

# Mark Scheme (Results)

# January 2016

## International Advanced Level in Core Mathematics C12 (WMA01/01)





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### General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

## **General Instructions for Marking**

- 1. The total number of marks for the paper is 125.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{\text{will be used for correct ft}}$
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- **\*** The answer is printed on the paper or ag- answer given
- L or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the

subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.
- 8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

### General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

#### Method mark for solving 3 term quadratic:

- 1. Factorisation
  - $(x^{2} + bx + c) = (x + p)(x + q)$ , where |pq| = |c|, leading to x = ...

 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where |pq| = |c| and |mn| = |a|, leading to  $x = \dots$ 

#### 2. Formula

Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*).

#### 3. Completing the square

Solving  $x^2 + bx + c = 0$ :  $(x \pm \frac{b}{2})^2 \pm q \pm c$ ,  $q \neq 0$ , leading to  $x = \dots$ 

#### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

#### Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### Exact answers

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

#### Answers without working

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

## January 2016 Core Mathematics C12 Mark Scheme

Question Number	Scheme	Marks
1(a)	$u_2 = 2 \times 2 - 6 = -2$ , $u_3 = 2 \times (-2) - 6 = -10$ or $u_3 = 2 \times (2 \times 2 - 6) - 6 = -10$	M1 A1
		[2]
(b)	$\sum_{i=1}^{4} u_i = 2 + (-2) + (-10)$	M1
	+ (-26)	A1ft
	= -36	A1
		[3]
	Notes	5 marks
(a)	M1: Attempt to use the given formula correctly at least once. This may be implied by a correct $u_2$ or a value for $u_3$ which follows through from their $u_2$ or implied by correct answer for $u_3$ A1: $u_3$ correct and no incorrect work seen	value for
(b)	M1: Uses sum of the 3 numerical terms from part (a) (may be implied by correct answer for the Attempting to sum an AP here is M0. A1ft: obtains $u_4$ correctly (may be attempted in part (a)) and adds to sum of the first three term (a) A1: -36 cao (-36 implies both A marks)	
	Special Cases:Some candidates attempt $u_2 + u_3 + u_4 + u_5$ in part (b) – allow M1 onlySome candidates mis-copy one of their terms from part (a) into part (b) – allow M1 only	

Question Number	Scheme				
	Way 1:	W	ay 2:	Way 3:	
2(i)	$\frac{49}{\sqrt{7}} = \frac{7^2}{7^{\frac{1}{2}}} = 7^{2-\frac{1}{2}}$ $7\sqrt{7} = 7^{1+\frac{1}{2}}$		M1		
	( <i>a</i> =	$=)1\frac{1}{2}$ (oe) or	see answer = $7^{1\frac{1}{2}}$		A1
		/ 2 ( )			[2]
(ii)	Way 1:			Way 2:	
	$\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$		(15√2	$(\sqrt{18} - 4)$	M1
	$=\frac{\dots}{2}$		$=15\sqrt{36}$ -	$60\sqrt{2} + 20\sqrt{18} - 80$	B1
	$\frac{10}{\sqrt{18}-4} = 5\left(3\sqrt{2}+4\right) = 15\sqrt{2}$	$\sqrt{2}$ + 20 *		$\frac{60\sqrt{2} + 60\sqrt{2} - 80}{\frac{10}{8} - 4} = 15\sqrt{2} + 20*$	A1cso
					[3]
		Not			5 marks
(i)	Way 1:		Vay 2:	Way 3:	
		and adds the A1: cao (ar low work with	fraction to $7\sqrt{7}$ ir powers of 7 nswer only is 2 man inexact decimals for		o obtain a
(ii)	$49 \times 7^{-2} = 18$ Way 1:	$0.52 \Rightarrow \log 18.$	$52 = 1.4999 \Rightarrow c$	$\frac{u = 1.5 \text{ scores M1A0}}{\text{Way 2:}}$	
	M1: Multiply numerator and denot $\sqrt{18} + 4$ or equivalent. The statem $\frac{10(\sqrt{18} + 4)}{(\sqrt{18} - 4)(\sqrt{18} + 4)}$ is sufficient b allow $\frac{10(\sqrt{18} + 4)}{\sqrt{18} - 4(\sqrt{18} + 4)}$ unless m brackets are implied by subsequent B1: Correctly obtains $\pm 2$ in the det (Must follow M1 – i.e. treat as A implied by e.g. $\frac{10(\sqrt{18} + 4)}{18 - 16} = 5($ A1: Correct result with no errors st $\sqrt{18} = 3\sqrt{2}$ used before their find Note that for Way 1, correct work $5\sqrt{18} + 20$ followed by $15\sqrt{2} + 2$	thent but do not nissing at work. nominator 1). May be $\sqrt{18} + 4$ seen and hal answer. i leading to	at least 3 (not nec B1: All 4 terms c A1) A1: Obtains 10 w implied by e.g. 2	expand $(15\sqrt{2} + 20)(\sqrt{18} - \frac{1}{2})$ cessarily correct) terms correct ( <b>Must follow M1</b> – i with no errors and $\sqrt{18} = 3\sqrt{2}$ $20\sqrt{18} = 60\sqrt{2}$ and concluins nswer i.e. not just $10 = 10$	.e. treat as $\sqrt{2}$ seen or

Question Number	Scheme	Marks
3.	$\int \left( 6x - 3 - \frac{2}{\sqrt{x}} \right) dx = \frac{6x^2}{2} - 3x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + (c)$	M1 A1 A1
	$\left[\frac{6x^2}{2} - 3x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + (c)\right]_1^4 = (28) - (-4) = 32$	M1 A1
		[5]
		5 marks
	Notes	
	M1: Attempt to integrate original $f(x)$ – at least one power increased $x^n \rightarrow x^{n+1}$ A1: Two of the three terms correct un-simplified or simplified (Constant not required) A1: All three terms correct un-simplified or simplified (Constant not required) M1: Substitutes limits 4 and 1 into their 'changed' function and subtracts the right way round A1: 32 cao (32 + c is A0) The question requires the use of calculus so a correct answer only scores no marks)	

Question Number	Scheme	Marks
4.	$a + 3d = 3$ <b>OR</b> $\frac{6}{2}(2a + 5d) = 27$ $a + 3d = 3$ <b>AND</b> $\frac{6}{2}(2a + 5d) = 27$	M1 A1
	$a + 3d = 3$ <b>AND</b> $\frac{6}{2}(2a + 5d) = 27$	Al
	Eliminates one variable to find <i>a</i> or <i>d</i> from 2 equations in <i>a</i> and <i>d</i>	dM1
	Obtains $a = 12$ or $d = -3$	A1
	Obtains $a = 12$ and $d = -3$	A1
		[6] 6 marks
	Notes	
	<b>M1A1:</b> Writes down a correct (possibly un-simplified) equation for $4^{th}$ term <b>or</b> for sum of the first Allow the individual terms to be added for the sum e.g. $a + a + d + a + 2d + a + 3d + a + 4d + a + 4$	5d = 27 simplified)

Question number			S	Scheme				Marks
5(a)				through from O	of a positive sin O with at least Condone diffe ow the x-axis.	one complet	e cycle	B1
				shown	shape with one from O to $\frac{3\pi}{2}$ ) $\frac{\pi}{2}$ and $\pi$	-		B1
								[2]
(b)	x	0		$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$		
	У	0		0.5	0.866	1		
	May be in	plied by use		s $\frac{1}{2} \times \frac{\pi}{12}$ $\frac{1}{2}h = \frac{1}{2}$	$\frac{1}{2} \left(\frac{\pi}{6} - \frac{\pi}{12}\right) = \frac{1}{2}$	·(0.261)		B1
		{(	0+1)+	+2(0.5+	0.866)}			M1
		0.48	885176	576 <b>a</b> v	vrt 0.49			A1
								[3] 5 marks
				N	lotes			•
(a)	Notes as above <b>B1</b> : Correct shape with positive gradient through <i>O</i> <b>B1</b> : Need not see endpoints labelled. Ignore any part of the curve to the left of the origin but if the curve extends beyond $x = \frac{3\pi}{2}$ then then $x = \frac{3\pi}{2}$ must be labelled on the diagram. Labels for $\frac{\pi}{2}$ and $\pi$ may be on the diagram or in the text but not just in a table of values and must be in radians not degrees. (Allow awrt 1.57 and 3.14) The amplitudes must not be significantly different above and below the <i>x</i> -axis.						s for $\frac{\pi}{2}$ and	
<b>(b)</b>	<b>B1:</b> Need $\frac{1}{2}$ of $\frac{\pi}{12}$							
	12 M1: requires first bra additional values fro If values used in brac A1: for awrt 0.49 Separate trapezia ma	acket to contain the two in the two in whets are $x$ values are	ain firs the tab alues in	t plus las ble. stead of	t values <b>and</b> se <i>y</i> values this sc	ores M0.		eno
	Special Case: Bracketing mistake: i.e. $\frac{\pi}{24}(0+1)+2(0.5+0.866)$ (=2.86)							
	scores <b>B1 M1 A0</b> un full marks can be giv <b>Need to see trapeziu</b>	en).		-			n done cor	rectly (then

Question Number	Scheme	Marks	
6.	$f(x) = x^3 + x^2 - 12 x - 18$		
(a)	Attempts f(±3)	M1	
	$\{f(-3)=\}$ 0 so $(x + 3)$ is a factor of $f(x)$ .	A1	
		[2]	
(b)	$x^3 + x^2 - 12x - 18 = (x+3)(x^2 + \dots$	M1	
	$x^{3} + x^{2} - 12x - 18 = (x+3)(x^{2} - 2x - 6)$ or $x^{3} + x^{2} - 12x - 18 = (x+3)(x - 1 + \sqrt{7})(x - 1 - \sqrt{7})$ oe	A1	
		[2]	
(c)	(x =) -3	B1	
	$x = \frac{2 \pm \sqrt{4 + 24}}{2} = 1 \pm \sqrt{7} \text{ or by completion of square } (x - 1)^2 = 7 \text{ so } x = 1 \pm \sqrt{7}$ or $(x - 1 + \sqrt{7})(x - 1 - \sqrt{7}) = 0 \Rightarrow x = 1 \pm \sqrt{7}$	M1 A1	
		[3]	
		7 marks	
	Notes		
(a)	M1: As on scheme – must use the <u>factor theorem</u> A1: for seeing 0 and conclusion which may be in a preamble and may be minimal e.g. QED, proven, true, tick etc. There must be no obvious errors but need to see at least $(-3)^3 + (-3)^2 - 12(-3) - 18 = 0$ for A1 but allow invisible brackets e.g. $-3^3 + -3^2 - 12(-3) - 18 = 0$ provided there are no obvious errors.		
(b)	M1: Uses $(x + 3)$ as a factor and obtains correct first term of quadratic factor by division or any method e.g. comparing coefficients or finding roots and factorising A1: Correct quadratic and writes $(x+3)(x^2-2x-6)$ or $(x+3)(x-1+\sqrt{7})(x-1-\sqrt{7})$ oe Note that this work may be done in part (a) and the result re-stated here.		
(c)	<b>B1:</b> States -3 <b>M1:</b> Method for finding their roots. Allow the usual rules applied to their quadratic. <b>This mar</b> finding the roots and not for just finding factors. You may need to check their roots if no shown e.g. if they give decimal answers (3.645, -1.645)		
	A1: need both roots. Correct answer implies M mark. Allow $x = \frac{2 \pm \sqrt{28}}{2}$		
	If they give extra roots e.g. $x = -3$ , $-1$ , $\frac{2 \pm \sqrt{28}}{2}$ , lose the final A mark (B1M1A0)		

Question Number	Scheme	Marks
7(a)	$(1+kx)^8 = 1 + {8 \choose 1}(kx) + {8 \choose 2}(kx)^2 + {8 \choose 3}(kx)^3 \dots$	M1
	$=1+8kx,+28k^2x^2,+56k^3x^3+$	B1, A1, A1
(b)	1512	[4]
(0)	Sets "56 $k^3$ " = 1512 and obtains $k^3 = \frac{1512}{56}$	M1 A1
	So <i>k</i> = 3	A1
		[3]
	NT /	7 marks
(a)	<b>Notes</b> M1: The method mark is awarded for an attempt at the Binomial expansion to get the third a	
	term. The <b>correct</b> binomial coefficient needs to be combined with the correct power of x. Ign errors and omission of or incorrect powers of k. Accept any notation for ${}^{8}C_{2}$ or ${}^{8}C_{3}$ , e.g. $\begin{pmatrix} 8\\ 2 \end{pmatrix}$ 28 or 56 from Pascal's triangle. This mark may be given if no working is shown, but either or both of $28k^{2}x^{2}$ and $56k^{3}x^{3}$ is <b>B1:</b> This is for $1 + 8kx$ and not for just $1 + \begin{pmatrix} 8\\ 1 \end{pmatrix} (kx)$ <b>A1:</b> is cao and is for $28k^{2}x^{2}$ or for $28(kx)^{2}$	$\binom{8}{3}$ or $\binom{8}{3}$ or
	A1: is cao and is for $56k^3x^3$ or for $56(kx)^3$	
	Any extra terms in higher powers of x should be ignored. Allow terms separated by commas or given as a list for all the marks.	
(b)	M1: Sets their coefficient of $x^3 = 1512$ and obtains $k^n =$ where <i>n</i> is 1 or 3	
	A1: $k^3 = \frac{1512}{56}$ or equivalent e.g. 27 (May be implied by their final answer)	
	A1: $k = 3 \operatorname{cao}(\pm 3 \operatorname{is} A0)$	
	Note (b) can be marked independently of part (a) so part (a) might be incorrect or not	attempted but
	they have $56k^3 = 1512$ etc. in (b)	

Question Number	Scheme	Marks
	$7\sin x = 3\cos x$	
<b>8</b> (a)	$(\tan x =)\frac{3}{7}$	B1
		[1]
(b)	$\tan\left(2\theta+30\right)=\frac{3}{7}$	B1ft
	$\tan^{-1}\frac{3}{7}(\alpha)$	M1
	One of $\theta$ = awrt 87 or awrt 177 or awrt 267 or awrt 357	Al
	Follow through any of their final $\theta$ 's for $\theta \pm 90n$ within range	A1ft
	All of $\theta = 86.6, 176.6, 266.6, 356.6$	A1
		[5]
		6 marks
	Notes	
(a)	<b>B1:</b> $(\tan x =)\frac{3}{7}$ or exact equivalent so accept recurring decimal (0.428571) but not roun	nded answer
(b)	<b>B1ft:</b> Correct equation as shown or follow through their value for tan <i>x</i> from part (a). Must	be
	$\tan(2\theta+30) = \dots$ but $2\theta+30$ may be implied later by an attempt to subtract 30 and then	divide by 2.
	If the processing is unclear or incorrect and $2\theta + 30$ is never seen, score B0 here.	
	<b>M1:</b> Finds arctan of their $\frac{3}{7}$ . Could be implied by their value e.g. 23.19 or just $\tan^{-1} \frac{3}{7}$ "	
	A1: For one of either $\theta$ = awrt 87 or awrt 177 or awrt 267 or awrt 357	
	A1ft: Follow through any of their final answers to which an integer multiple of 90 has been subtracted to give another solution in range but not for adding a multiple of 90 to just $\alpha$ .	added or

Question Number	Scheme	Marks			
9.(a)	$130000 \times (1.02) = 132600 *$ or $2\% = 2600$ and $130000 + 2600 = 132600 *$	B1			
		[1]			
(b)	( <i>r</i> =) 1.02	B1			
(c)		[1]			
	Uses $130000 \times (1.02)^{N-1} > 260000$ or $130000 \times (1.02)^{N-1} = 260000$	M1			
	So $(1.02)^{N-1} > 2$	A1			
	$(N-1)\log_{10}(1.02) > \log_{10} 2$ or $(N-1)\log_{10}(1.02) = \log_{10} 2$ or $(N-1) > \log_{1.02} 2$ or $(N-1) = \log_{1.02} 2$	M1			
	$N > \frac{\log_{10} 2}{\log_{10} (1.02)} + 1*$	Alcso			
		[4			
(d)	(N=) 37	B1			
		[1			
	Notes	7 mark			
<b>(a)</b>	<b>B1:</b> A reason must be provided for this mark as the answer is printed.				
	Allow both $130000 \times (1+2\%)$ and $130000 \times (102\%)$ as both give the correct answer when entered this				
	way on a calculator. But not 130000×1+2%				
(b)	<b>B1:</b> For 1.02 oe e.g. allow $\frac{51}{50}$				
(c)	<b>M1:</b> Correct inequality or equality – may use <i>r</i> or their <i>r</i> or 1.02 and may use <i>N</i> or <i>n</i> . <b>A1:</b> $(1.02)^{N-1} > 2$ cao. Allow $(1.02)^{n-1} > 2$				
	M1: Correct use of logs power rule on their previous line which must have come from using t of a GP. Condone missing brackets for this mark e.g. $N - 1\log_{10} (1.02) > \log_{10} 2$ . (May follow instead of > or use of r instead of 1.02 or use of N instead of N - 1). These cases can get MOAU the base to be absent or just 'ln' for this mark. If the inequality sign is reversed at this point, st M1.	use of = 0M1. Allow			
	A1*: Answer is <b>exactly</b> as printed ( <b>including the bases</b> ) and <b>all</b> inequality work should be corprevious marks scored and <b>no missing brackets earlier</b> . Allow this mark to score from a corr line provided the power rule is used. So fully correct work leading to				
	$(N-1)\log_{10}(1.02) > \log_{10} 2 \Longrightarrow N > \frac{\log_{10} 2}{\log_{10}(1.02)} + 1$ scores the final M1A1 but				
	$(1.02)^{N-1} > 2 \Rightarrow N > \frac{\log_{10} 2}{\log_{10} (1.02)} + 1$ scores M0A0 (no explicit use of power rule)				
(d)	<b>B1:</b> Only need $N = 37$ – may follow trial and error or uses logs to a different base. Do not allow $N \ge 37$ or $N > 37$ or $N = 37.0$				

Question Number	Scheme	Marks
	$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000$ $\frac{dy}{dx} = 12 \times \frac{5}{4}x^{\frac{1}{4}} - \frac{10}{18}x$	
10.(a)	$\frac{dy}{dx} = 12 \times \frac{5}{4} x^{\frac{1}{4}} - \frac{10}{18} x$	M1 A1
(b)		[2]
(0)	Put $12 \times \frac{5}{4} x^{\frac{1}{4}} - \frac{10}{18} x = 0$ so $x^n = k  (n \in \Box, k \neq 0)$	M1
	$\therefore x = ()^{\frac{4}{3}}$ $\therefore x = 81$	dM1
	$\therefore x = 81$ (Ignore x = 0 if given as a second solution)	A1
	So $y = 12(81)^{\frac{5}{4}} - \frac{5}{18}(81)^2 - 1000$ i.e. $y = 93.5$	dM1A1
		[5]
(c)	$\frac{d^2 y}{dx^2} = -\frac{15}{4}x^{-\frac{3}{4}} - \frac{5}{9}$	B1ft
	Substitutes their non-zero x (positive or negative) into their second derivative.	M1
	Obtains maximum after correctly substituting 81 into correct second derivative to give correct	
	negative quantity $-\frac{15}{36}$ o.e. or decimal e.g0.4 (see note below) and considers negative	
	sign deducing maximum. $d^2 = 15$	A1
	Note that a correct second derivative followed by $x = 81 \Rightarrow \frac{d^2 y}{dx^2} = \frac{15}{4} 81^{-\frac{3}{4}} - \frac{5}{9} = -\frac{5}{12}$ therefore	
	maximum scores B1M1A0 here.	
		[3 10 marks
	Natas	
	Notes	
<b>(a)</b>	<b>M1:</b> Attempt to differentiate – power reduced by one $x^n \rightarrow x^{n-1}$ (but not just 1000 $\rightarrow$ 0) <b>A1:</b> Two correct terms and no extra terms. Terms may be un-simplified.	
(a) (b)	<b>M1:</b> Attempt to differentiate – power reduced by one $x^n \rightarrow x^{n-1}$ (but not just 1000 $\rightarrow$ 0)	
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 $\to$ 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where <i>n k</i> is non-zero	ark can
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just $1000 \to 0$ ) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where $n$ k is non-zero dM1: Correct processing to obtain a value for $x$ . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of $x$ . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} (a - bx^{\frac{3}{4}}) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$	ark can have the
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 $\to$ 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where <i>n k</i> is non-zero dM1: Correct processing to obtain a value for <i>x</i> . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must <b>correct powers of</b> <i>x</i> .	ark can have the
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	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 $\to$ 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where $n$ k is non-zero dM1: Correct processing to obtain a value for $x$ . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of $x$ . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} \left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for $x$ into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao	ark can have the $x = \sqrt[3]{k}$ he first
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 $\to$ 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where $n$ k is non-zero dM1: Correct processing to obtain a value for $x$ . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must <b>correct powers of</b> $x$ . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} (a - bx^{\frac{1}{4}}) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their <b>positive</b> value for $x$ into $y = \dots$ and <b>not into</b> $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao If $x = 81$ appears from <b>no working</b> following a correct derivative score M1M0A0 then allow fu B1ft: Correct follow through second derivative M1: Substitutes their non-zero $x$ ( <b>positive or negative</b> ) into their second derivative.	ark can have the $x = \sqrt[3]{k}$ he first
<b>(b)</b>	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just $1000 \to 0$ ) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where $n$ k is non-zero dM1: Correct processing to obtain a value for $x$ . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of $x$ . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} \left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for $x$ into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao If $x = 81$ appears from no working following a correct derivative score M1M0A0 then allow fu B1ft: Correct follow through second derivative M1: Substitutes their non-zero $x$ (positive or negative) into their second derivative. Note: Solving $\frac{d^2y}{dx^2} = 0$ is M0	ark can have the $x = \sqrt[3]{k}$ he first <u>all recovery</u>
(b)	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 $\to$ 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where $n$ k is non-zero dM1: Correct processing to obtain a value for $x$ . (Dependent on the first method mark). This m only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must <b>correct powers of</b> $x$ . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} (a - bx^{\frac{1}{4}}) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their <b>positive</b> value for $x$ into $y = \dots$ and <b>not into</b> $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao If $x = 81$ appears from <b>no working</b> following a correct derivative score M1M0A0 then allow fu B1ft: Correct follow through second derivative M1: Substitutes their non-zero $x$ ( <b>positive or negative</b> ) into their second derivative.	ark can have the $x = \sqrt[3]{k}$ he first <u>all recovery</u>

Question Number	Scheme	Marks
11(a)	$16^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos \angle YXZ$	M1
	$\cos \angle YXZ = \frac{10^2 + 12^2 - 16^2}{2 \times 10 \times 12}$ or $\frac{-12}{240}$ or $-0.05$	A1
	$\angle BOC = 1.62(08)$ (N.B. 92.87 degrees is A0)	A1
		[3]
(b)	Uses $s = 5\theta$ with their $\theta$ from part (a)	M1
	awrt 8.1	A1
	Perimeter = $r\theta + 28$ , = 28 + their arc length	M1
	awrt 36.1	A1
(a)		[4]
(c)	area of sector $=\frac{1}{2}(5)^2 \theta$	B1ft
	area of triangle = $\frac{1}{2}$ 10×12sin $\theta$ (= 59.92 or 59.93)	B1ft
	Area of shaded region = $\frac{1}{2} \times 10 \times 12 \sin \theta - \frac{1}{2} (5)^2 \theta = 59.9 20.2$ = 39.7 (cm <sup>2</sup> )	M1 A1
		[4] (11 marks)
	Notes	
(a)	M1: Uses cosine rule – must be a correct statement A1: Correct value or correct numerical expression for $\cos \angle YXZ$ A1: accept awrt 1.62 and must be seen in part (a) (answer in degrees is A0 (92.865))	
(b)	M1: Uses $s = 5\theta$ with their $\theta$ in radians, or correct formula for degrees if working in degrees	5
	<ul> <li>A1: Accept awrt 8.1 (may be implied by their perimeter)</li> <li>M1: Adds their arc length to 28 or (16 + 7 + 5)</li> <li>A1: Accept awrt 36.1 do not need units (ignore any given)</li> </ul>	
(c)	B1ft: This formula used with their $\theta$ in radians or correct formula for degrees B1ft: Correct formula for area used – may use half base times height (may be implied by a co (59.9)) M1: Subtracts their sector area from their triangle area <b>this way round</b> . A1: awrt 39.7 – do not need units (ignore any given)	orrect answer
	Alternative approach to finding angle YXZ and area of triangle:	
	Let foot of perpendicular from X to YZ be W and $XW = h$ and $YW = x$ so $WZ = 16 - x$ :	
	$h^{2} + x^{2} = 100,  h^{2} + (16 - x)^{2} = 144 \implies x = \frac{53}{8}, h = \frac{3\sqrt{399}}{8}$ M1: Correct work leading to values	of <i>x</i> and <i>h</i>
	$\angle YXZ = \sin^{-1}\left(\frac{53}{80}\right) + \sin^{-1}\left(\frac{25}{32}\right) = 1.62$ A1:Correct expression for $\angle YXZ$ , A1: awrt 1.62	
	The B1 for the triangle area in (c) can then score for $\frac{1}{2} \times 16 \times "\frac{3\sqrt{399}}{8}"$ . Note this is $3\sqrt{399}$	

Question Number	Scheme	Marks
1 (unite of	(a) and (b) can be marked together	
12(a)	$f(x) = \frac{16 + 24\sqrt{x} + 9x}{x}$	M1
	$f(x) = 16x^{-1} + 24x^{-\frac{1}{2}} + 9$	M1A1A1
(b)	$f'(x) = -16x^{-2} - 12x^{-\frac{3}{2}}$	[4] M1 A1
()		[2]
(c)	When $x = 4$ , $y = 25$	B1
	f'(4) = $-1 - \frac{12}{8} = -2\frac{1}{2}$ Equation of tangent is $y - 25 = -\frac{5}{2}(x - 4)$	M1
	Equation of tangent is $y-25 = -\frac{5}{2}(x-4)$	M1 A1
		[4]
		10 marks
	Notes	
(b)	M1: Divides at least one term in numerator by x correctly <u>following an attempt at ex</u> $\frac{16}{x}$ A1: Two correct terms A1: All terms correct M1: Evidence of differentiation $x^n \rightarrow x^{n-1}$ of an expression of the form $Ax^{-1}$ or B: $x^k \rightarrow x^{k-1}$ ( $k \neq 1$ ) and not inst $C \rightarrow 0$ . Differentiating ten and better corrected in	$x^k$ so $x^{-1} \rightarrow x^{-2}$ or
	$x^k \rightarrow x^{k-1}$ $(k \neq 1)$ and not just $C \rightarrow 0$ . Differentiating top and bottom separately is Note this is a hence and so attempts at e.g. use of the quotient rule scores M0. A1: cao and cso (May be un-simplified) Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give scores M1A0 if otherwise correct.	
(c)		
(0)	<b>B1:</b> 25 only <b>M1:</b> Substitute $x = 4$ into their derived function <b>M1:</b> Uses their "25" and their "gradient" which has come from calculus ( <b>not the nor</b> x = 4 to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value <b>A1:</b> any correct form e.g.	
(0)	<b>B1:</b> 25 only <b>M1:</b> Substitute $x = 4$ into their derived function <b>M1:</b> Uses their "25" and their "gradient" which has come from calculus ( <b>not the nor</b> x = 4 to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value <b>A1:</b> any correct form e.g. $y = -\frac{5}{2}x + 35$ , $5x + 2y - 70 = 0$	
(c)	<b>B1:</b> 25 only <b>M1:</b> Substitute $x = 4$ into their derived function <b>M1:</b> Uses their "25" and their "gradient" which has come from calculus ( <b>not the nor</b> x = 4 to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value <b>A1:</b> any correct form e.g.	ie for c

Question Number	Scheme	Marks
13(a)	$3kx^{2} + (8k+6)x + 9k - 2 = 0 \text{ or } 3kx^{2} + 8kx + 6x + 9k - 2 = 0$	B1
	Uses $b^2 - 4ac$ with $a = 3k$ , $b = 8k \pm 6$ and $c = 9k \pm 2$	M1
	$-44k^2 + 120k + 36 < 0$ or $36 < 44k^2 - 120k$ o.e.	Al
	Reached with no errors	AI
	$11k^2 - 30k - 9 > 0*$	A1*
( <b>b</b> )		[4]
<b>(b)</b>	Attempts to solve $11k^2 - 30k - 9 = 0$ to give $k =$	M1
	$\Rightarrow$ Critical values, $k = 3, -\frac{3}{11}$	A1
	$\Rightarrow \text{Critical values, } k = 3, -\frac{3}{11}$ $k > 3(\text{or}) \ k < -\frac{3}{11}$	M1 A1cao
		[4]
	Notes	8 marks
	the '= 0' may be implied by use of a correct discriminant. M1: Attempts $b^2 - 4ac$ with $a = 3k$ , $b = 8k \pm 6$ and $c = 9k \pm 2$ or uses quadratic formul seen to solve their equation or uses $b^2 = 4ac$ or e.g. $b^2 < 4ac$ . There must be no x's. A1: Obtains a correct three term quadratic inequality that is not the printed answer w A1: Correct answer with no errors	
(b)	M1: Uses factorisation, formula, or completion of square method to find two values for a correct answers with no obvious method for <u>the given</u> three term quadratic	<i>k</i> or finds two
	<b>A1:</b> Obtains $k = 3, -\frac{3}{11}$ accept awrt - 0.272	
	<b>M1: Chooses outside region (</b> $k <$ Their Lower Limit $k >$ Their Upper Limit ) for a 3 inequality. Do not award simply for diagram or table.	term quadratic
	A1: $k > 3$ (or) $k < -\frac{3}{11}$ must be exact here but allow $-0.27$ for $-\frac{3}{11}$ .	
	Allow other notation such as $\left(-\infty, -\frac{3}{11}\right) \cup \left(3, \infty\right)$	
	$k > 3$ and $k < -\frac{3}{11}$ and $-\frac{3}{11} > k > 3$ score M1A0	
	ISW if possible e.g. $k > 3$ , $k < -\frac{3}{11}$ followed by $-\frac{3}{11} > k > 3$ can score M1A1	
	$k > 3$ , $k > -\frac{3}{11}$ followed by $k > 3$ (or) $k < -\frac{3}{11}$ can score M1A1	
	Allow (b) to be solved in terms of x for the first 3 marks but the final A mark needs the re <b>Fully correct answer with no working scores full marks.</b>	egions in terms of $k$ .
	Answers that are otherwise correct but use $\leq, \geq$ lose final mark.	

Question Number	Scheme	Marks
14(i)	$\log_{a} x + \log_{a} 3 = \log_{a} 27 - 1  \text{so}  \log_{a} \frac{3x}{27} = -1$ Or $\log_{a} x + \log_{a} 3 = \log_{a} 27 - \log_{a} a  \text{so}  \log_{a} 3x = \log_{a} \frac{27}{a}$ Or $\log_{a} x + 1 = \log_{a} 27 - \log_{a} 3 = \log_{a} 9  \text{so}  \log_{a} ax = \log_{a} 9$	M1 A1
	$\frac{3x}{27} = a^{-1}$	M1
	$x = 9a^{-1}  \text{or}  \frac{9}{a}$	A1
		[4]
(ii)	$x^2 - 7x + 12 = 0$ and attempt to solve to give $x =$ or $\log_5 y =$ (implied by correct answers)	M1
	x (or $\log_5 y$ ) = 3 and 4	A1
	$y = 5^3$ or $5^4$	dM1
	y = 5 or $5y = 125$ and $625$	A1
	<i>y</i> = 125 <b>and</b> 025	[4]
		8 marks
	Notes	
(i)	M1: Uses sum or difference of logs correctly e.g.	·
	$\log x + \log 3 = \log 3x$ or $\log 27 - \log 3 = \log 9$ or $\log 27 - \log x = \log \frac{27}{x}$ etc.	
	or writes 1 as $\log_a a$	
	<ul><li>A1: Uses two rules correctly to obtain correct log equation</li><li>M1: Removes logs correctly to obtain an equation connecting <i>x</i> and <i>a</i></li><li>A1: Correct simplified answer</li></ul>	
	Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27 - 1)$ . This can score a maximum of	1 out of 4 if
	they have $\log x + \log 3 = \log 3x$	
	Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a 3x}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores M1A0M0A0	
	Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a x \log_a 3}{\log_a 27} = -1 \Longrightarrow \frac{3x}{27} = a^{-1}$ etc. scores <b>no man</b>	•ks
(ii)	<ul> <li>M1: Recognise and attempt to solve quadratic</li> <li>A1: Obtain both 3 and 4 (Both correct implies M1A1)</li> <li>dM1: Uses powers correctly to find a value for y (Dependent on first method mark)</li> </ul>	

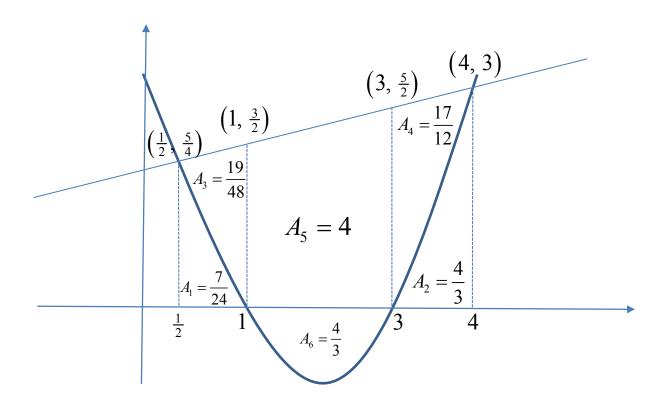
Question Number	Scheme	Marks
15 (a)	Mid-point of $AB = (2, -3)$	M1 A1
	$(r^2) = (12 - "2")^2 + (2 - "-3")^2$ or $(r^2) = (-8 - "2")^2 + (-8 - "-3")^2$ or $(d^2) = (-8 - 12)^2 + (-8 - 2)^2$	M1
	$r^2 = 125$	A1
	$"125" = (x \pm "2")^{2} + (y \pm "-3")^{2}$	M1
	$125 = (x-2)^2 + (y+3)^2$	A1
		[6]
(b)	gradient from "(2, -3)" to (4, 8) = $\frac{8 - " - 3"}{4 - "2"}$ , $\left(=\frac{11}{2}\right)$	M1
	$ZM$ has gradient $-\frac{1}{m}$ $\left(=-\frac{2}{11}\right)$	M1
	Either: $y - 8 = "-\frac{2}{11}"(x - 4)$ or: $y = "-\frac{2}{11}"x + c$ and $8 = "-\frac{2}{11}"(4) + c \implies c = "8\frac{8}{11}"$	ddM1
	2x + 11y - 96 = 0	Al
		[4]
		(10marks)
	Notes	
	A1: cao M1: Finds radius or radius <sup>2</sup> , diameter or diameter <sup>2</sup> using any valid method – probably distance from centre to one of the points. Need not state $r = \dots$ so ignore lhs – you are just looking for correct use of Pythagoras with or without the square root so ignore how they reference it for this mark. A1: for any equivalent $r^2 = 125$ or $r = \sqrt{125}$ (11.18) etc. Their numeric answer must be identified here as either r or $r^2$ (may be implied by their equation). If they halve it or double it, this is M1 A0. M1: Attempt to use a true equation for circle with their centre and radius or the letter r, allow sign slips in brackets but do not allow use or r instead of $r^2$ in the equation. So must be using $r^2 = (\mathbf{x} \pm)^2 + (\mathbf{y} \pm)^2$	
	A1: correct answer only (Allow $(5\sqrt{5})^2$ instead of 125 but not $5\sqrt{5}^2$ )	
(b)	<ul> <li>M1: States or uses gradient equation correctly with their centre and (4, 8). Must be using the (4, 8). If no method is shown and gradient incorrect for their values score M0.</li> <li>M1: Finds negative reciprocal. Follow through their gradient ddM1: Correct straight line method with (4, 8) and perpendicular gradient. Dependent on both method marks having been scored.</li> <li>A1: cao – accept multiples of this equation (Note integer coefficients not required) A common error here is to use the diameter to find the gradient. This usually scores M0M1ddI just one mark for the perpendicular gradient rule.</li> </ul>	<b>h</b> previous
	(b) Alternative uses implicit differentiation: e.g.	``
	$125 = (x-2)^2 + (y+3)^2 \Rightarrow 2(x-2) + 2(y+3)\frac{dy}{dx} = 0$ M1(correct implicit differentiation $\frac{dy}{dx} = 2 - x - 2 - 4$	1) oe
	$\Rightarrow \frac{dy}{dx} = \frac{2-x}{y+3} = \frac{2-4}{8+3}$ M1(Substitution)	
	Then follow the scheme.	

Question Number	Scheme	Marks
16(a)	$\frac{1}{2}x + 1 = x^2 - 4x + 3$	M1
	$2x^2 - 9x + 4 = 0 \implies x = \frac{1}{2}$ or $x = 4$	dM1 A1
	y = 5/4 or $y = 3$	dM1 A1
(b)	Curve meets x-axis at $x = 3$ and at $x = 1$ (No need to see $y = 0$ )	[5] M1 A1
(0)	Curve meets x-axis at $x = 5$ and at $x = 1$ (no need to see $y = 0$ )	[2]
	NOTE that the subscripted A's refer to areas on the diagram given at the end of the scheme. All the method marks are for <u>their</u> $x = 1/2$ , 4, 1 and 3	
(c) Way 1	$\int x^2 - 4x + 3  \mathrm{d}x = \frac{1}{3}x^3 - 2x^2 + 3x$	M1 A1
-	Use limits 1 and $\frac{1}{2} \left[ \left( \frac{1}{3}(1)^3 - 2(1)^2 + 3 \times 1 \right) - \left( \frac{1}{3} \left( \frac{1}{2} \right)^3 - 2 \cdot \left( \frac{1}{2} \right)^2 + 3 \times \left( \frac{1}{2} \right) \right) \right] A_1$	M1
	Use limits 4 and 3 $[(\frac{1}{3}(4)^3 - 2(4)^2 + 3 \times (4)) - (\frac{1}{3}(3)^3 - 2(3)^2 + 3 \times (3))] A_2$	M1
	Area of trapezium =	
	$\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{5}{4}+3) \times (4-\frac{1}{2}) = \dots \text{ or } \int_{\frac{1}{2}}^{4} (\frac{1}{2}x+1) dx = \left[\frac{1}{4}x^2 + x\right]_{\frac{1}{2}}^{4} = (4+4) - (\frac{1}{16}+\frac{1}{2}) = \dots$	M1
	7.4375 $(7\frac{7}{16})$ $(\frac{119}{16})$ (may be implied by correct final answer)	A1
	Uses correct combination of correct areas. Area of region = Area of trapezium $-A_1 - A_2$ Dependent on all previous method marks	ddddM1
	$= 7.4375 - \frac{7}{24} - \frac{4}{3} = \frac{93}{16} \text{ or } 5.8125$	A1
(-)		[8]
(c) Way 2	Alternative method using "line – curve" and subtracting area below x- axis $\int -x^2 + \frac{9}{2}x - 2dx = -\frac{x^3}{3} + \frac{9}{4}x^2 - 2x \text{ or } \int x^2 - \frac{9}{2}x + 2dx = \frac{x^3}{3} - \frac{9}{4}x^2 + 2x$	M1A1
	Use limits $\frac{1}{2}$ and 4 on this <i>subtracted</i> integration $(A_3 + A_4 + A_5 + A_6) = 6\frac{2}{3} + \frac{23}{48} = \dots$	M1
	$\pm \int x^2 - 4x + 3dx = \pm \left(\frac{1}{3}x^3 - 2x^2 + 3x\right)$	M1
	Use limits 1 and 3 on their integrated curve to obtain $A_6 = \pm \frac{4}{3}$	M1A1
	Uses correct combination of correct areas. Area of region = $(A_3 + A_4 + A_5 + A_6) - A_6$	ddddM1
	<b>Dependent on all previous method marks</b> $6\frac{2}{3} + \frac{23}{48} - \frac{4}{3} = \frac{93}{16}$	A1
	3 40 3 10	[8]
(c) Way 2	Alternative method using "line – curve" for areas $A_3$ and $A_4$ and adding smaller trapezium	
Way 3	$\int -x^2 + \frac{9}{2}x - 2dx = -\frac{x^3}{3} + \frac{9}{4}x^2 - 2x \text{ or } \int x^2 - \frac{9}{2}x + 2dx = \frac{x^3}{3} - \frac{9}{4}x^2 + 2x$	M1A1
	Use limits 1 and $\frac{1}{2} \left[ \left( -\frac{1}{3}(1)^3 + \frac{9}{4}(1)^2 - 2 \times 1 \right) - \left( -\frac{1}{3}(\frac{1}{2})^3 + \frac{9}{4}(\frac{1}{2})^2 - 2 \times \frac{1}{2} \right] A_3$	M1
	Use limits 4 and 3 $\left[\left(-\frac{1}{3}(4)^3 + \frac{9}{4}(4)^2 - 2 \times 4\right) - \left(-\frac{1}{3}(3)^3 + \frac{9}{4}(3)^2 - 2 \times 3\right] A_4$	M1
	Area of trapezium = $\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{3}{2} + \frac{5}{2}) \times (3-1) = \dots$ or $\int_{1}^{3} (\frac{1}{2}x+1)dx = \left[\frac{1}{4}x^{2} + x\right]_{1}^{3} = (\frac{9}{4}+3) - (\frac{1}{4}+1) = \dots$	M1
	= 4	A1
	Uses correct combination of correct areas. Area of region = $A_3 + A_4 + A_5$ Dependent on all previous method marks	ddddM1
	$\frac{19}{48} + \frac{17}{12} + 4 = \frac{93}{16}$	A1
		[8]

(c) Way 4	Alternative method: Finds area of larger trapezium and subtracts A <sub>1</sub> + A <sub>2</sub> which is found by integrating quadratic between ½ and 4 and adding area below <i>x</i> -axis	
	$\int x^2 - 4x + 3  \mathrm{d}x = \frac{1}{3}x^3 - 2x^2 + 3x$	M1 A1
	Use limits 4 and $\frac{1}{2} \left[ \left( \frac{1}{3} (4)^3 - 2(4)^2 + 3 \times 4 \right) - \left( \frac{1}{3} \left( \frac{1}{2} \right)^3 - 2 \cdot \left( \frac{1}{2} \right)^2 + 3 \times \left( \frac{1}{2} \right) \right) \right] A_1 + A_2 - A_6$	M2
	<b>AND</b> Use limits 3 and 1 $\pm [(\frac{1}{3}(3)^3 - 2(3)^2 + 3 \times 3) - (\frac{1}{3}(1)^3 - 2.(1)^2 + 3 \times (1))] \pm A_6$	1112
	Area of trapezium = $\frac{4}{4}$	
	$\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{5}{4}+3) \times (4-\frac{1}{2}) = \dots \text{ or } \int_{\frac{1}{2}}^{\frac{1}{2}} (\frac{1}{2}x+1)dx = \left[\frac{1}{4}x^2 + x\right]_{\frac{1}{2}}^{\frac{4}{2}} = (4+4) - (\frac{1}{16}+\frac{1}{2}) = \dots$	M1
	7.4375 $(7\frac{7}{16})$ (may be implied by correct final answer)	A1
	Uses correct combination of correct areas. Area of region = $7.4375 - (A_1 + A_2 - A_6 + A_6)$	ddddM1
	Dependent on all previous method marks	
	$= 7.4375 - \left(\frac{7}{24} + \frac{4}{3}\right) = \frac{93}{16}$	A1
		[8]
	Notes	15 marks
(a)	M1: Puts equations equal or finds x in terms of y and substitutes or substitutes for x dM1: Solves three term quadratic in x to obtain $x =$ or in y to obtain $y =$ (Dependent on find A1: Both answers correct dM1: Obtains at least one value for y or x (Dependent on first M) A1: Both correct	irst M)
	Note: Allow candidates to obtain $x^2 - \frac{9}{2}x + 2 = 0$ and solve as $(2x-1)(x-4) = 0 \Rightarrow x = \frac{1}{2}, 4$	
	The coordinates do not need to be 'paired'	
(b)	M1: Attempts to solve $0 = x^2 - 4x + 3$ according to the usual rules A1: cao Attempts by T&L can seere both marks for $x = 1$ and $x = 2$ . If one solution is obtained by this a	ooro M140
	Attempts by T&I can score both marks for $x = 1$ and $x = 3$ . If one solution is obtained by this, s For (c) do not allow 'mixed' methods. For their strategy, they must be finding the appro	
	but apply the method for the scheme that gives the most credit for the candidate	•
(c) Way 1	M1: Attempt at integration of the given quadratic expression $(x^n \rightarrow x^{n+1} \text{ at least once})$ A1: Correct integration of the given quadratic expression M1: Finds area of $A_1$ M1: Finds area of $A_2$ M1: Finds area of appropriate trapezium A1: Correct area of trapezium 7.4375 $(7\frac{7}{16})$	
	<b>ddddM1:</b> correct final combination <b>A1:</b> any correct form of this exact answer	
(c)	M1: Attempt at integration of ±(the given quadratic expression – the given line) $(x^n \rightarrow x^{n+1} a)$	t least once)
Way 2	A1: Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0 M1: Uses the limits $\frac{1}{2}$ and 4 on their <i>subtracted</i> integration M1: Attempts to integrate curve M1: Uses the limits 1 and 3 on the integrated curve C A1: Obtains $A_6 = \pm \frac{4}{3}$	
	<b>ddddM1:</b> correct final combination <b>A1:</b> any correct form of this exact answer Note: A common error with this method is to use the limits $\frac{1}{2}$ and 4 on their <i>subtracted</i> integrat stop (this should give an area of $\frac{343}{48}$ ). This will usually score 3/8 in (c)	tion and then

(c) Way 3	<b>M1:</b> Attempt at integration of $\pm$ (the given quadratic expression – the given line) $(x^n \rightarrow x^{n+1} \text{ at least once})$		
Way 5	A1: Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor simplified. If there are sign errors when subtracting before valid etternation access M1A0		
	simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0 <b>M1:</b> Uses the limits <sup>1</sup> / <sub>2</sub> and 1 on their <i>subtracted</i> integration		
	M1: Uses the limits 4 and 3 on their <i>subtracted</i> integration		
	M1: Finds area of appropriate trapezium		
	A1: Correct area of trapezium 4		
	ddddM1: correct final combination		
	A1: any correct form of this exact answer		
(c) Way 4	<b>M1:</b> Attempt at integration of the given quadratic expression $(x^n \rightarrow x^{n+1} \text{ at least once})$		
vvuy -	A1: Correct integration of the given quadratic expression		
	<b>M2:</b> Finds area of $A_1 + A_2 - A_6$ by using the limits $\frac{1}{2}$ and 4 and finds area of $A_6$ by using the limits 1 and 3		
	M1: Finds area of appropriate trapezium		
	A1: Correct area of trapezium 7.4375 $(7\frac{7}{16})$		
	ddddM1: correct final combination		
	A1: any correct form of this exact answer		

#### **Diagram for Question 16**



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