |
| | Just saying 0.914 is strong +ve correlation between amount spent and time shopping and 0.178 is weak correlation ...scores B0B0. | |
| (f) | B1g for a sensible, practical suggestion showing that other factors might affect the amount spent. E.g. different day (weekend vs weekday) or time of day (time spent queuing if busy) | |

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|-----------------|---|--|
| 2. (a) |  <p style="text-align: right;">Correct tree shape</p> <p>A, B and C and 0.35 and 0.25</p> <p>D (x3) and $0.03, 0.06, 0.05$ (May be implied by seeing $P(A \cap D)$ etc at the ends)</p> | <p>M1</p> <p>A1</p> <p>A1 (3)</p> |
| (b)(i) | $P(A \cap D) = 0.35 \times 0.03, = \underline{\mathbf{0.0105}}$ or $\frac{21}{2000}$ | <p>M1, A1</p> |
| (ii) | $P(D) = (i) + 0.25 \times 0.06 + (0.4 \times 0.05)$ $= \underline{\mathbf{0.0455}}$ or $\frac{91}{2000}$ | <p>$P(C) = 0.4$ (anywhere) B1</p> <p>M1</p> <p>A1 (5)</p> |
| (c) | $P(C D) = \frac{P(C \cap D)}{P(D)}, = \frac{0.4 \times 0.05}{(ii)}$ $= 0.43956... \text{ or } \frac{40}{91}$ | <p>M1, A1ft</p> <p>$\underline{\mathbf{0.44}}$ or awrt $\underline{\mathbf{0.440}}$ A1 (3)</p> <p style="text-align: right;">11 marks</p> |
| (a) | <p>M1 for tree diagram, 3 branches and then two from each. At least one probability attempted.</p> <p>(b) 1st M1 for 0.35×0.03. Allow for equivalent from <u>their</u> tree diagram.</p> <p>B1 for $P(C) = 0.4$, can be in correct place on tree diagram or implied by 0.4×0.05 in $P(D)$.</p> <p>2nd M1 for all 3 cases attempted and <u>some</u> correct probabilities seen, including +. Can ft their tree. Condone poor use of notation if correct calculations seen. E.g. $P(C D)$ for $P(C \cap D)$.</p> <p>(c) M1 for attempting correct ratio of probabilities. There must be an attempt to substitute some values in a correct formula. If no correct formula and ration not correct ft score M0. Writing $P(D C)$ and attempting to find this is M0. Writing $P(D C)$ but calculating correct ratio – ignore notation and mark ratios.</p> <p>A1ft must have their 0.4×0.05 divided by their (ii). If ratio is incorrect ft (0/3) unless correct formula seen and part of ratio is correct then M1.</p> | |

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|-----------------|---|----------------|----------------|----------------|----------------|-----------------|---|---|------------|----------------|----------------|----------------|----------------|----------------|-----------------|--|
| 3. (a) | <p>N.B. Part (a) doesn't have to be in a table, could be a list $P(X = 1) = \dots$ etc</p> <table border="1" data-bbox="293 349 968 510"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>$P(X = x)$</td> <td>$\frac{1}{36}$</td> <td>$\frac{3}{36}$</td> <td>$\frac{5}{36}$</td> <td>$\frac{7}{36}$</td> <td>$\frac{9}{36}$</td> <td>$\frac{11}{36}$</td> </tr> </table> <p>0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306 (Accept awrt 3 s.f)</p> <p>(b) $P(3) + P(4) + P(5) = \frac{21}{36}$ or $\frac{7}{12}$ or awrt 0.583</p> <p>(c) $E(X) = \frac{1}{36} + 2 \times \frac{3}{36} + \dots = \frac{161}{36}$ or 4.472 or $4\frac{17}{36}$</p> <p>(d) $E(X^2) = \frac{1}{36} + 2^2 \times \frac{3}{36} + \dots = \frac{791}{36}$ or full expression or $21\frac{35}{36}$ or awrt 21.97</p> <p>$\text{Var}(X) = \frac{791}{36} - \left(\frac{161}{36}\right)^2$, = 1.9714... *</p> <p>(e) $\text{Var}(2 - 3X) = 9 \times 1.97$ or $(-3)^2 \times 1.97$, = 17.73 awrt 17.7 or $\frac{2555}{144}$</p> | x | 1 | 2 | 3 | 4 | 5 | 6 | $P(X = x)$ | $\frac{1}{36}$ | $\frac{3}{36}$ | $\frac{5}{36}$ | $\frac{7}{36}$ | $\frac{9}{36}$ | $\frac{11}{36}$ | <p>B1, B1, B1 (3)</p> <p>M1, A1 (2)</p> <p>M1, A1 (2)</p> <p>M1, A1 (4)</p> <p>M1, A1 (2)</p> <p>13 marks</p> |
| x | 1 | 2 | 3 | 4 | 5 | 6 | | | | | | | | | | |
| $P(X = x)$ | $\frac{1}{36}$ | $\frac{3}{36}$ | $\frac{5}{36}$ | $\frac{7}{36}$ | $\frac{9}{36}$ | $\frac{11}{36}$ | | | | | | | | | | |
| | <p>(a) 1st B1 for $x = 1, \dots, 6$ and at least one correct probability N.B. $\frac{3}{36} = \frac{1}{12}$ and $\frac{9}{36} = \frac{1}{4}$ 2nd B1 for at least 3 correct probabilities 3rd B1 for a fully correct probability distribution.</p> <p>(b) M1 for attempt to add the correct three probabilities, ft their probability distribution</p> <p>(c) M1 for a correct attempt at $E(X)$. Minimum is as printed. Exact answer only scores M1A1. [Division by 6 at any point scores M0, no ISW. Non-exact answers with no working score M0.]</p> <p>(d) 1st M1 for a correct attempt at $E(X^2)$. Minimum as printed. $\frac{791}{36}$ or awrt 21.97 scores M1A1. 2nd M1 for their $E(X^2) - (\text{their } E(X))^2$. 2nd A1 cso needs awrt 1.97 <u>and</u> $\frac{791}{36} - \left(\frac{161}{36}\right)^2$ or $\frac{2555}{1296}$ or any fully correct expression seen. Can accept <u>at least 4 sf</u> for both. i.e. 21.97 for $\frac{791}{36}$, 4.472 for $\frac{161}{36}$, 20.00 for $\left(\frac{161}{36}\right)^2$.</p> <p>(e) M1 for correct use of $\text{Var}(aX + b)$ formula or a <u>full</u> method. NB $-3^2 \times 1.97$ followed by awrt 17.7 scores M1A1 <u>BUT</u> $-3^2 \times 1.97$ alone, or followed by -17.7, scores M0A0.</p> | | | | | | | | | | | | | | | |

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|-----------------|---|---|
| 4. (a) | Positive skew (both bits) | B1 (1) |
| (b) | $19.5 + \frac{(60-29)}{43} \times 10, = 26.7093\dots$ (N.B. Use of 60.5 gives 26.825... so allow awrt 26.8) | awrt 26.7 M1, A1 (2) |
| (c) | $\mu = \frac{3550}{120} = 29.5833\dots$ or $29\frac{7}{12}$ $\sigma^2 = \frac{138020}{120} - \mu^2$ or $\sigma = \sqrt{\frac{138020}{120} - \mu^2}$ $\sigma = 16.5829\dots$ or ($s = 16.652\dots$) | awrt 29.6 B1 M1 awrt 16.6 (or $s = 16.7$) A1 (3) |
| (d) | $\frac{3(29.6 - 26.7)}{16.6}$ $= 0.52\dots$ (N.B. 60.5 in (b) ...awrt 0.499[or with s awrt 0.497]) | M1A1ft awrt 0.520 (or with s awrt 0.518) A1 (3) |
| (e) | $0.520 > 0$ So it is consistent with (a) | correct statement about their (d) being >0 or <0 ft their (d) B1ft dB1ft (2) |
| (f) | Use <u>Median</u> Since the data is skewed <u>or</u> less affected by outliers/extreme values | B1 dB1 (2) |
| (g) | If the data are <u>symmetrical</u> or <u>skewness is zero</u> or <u>normal/uniform distribution</u> (“mean =median” or “no outliers” or “evenly distributed” all score B0) | B1 (1) 14 marks |
| (b) | M1 for $(19.5 \text{ or } 20) + \frac{(60-29)}{43} \times 10$ or better. Allow 60.5 giving awrt 26.8 for M1A1 Allow their $0.5n$ [or $0.5(n+1)$] instead of 60 [or 60.5] for M1. | |
| (c) | M1 for a correct expression for σ, σ^2, s or s^2 . NB $\sigma^2 = 274.99$ and $s^2 = 277.30$ Condone poor notation if answer is awrt 16.6 (or 16.7 for s) | |
| (d) | M1 for attempt to use this formula using their values to any accuracy. Condone missing 3. 1 st A1ft for using their values to at least 3sf. Must have the 3. 2 nd A1 for using accurate enough values to get awrt 0.520 (or 0.518 if using s) NB Using only 3 sf gives 0.524 and scores M1A1A0 | |
| (e) | 1 st B1 for saying or implying correct sign for their (d). B1g and B1ft. Ignore “correlation” if seen. 2 nd B1 for a comment about consistency with their (d) and (a) being positive skew, ft their (d) only This is dependent on 1 st B1: so if (d) >0 , they say yes, if (d) <0 they say no. | |
| (f) | 2 nd B1 is dependent upon choosing median. | |

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|-----------------|---|-------------------------|
| 5. (a) | Time is a <u>continuous</u> variable <u>or</u> data is in a <u>grouped</u> frequency table | B1 (1) |
| (b) | Area is proportional to frequency <u>or</u> $A \propto f$ <u>or</u> $A = kf$ | B1 (1) |
| (c) | $3.6 \times 2 = 0.8 \times 9$ <p>1 child represented by 0.8</p> | M1 dM1 A1 cso (3) |
| (d) | $(\text{Total}) = \frac{24}{0.8}, = \underline{\underline{30}}$ | M1, A1 (2) |
| 7 marks | | |
| (b) | <p>1st B1 for one of these correct statements. “Area proportional to frequency density” or “Area = frequency” is B0</p> | |
| (c) | <p>1st M1 for a correct combination of any 2 of the 4 numbers: 3.6, 2, 0.8 and 9 e.g. 3.6×2 or $\frac{3.6}{0.8}$ or $\frac{0.8}{2}$ etc BUT e.g. $\frac{3.6}{2}$ is M0 2nd M1 dependent on 1st M1 and for a correct combination of 3 numbers leading to 4th. May be in separate stages but must see all 4 numbers A1cso for fully correct solution. Both Ms scored, no false working seen and <u>comment required</u>.</p> | |
| (d) | M1 for $\frac{24}{0.8}$ seen or implied. | |

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|-----------------|--|---------------------------------|
| 6. (a) | Used to simplify <u>or</u> represent a real world problem Cheaper <u>or</u> quicker <u>or</u> easier (than the real situation) <u>or</u> more easily modified To improve understanding of the real world problem Used to predict outcomes from a real world problem (idea of predictions) | (any two lines) B1 B1 (2) |
| (b) | (3 or 4) Model used to make predictions. (Idea of predicted values based on the model) | B1 |
| | (4 or 3) (Experimental) data collected | B1 |
| | (7) Model is refined. | B1 (3) |
| 5 marks | | |
| (a) | 1 st B1 For one line 2 nd B1 For a second line Be generous for 1 st B1 but stricter for B1B1 | |
| (b) | 1 st & 2 nd B1 These two points can be interchanged. Idea of values from (experimental) data and predicted values based on the model. 1 st B1 for predicted values from model e.g. “model used to gain suitable data” 2 nd B1 for data collected. Idea of experimental data but “experiment” needn’t be explicitly seen | |
| | 3 rd B1 This should be stage 7. Idea of refinement or revision or adjustment | |

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|-----------------|---|---|
| 7. (a) | $P(X < 91) = P\left(Z < \frac{91-100}{15}\right)$ $= P(Z < -0.6)$ $= 1 - 0.7257$ $= 0.2743$ | Attempt standardisation M1 A1 M1 A1 (awrt 0.274) (4) |
| (b) | $1 - 0.2090 = 0.7910$ $P(X > 100+k) = 0.2090 \quad \text{or} \quad P(X < 100+k) = 0.7910 \quad (\text{May be implied})$ $\frac{100+k-100}{15} = 0.81 \quad (\text{ft their } z = 0.81, \text{ but must be } z \text{ not prob.})$ $\underline{k = 12}$ | 0.791 B1 M1 B1 M1, A1ft A1 cao (6) 10 marks |
| (a) | 1 st M1 for attempting standardisation. $\pm \frac{(91-\mu)}{\sigma \text{ or } \sigma^2}$. Can use of 109 instead of 91. Use of 90.5 etc is M0 1 st A1 for -0.6 (or +0.6 if using 109) 2 nd M1 for 1 - probability from tables. Probability should be > 0.5 | |
| (b) | 1 st B1 for 0.791 seen or implied. 1 st M1 for a correct probability statement, but must use X or Z correctly. Shown on diagram is OK 2 nd B1 for awrt 0.81 seen (or implied by correct answer - see below) (Calculator gives 0.80989...) 2 nd M1 for attempting to standardise e.g. $\frac{100+k-100}{15}$ or $\frac{k}{15}$ $\frac{X-100}{15}$ scores 2 nd M0 until the 100+ k is substituted to give k, but may imply 1 st M1 if k= 112.15 seen 1 st A1ft for correct equation for k (as written or better). Can be implied by k = 12.15 (or better) 2 nd A1 for k = 12 only. <u>Answers only</u> k = 112 or 112.15 or better scores 3/6 (on EPEN give first 3 marks) k = 12.15 or better (calculator gives 12.148438...) scores 5/6 (i.e loses last A1 only) k = 12 (no incorrect working seen) scores 6/6 NB Using 0.7910 instead of 0.81 gives 11.865 which might be rounded to 12. This should score no more than B1M1B0M1A0A0. | |