Question	Scheme	Marks	AOs
12(a)	$u=1+\sqrt{x}\Rightarrow x=(u-1)^2\Rightarrow\dfrac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\displaystyle\int\frac{x}{1+\sqrt{x}}dx=\int\frac{(u-1)^2}{u}2(u-1)du$ or $\displaystyle\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times2x^{\frac{1}{2}}du=\int\frac{2x^{\frac{3}{2}}}{u}du=\int\frac{2(u-1)^3}{u}du$	M1	2.1
	$\displaystyle\int_0^{16}\frac{x}{1+\sqrt{x}}dx=\int_1^5\frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$\displaystyle2\int\frac{u^3-3u^2+3u-1}{u}du=2\int\left(u^2-3u+3-\frac{1}{u}\right)du=...$	M1	3.1a
	$=(2)\left[\dfrac{u^3}{3}-\dfrac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\dfrac{5^3}{3}-\dfrac{3(5)^2}{2}+3(5)-\ln5-\left(\dfrac{1}{3}-\dfrac{3}{2}+3-\ln1\right)\right]$	dM1	2.1
	$=\dfrac{104}{3}-2\ln5$	A1	1.1b
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct expression for $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$ (or u') or dx in terms of du or du in terms of dx

M1: Complete method using the given substitution.

This needs to be a correct method for their $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$ leading to an integral in terms of u only (ignore any limits if present) so for each case you need to see:

$$\frac{dx}{du}=f(u)\rightarrow\int\frac{x}{1+\sqrt{x}}dx=\int\frac{(u-1)^2}{u}f(u)du$$

$$\frac{du}{dx}=g(x)\rightarrow\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times\frac{du}{g(x)}=\int h(u)du.$$ In this case you can condone slips with coefficients e.g. allow $\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}\rightarrow\displaystyle\int\frac{x}{1+\sqrt{x}}dx=\int\frac{x}{u}\times\frac{x^{\frac{1}{2}}}{2}du=\int h(u)du$

Question	Scheme	Marks	AOs
7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times5^2 - 20\times5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, −23) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative 1:		
	$\pm\int \left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 + \frac{1}{2}\times5(23+13)$ $= -\frac{455}{12} + 90$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
	(c) Alternative 2:		
	$\int \left(x^3 - 10x^2 + 27x\right) dx = \left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2\right]_0^5 - \frac{1}{2}\times5\times10$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
			(9 marks)

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k}\quad\left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right) \times \frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to k.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x-k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x = Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x \to x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

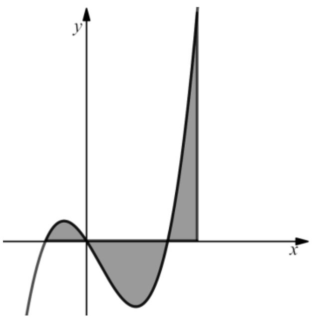
A1*: Proceeds to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**.

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1	2.1
		A1	1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}\ \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1	1.1b
		A1	1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2) \quad \{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \mathrm{g}'(x))$, where $\mathrm{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = \ldots$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$
Note:	Allow $x = \text{awrt } 8.15$
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0,\ B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \rightarrow \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \rightarrow x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2} = \frac{20}{3}$ oe for this mark

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15$ *	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of *x*.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to *A* and *B*	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x$ Or $[..............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

Question	Scheme	Marks	AOs
5	States $\left\{ \lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\ \delta x \text{ is} \right\}$ $\int_4^9 \sqrt{x}\, dx$	B1	1.2
	$= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_4^9$	M1	1.1b
	$= \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\, dx$ with or without the 'dx'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}} \right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\, dx - \int_1^3 \sqrt{x}\, dx$ or for $\int_3^9 \sqrt{x}\, dx$ without reference to a correct $\int_4^9 \sqrt{x}\, dx$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\, dx = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3} x^{\frac{3}{2}} + c \right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\, dx$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Question	Scheme	Marks	AOs
10 (a)	$x=u^2+1 \Rightarrow \mathrm{d}x=2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int \frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)}=\int \frac{3\times 2u\ \mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p=2, q=3$	B1	1.1b
	$=\int \frac{3\times 2\not{u}\ \mathrm{d}u}{u^{\not{2}}(3+2u)}=\int \frac{6\ \mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)}=\frac{A}{u}+\frac{B}{3+2u} \Rightarrow A=..., B=...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6\ \mathrm{d}u}{u(3+2u)}=2\ln u-2\ln(3+2u) \quad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u=\text{"3"}, u=\text{"2"}$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3-2\ln 9)-(2\ln 2-2\ln 7)=2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
		(10 marks)	
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $\mathrm{d}x=2u\mathrm{d}u$ or exact equivalent. E.g. $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$, $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x=u^2+1$, including $\mathrm{d}x \to ...u\mathrm{d}u$ to form an integrand in terms of u. Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p=2, q=3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B.

A1: Correct PF $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$ (Not scored for just the correct values of A and B)

dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns. Look for $P\ln u+Q\ln(3+2u)$

A1ft: Correct integration for their $\frac{A}{u}+\frac{B}{3+2u} \to A\ln u+\frac{B}{2}\ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k\ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u's back to x's and use limits of 5 and 10.

A1: Proceeds to $\ln\frac{49}{36}$. Answers without working please send to review.

Question	Scheme	Marks	AOs
6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=...,B=...,C=...$ **or** $x+2\overline{)x^2+8x-3}$ giving quotient $x+6$: $\underline{x^2+2x}$, $6x-3$, $\underline{6x+12}$, -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+8x-3}{x+2}\,dx=\int x+6-\frac{15}{x+2}\,dx=...-15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2)\quad(+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,dx=\left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ $=(18+36-15\ln 8)-(0+0-15\ln 2)$ $=18+36-(15-45)\ln 2$ or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	$=54-30\ln 2$	A1	1.1b
		(4)	
			(7 marks)

Question	Scheme	Marks	AOs
8	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$f(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots$ (6)	dM1	1.1b
	$(f(x)=)\,2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x)=)(x+4)(2x^2-5x-3)$ $(f(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Question Number	Scheme	Marks
3(a)	$4x^3+2x^2+17x+8 \equiv (Ax+B)(x^2+4)+Cx+D$	
	Compare x^3 terms: A=4	B1
	Compare x^2 terms: B=2	B1
	Compare either x term or constant term: 4A+C =17 or 4B+D =8	
	$\Rightarrow C = ..\text{or } D = ..$	M1
	$\Rightarrow C=1, D=0$	A1
		(4)
(b)	$\int_1^4 \frac{4x^3+2x^2+17x+8}{x^2+4}\,dx = \int_1^4 4x+2+\frac{x}{x^2+4}\,dx$	M1
	$=\left[2x^2+2x,+\frac{1}{2}\ln(x^2+4)\right]_1^4$	M1, M1A1
	$=\left[2\times16+2\times4+\frac{1}{2}\ln(20)\right]-\left[2\times1+2\times1+\frac{1}{2}\ln(5)\right]$	
	$=36+\frac{1}{2}\ln\left(\frac{20}{5}\right)$	dM1
	$=36+\ln(2)$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
4. (a)	$\left\{\int (2x+3)^{12}\,\mathrm{d}x\right\} = \dfrac{(2x+3)^{13}}{(13)(2)}\ \{+c\}$ $\pm\lambda(2x+3)^{13}$	M1
	$\dfrac{(2x+3)^{13}}{(13)(2)}\ \{+c\}$ (Ignore '+ c')	A1
		[2]
(b)	$\left\{\int \dfrac{5x}{4x^2+1}\,\mathrm{d}t\right\} = \dfrac{5}{8}\ln(4x^2+1)\ \{+c\}$ or $\dfrac{5}{8}\ln(x^2+\frac{1}{4})\ \{+k\}$	M1
		A1
		[2]
		4

Notes

(a) **M1**: Gives $\pm\lambda(2x+3)^{13}$ where λ is a constant or $\pm\mu(x+\frac{3}{2})^{13}$

A1: Coefficient does not need to be simplified so is awarded for $\dfrac{(2x+3)^{13}}{(13)(2)}$ or for $\dfrac{2^{12}}{13}(x+\frac{3}{2})^{13}$ i.e. $\dfrac{4096}{13}(x+\frac{3}{2})^{13}$

Ignore subsequent errors and condone lack of constant c

N.B. If a binomial expansion is attempted, then it needs all thirteen terms to be correctly integrated for M1A1

(b) **M1**: Gives $\pm\mu\ln(4x^2+1)$ where μ is a constant or $\pm\mu\ln(x^2+\frac{1}{4})$ or indeed $\pm\mu\ln(k(4x^2+1))$

May also be awarded for $\dfrac{5}{8}\ln(4x+1)$ or $\dfrac{5}{8}\ln(x^2+1)$, where coefficient 5/8 is correct and there is a slip writing down the bracket.

It may also be given for $\pm\mu\ln(u)$ where u is clearly defined as $(4x^2+1)$ or equivalent substitutions such as $\pm\mu\ln(4u+1)$ where $u=x^2$

A1: $\dfrac{5}{8}\ln(4x^2+1)$ or $\dfrac{5}{8}\ln(x^2+\frac{1}{4})$ o.e. The modulus sign is not needed but allow $\dfrac{5}{8}\ln\left|4x^2+1\right|$

Also allow $0.625\ln(4x^2+1)$ and condone lack of constant c

N.B. $\dfrac{5}{8}\ln 4x^2+1$ with no bracket can be awarded M1A0

Question Number	Scheme	Marks
4	$x = 2\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 2\cos\theta$	B1
	$\displaystyle\int \frac{1}{\left(4-x^2\right)^{3/2}}dx = \int \frac{1}{\left(4-4\sin^2\theta\right)^{3/2}} 2\cos\theta(d\theta)$	M1
	$\displaystyle = \int \frac{1}{4}\sec^2\theta(d\theta)\ OR\ \int \frac{1}{4}\times\frac{1}{\cos^2\theta}(d\theta)$	M1
	$= \dfrac{1}{4}\tan\theta$	dM1A1
	Uses limits 0 and $\dfrac{\pi}{3}$ in their integrated expression	
	$= \left[\dfrac{1}{4}\tan\theta\right]_0^{\frac{\pi}{3}} = \dfrac{\sqrt{3}}{4}$	M1A1
		(7 marks)

B1 States either $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$. Condone $x' = 2\cos\theta$

M1 Attempt to produce integral in just θ by substituting $x = 2\sin\theta$ and using $dx = \pm A\cos\theta\ (d\theta)$
You may condone a missing $d\theta$

M1 Uses $1-\sin^2\theta = \cos^2\theta$ and simplifies integral to $\displaystyle\int C\sec^2\theta(\, d\theta)$ or $\displaystyle\int \frac{C}{\cos^2\theta}(\, d\theta)$

Again you may condone a missing $d\theta$

dM1 Dependent upon previous M1 for $\displaystyle\int \sec^2\theta \to \tan\theta$

A1 $\dfrac{1}{4}\tan\theta(+c)$. No requirement for the $+c$

M1 Changes limits in x to limits in θ of 0 and $\dfrac{\pi}{3}$, then subtracts their integrated expression either way around. The subtraction of 0 can be implied if f (0) =0. If the candidate changes the limits to 0 and 60 (degrees) it scores M0, A0. Alternatively they could attempt to change their integrated expression in θ back to a function in x and use the original limits. Such a method would require seeing either $\cos\theta = \sqrt{1-\dfrac{x^2}{4}}$ or $\tan\theta = \dfrac{x}{2}\Big/\sqrt{1-\dfrac{x^2}{4}}$

A1 $\dfrac{\sqrt{3}}{4}$.

Question Number	Scheme	Marks
5.(i)	$\frac{dy}{dx}=\frac{(x+1)-x}{(x+1)^2}=\frac{1}{(x+1)^2}$ or $y=\frac{x}{x+1}=1-\frac{1}{x+1}\Rightarrow\frac{dy}{dx}=\frac{1}{(x+1)^2}$	M1
	(see notes for further methods)	
	$\frac{1}{(x+1)^2}=\frac{1}{4}$ *or* $(x+1)^2=4$ *or* $x^2+2x+1=4$	A1
	$x=1,-3$	M1 A1
		(4)
(ii)	$\int\frac{t+1}{t}\,dt=\int 1+\frac{1}{t}\,dt=t+\ln t\ \ (+c)$ see notes for integration by parts.	M1A1
	$[t+\ln t]_a^{2a}=\ln 7\Rightarrow 2a+\ln 2a-a-\ln a=\ln 7$	
	$a+\ln\left(\frac{2a}{a}\right)=\ln 7\Rightarrow a=\ln\left(\frac{7}{2}\right)$ or $a=\ln 7-\ln 2$	dM1A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
8(a)	**Either** $f(\theta)=9\cos^2\theta+\sin^2\theta=9\cos^2\theta+1-\cos^2\theta$	M1
	$=8\cos^2\theta+1=8\frac{(\cos 2\theta+1)}{2}+1$	M1
	$=5+4\cos 2\theta$	A1 [3]
	Or $f(\theta)=9\frac{(\cos 2\theta+1)}{2}+1\frac{(1-\cos 2\theta)}{2}$	M1 M1
	$=5+4\cos 2\theta$	A1 [3]
(b)	**Either :Way1** splits as $\int_0^{\frac{\pi}{2}} a\theta^2\,d\theta+\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta$	
	$\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta=\;...\theta^2\sin 2\theta\pm\int...\theta\sin 2\theta d\theta$	M1
	$==..\theta^2\sin 2\theta\pm..\theta\cos 2\theta\pm\int..\cos 2\theta d\theta$	dM1
	Integral $=\underline{\underline{\left[2\theta^2\sin 2\theta+2\theta\cos 2\theta-\sin 2\theta\right]}}+\frac{5}{3}\theta^3$	$\underline{\underline{\text{A1}}}$ **B1ft**
	Use limits to give $\left[\frac{5\left(\frac{\pi}{2}\right)^3}{3}-\pi\right]-[0] \quad = \quad \left[\frac{5\pi^3}{24}-\pi\right]$	ddM1 A1 [6]
		(9 marks)
1st 4 marks	**Or: Way 2** $\int_0^{\frac{\pi}{2}}\theta^2 f(\theta)d\theta=\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta=$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\int..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(...\theta\pm..\sin 2\theta)-..\theta\left(..\theta^2\pm...\cos 2\theta\right)\pm\int.\left(..\theta^2\pm...\cos 2\theta\right)d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-2\theta\left(\frac{5\theta^2}{2}-\cos 2\theta\right)+\left(\frac{5\theta^3}{3}-\sin 2\theta\right)$	A1 **B1ft**
1st 4 marks	**Or: Way 3 Way 2 that goes back to Way One**	
	$\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta=\theta^2(..\theta\pm..\sin 2\theta)-\int..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int...\theta^2 d\theta\right)\pm\int..\theta\sin 2\theta d\theta$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int...\theta^2 d\theta\right)\pm..\theta\cos 2\theta\pm\int..\cos 2\theta d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-\frac{10}{3}\theta^3+2\theta\cos 2\theta-\sin 2\theta$	A1 **B1ft**

Question Number	Scheme	Notes	Marks
	Note that 2^x can be replaced by $e^{x\ln 2}$ throughout and allow omission of "dx" throughout		
5	$\int x2^x\,dx = x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\,dx$	M1: Integrates by parts the right way around to obtain an expression of the form $ax2^x - \int b2^x\,dx$. Allow a = 1 and/or b = 1.	M1A1
		A1: $x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\,dx$ (Does not need to be seen all on one line)	
	$\int x2^x\,dx = x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	dM1: Completes to obtain an expression of the form $... - k2^x$	dM1A1
		A1: $x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	
	$\left[x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}\right]_0^2 = \left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - \left(\frac{0\times 2^0}{\ln 2} - \frac{2^0}{(\ln 2)^2}\right)$ Uses the limits 0 and 2 and subtracts the right way round. F(0) may be implied by e.g. $\frac{1}{(\ln 2)^2}$ But $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - (0)$ or just $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right)$ is ddM0		ddM1
	$\left(= \frac{8}{\ln 2} - \frac{4}{(\ln 2)^2} + \frac{1}{(\ln 2)^2}\right)$		
	$= \frac{8\ln 2 - 3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256 - 3}{(\ln 2)^2}, \frac{\ln 2^8 - 3}{(\ln 2)^2}$ Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1
			(6 marks)

	Alternative by substitution:		
	$u=2^x \Rightarrow \int x2^x\,\mathrm{d}x=\int \frac{\ln u}{\ln 2}.u.\frac{1}{u\ln 2}\,\mathrm{d}u=\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u$		
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u=\frac{1}{(\ln 2)^2}\left(u\ln u-\int \mathrm{d}u\right)$	M1: Integrates by parts the right way around to obtain an expression of the form $au\ln u-\int b\,\mathrm{d}u$. Allow a = 1 and/or b = 1. A1: $\frac{1}{(\ln 2)^2}\left(u\ln u-\int \mathrm{d}u\right)$	M1A1
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u=\frac{1}{(\ln 2)^2}(u\ln u-u)$	dM1: Completes to obtain an expression of the form $...-ku$ A1: $\frac{1}{(\ln 2)^2}(u\ln u-u)$	dM1A1
	$\left[\frac{1}{(\ln 2)^2}(u\ln u-u)\right]_1^4=\frac{1}{(\ln 2)^2}(4\ln 4-4)-(\ln 1-1)$ Uses the limits 1 and 4 and subtracts the right way round.		M1
	$=\frac{4\ln 4-3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256-3}{(\ln 2)^2},\frac{\ln 2^8-3}{(\ln 2)^2}$, Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x \, dx = \frac{x^{-1}}{-1} \ln x - \int \frac{x^{-1}}{-1} \times \frac{1}{x} \, dx$	M1A1
	$= \frac{x^{-1}}{-1} \ln x + \int x^{-2} \, dx$	
	$= \frac{x^{-1}}{-1} \ln x + \frac{x^{-1}}{-1} (+c)$	M1A1
	$\int_1^e \frac{\ln x}{x^2} dx = \left[\frac{-1}{x} \ln x - \frac{1}{x}\right]_1^e = \left(\frac{-1}{e} \ln e - \frac{1}{e}\right) - \left(\frac{-1}{1} \ln 1 - \frac{1}{1}\right)$	M1
	$= 1 - \frac{2}{e}$	A1
		(6)
	Alternative by substitution:	
	$u = \ln x \Rightarrow \int \frac{\ln x}{x^2} dx = \int \frac{u}{e^{2u}} e^u \, du = \int u e^{-u} \, du$	
	$\int u e^{-u} \, du = -u e^{-u} - \int -e^{-u} \, du$	M1A1
	$\int u e^{-u} \, du = -u e^{-u} - e^{-u} (+c)$	M1A1
	$\int_1^e \frac{\ln x}{x^2} dx = \left[-u e^{-u} - e^{-u}\right]_0^1 = \left(-\frac{1}{e} - \frac{1}{e}\right) - (0 - 1)$	M1
	$= 1 - \frac{2}{e}$	A1

(Condone the lack of "dx" throughout)

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)

Must see an expression of the form $Ax^{-1} \ln x \pm B \int x^{-1} \times \frac{1}{x} \, dx$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\frac{x^{-1}}{-1} \ln x - \int \frac{x^{-1}}{-1} \times \frac{1}{x} \, dx$, $\left[-\frac{1}{x} \ln x\right]_1^e + \int \frac{1}{x^2} \, dx$

M1: It is for 'combining' their two terms in x correctly and integrating their resulting term by adding one to the power.

A1: A completely correct integral (simplified or un-simplified)

For students who substitute in limits early, look for e.g. $\left(\frac{e^{-1}}{-1} \ln e\right) - \left(\frac{1^{-1}}{-1} \ln 1\right) + \left[\frac{x^{-1}}{-1}\right]_1^e$

M1: It is for substituting in the limits 1 and e (either way round) and subtracting.

Question Number	Scheme	Marks
5(i)	$\int\left((3x+5)^9+e^{5x}\right)dx=\frac{(3x+5)^{10}}{30},+\frac{e^{5x}}{5}(+c)$	M1A1, B1
		(3)
(ii)	$\int \frac{x}{x^2+5}dx=\frac{1}{2}\ln\left(x^2+5\right)$	M1A1
	$\int_2^b \frac{x}{x^2+5}dx=\ln\left(\sqrt{6}\right)\Rightarrow\frac{1}{2}\ln\left\|b^2+5\right\|-\frac{1}{2}\ln\left\|2^2+5\right\|=\ln\left(\sqrt{6}\right)$	M1
	$\Rightarrow\ln\left(\frac{b^2+5}{9}\right)=\ln 6\Rightarrow b=7$	**dd**M1, A1
		(5)
		(8 marks)

(i)

M1: For an integral of the form $C(3x+5)^{10}$ or $C(3x+5)^{9+1}$ where C is a constant and no other powers of $(3x+5)$

A1: $\frac{(3x+5)^{10}}{30}$. No need for + c. Allow un-simplified e.g. $\frac{\frac{1}{3}(3x+5)^{10}}{10}$.

B1: $e^{5x}\rightarrow\frac{e^{5x}}{5}$

Mark each integration independently i.e. there is no need to see everything all on one line.

(ii)

M1: For an answer of the form $C\ln k\left(x^2+5\right)$ where C and k are constants. Allow log for ln.

A1: $\frac{1}{2}\ln k\left(x^2+5\right)$ or $\ln k\left(x^2+5\right)^{\frac{1}{2}}$ or $\frac{1}{2}\ln k\left|x^2+5\right|$. Allow log for ln.

M1: Substitutes in both 2 and b for x correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b=7$ only. $b=\pm7$ scores A0 unless the -7 is rejected.

<u>Note: May see integration by substitution in (ii)</u>

E.g. $u=x^2+5$

M1: $\int\frac{x}{x^2+5}dx=\int\frac{x}{u}\frac{du}{2x}=\frac{1}{2}\ln u$

For an answer of the form $C\ln k(u)$ where C is a constant

Allow log for ln as above.

A1: $\frac{1}{2}\ln ku$

M1: $\left[\frac{1}{2}\ln u\right]_9^{b^2+5}=\frac{1}{2}\ln\left(b^2+5\right)-\frac{1}{2}\ln 9=\ln\sqrt{6}$

Substitutes in both 9 and b^2+5 correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b=7$ only. $b=\pm7$ scores A0 unless the -7 is rejected.

Question Number	Scheme	Marks
8	$2+\ldots$	B1
	Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants	M1
	$\frac{3}{x}$ or $-\frac{1}{x-1}$ or $A = 3$ or $B = -1$	A1
	$\frac{3}{x}-\frac{1}{x-1}$	A1 **(B1 on Epen)**
	$\int_3^4 \frac{2x^2-3}{x(x-1)}\,dx = \int_3^4 \left(2+\frac{3}{x}-\frac{1}{x-1}\right)dx$	
	$=\left[2x+3\ln x-\ln(x-1)\right]_3^4$	M1 A1ft
	$=\left(8+3\ln 4-\ln 3\right)-\left(6+3\ln 3-\ln 2\right)=2+\ln\left(\frac{128}{81}\right)$	M1 A1cso
		(8 marks)

B1: 2 +…

M1: Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants

A1: $\frac{3}{x}$ or $-\frac{1}{x-1}$ or one correct constant

B1: $\frac{3}{x}-\frac{1}{x-1}$

M1: For $\int \frac{*}{x}+\frac{*}{x-1}\,dx \to p\ln mx+q\ln n(x-1)$ where *, p, q, m and n are constants.

A1ft: $2x+3\ln x-\ln(x-1)$. Follow through their "2", A and B so look for "2"x+A$\ln x$+B$\ln(x-1)$. This mark can be withheld if the brackets are missing unless subsequent work suggests their intended presence.

M1: For substituting in 3 and 4, subtracting either way around and using correct addition or subtraction log laws at least once.

A1: cso $2+\ln\left(\frac{128}{81}\right)$ or $2+\ln\left(1\frac{47}{81}\right)$ (Do not allow $2+\ln\left(\frac{2^7}{3^4}\right)$) $2+\ln\left(\frac{128}{81}\right)+c$ is also A0

Question Number	Scheme		Marks
6. (i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\ \{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\ \{+c\}\}$	*{Ignore subsequent working}*.	**[2]**
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ **or** $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y \sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^x\ \{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ **or** $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,dy = \int e^x\,dx$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^x\ \{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^x \to e^x$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^0 + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
			[7]
			12