Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5,\ x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4,\ x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • *H* is not likely to be a quadratic function in *x*	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

Question	Scheme	Marks	AOs
2(a)	$f(x)=(x-2)^2 \pm ...$	M1	1.2
	$f(x)=(x-2)^2+1$	A1	1.1b
		(2)	
(b)(i)	$P=(0, 5)$	B1	1.1b
(b)(ii)	$Q=(2, 1)$	B1ft	1.1b
		(2)	
		(4 marks)	
Notes			

(a)
M1: Achieves $(x-2)^2 \pm ...$ or states $a=-2$

A1: Correct expression $(x-2)^2+1$ ISW after sight of this

Condone $a=-2$ and $b=1$. Condone $(x-2)^2+1=0$

(b)
(i) B1: Correct coordinates for P. Allow to be expressed $x=0, y=5$
(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x=2, y=1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)
Score in any order if they state $P=(0, 5)$ and $Q=(2, 1)$

....

Allow part (b) to be awarded from a sketch. So award
First B1 from a sketch crossing the y-axis at 5
Second B1 from a sketch with minimum at $(2, 1)$

....

Question	Scheme	Marks	AOs
6 (a)	Attempts $P=100-6.25(15-9)^2$	M1	3.4
	$=-125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P>80 \Rightarrow (x-9)^2<3.2$ or $P=80 \Rightarrow (x-9)^2=3.2$	M1	3.1b
	$\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x=15$ into $P=100-6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the x = 15. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P=-125$ or states that $P<0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P=-125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. An explanation that it is not sensible as "the profit cannot be negative" , "the profit is negative" or "the company will not make any money" , "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets P = 0 and finds $x=5,13$ A1: States $15>13$ and states makes a loss

(b)

M1: Uses $P ... 80$ where ... is any inequality or "$=$"in $P=100-6.25(x-9)^2$ and proceeds to $(x-9)^2 ... k$ where k >0 and ... is any inequality or "$=$"

Eg. Condone $P<80$ in $P=100-6.25(x-9)^2 \Rightarrow (x-9)^2<k$ where k >0 If the candidate attempts to multiply out then allow when they achieve a form $ax^2+bx+c=0$

dM1: Award for solving to find the two positive values for x. Allow decimal answers

FYI correct answers are $\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$ Accept $\Rightarrow x=9\pm\sqrt{3.2}$

Condone incorrect inequality work $100-6.25(x-9)^2>80 \Rightarrow (x-9)^2>3.2 \Rightarrow x>9\pm\sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9-\sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound(s).

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once in these parts, withhold the final mark.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

Question	Scheme	Marks	AOs
9 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \{1200-3\times(4-20)^2\}-\{1200-3\times(3-20)^2\}$

A1: 93 tonnes or 93 t
(d)
For one mark

Shows an appreciation of the model

- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks

States the limitation on n and explains fully. (Total mass, not mass must be used)

- States that $n \leqslant 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that n cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \leqslant 20$ as the maximum total amount of tin mined is reached at 20 years

Question	Scheme	Marks	AOs
10	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k - 3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes:

B1: Explains why $k = 0$ gives no real roots
M1: Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark
M1: Attempts solution of quadratic inequality
A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Question	Scheme	Marks
11. (a)	$(13k-5)x^2-12kx-6=0$ or $(5-13k)x^2+12kx+6=0$	B1
	Uses b^2-4ac with $a=\pm 13k\pm 5, b=\pm 12k$ and $c=\pm 6$	M1
	And states $b^2-4ac>0$ with $a=\pm(13k-5), b=\pm 12k$ and $c=\pm 6$	A1ft
	Proceeds correctly with no errors to $6k^2+13k-5>0$ *	A1*
		[4]
(b)	Attempts to solve $6k^2+13k-5=0$ to give $k=$	M1
	$\Rightarrow$ Critical values, $k=\frac{1}{3}, \frac{-5}{2}$	A1
	$6k^2+13k-5>0$ gives $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$	M1 A1
		[4]
		8 marks
	Notes	

(a)

B1: Expresses equation as three term quadratic in *x*. $(13k-5)x^2-12kx-6=0$ **oe.**
The equals 0 may be implied by subsequent work. Allow $(5-13k)x^2+12kx+6=0$
Allow an equation of the form $13kx^2-5x^2-12kx-6(=0)$ as long as it is followed by $a=13k-5$........

M1: Attempts b^2-4ac with $a=\pm 13k\pm 5, b=\pm 12k$ and $c=\pm 6$
or uses quadratic formula to solve equation
or uses the discriminant on two sides of an equation or inequation e.g. $b^2=4ac$ or $b^2<4ac$

A1: Uses the discriminant condition, eg $b^2-4ac>0$ or $b^2>4ac$ with $a=\pm 13k\pm 5, b=\pm 12k$ and $c=\pm 6$

A1*: Proceeds to given answer with no errors. AG. Condone missing = 0 on the equation
Condone a solution where $(13k-5)x^2-12kx-6=0$ is followed by $144k^2+24(13k-5)>0$
Watch for $a=13k-5, b=+12k$ and $c=-6$ which does give the correct inequality but loses the final A1*

(b)

M1: Uses factorisation, formula, or completion of square method to find two values for k,
or finds two **correct** answers with no obvious method for **their** three term quadratic

A1: Obtains $k=\frac{1}{3}, \frac{-5}{2}$ accept -2.5, 0.333 (awrt) here but need exact answer for final A1.
Also condone $x=\frac{1}{3}, \frac{-5}{2}$ for this mark .

M1: Chooses outside region ($k<$ Their Lower Limit $k>$ Their Upper Limit) for appropriate 3 term quadratic inequality . Do not award simply for diagram or table.
Award if final answer is $k\geqslant\frac{1}{3}$ (or) $k\leqslant\frac{-5}{2}$ or $\frac{1}{3}<k<\frac{-5}{2}$
Condone *x* appearing instead of *k*

A1: $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$ $\left(k\neq\frac{5}{13}\right)$ must be exact and must be *k*.
Must be two separate inequalities and not be $k>\frac{1}{3}$ **and** $k<\frac{-5}{2}$

Question Number	Scheme	Marks
2(a)	$4(x-2) \leqslant 2x+1 \Rightarrow 4x-8 \leqslant 2x+1$ $\Rightarrow x \leqslant 4.5$	M1A1 **(2)**
(b)	$(2x-3)(x+5)>0$ Roots are $1.5, -5$ Chooses outsides $x<-5, x>1.5$	 B1 M1A1 **(3)**
(c)	$x<-5, 1.5<x \leqslant 4.5$	B1 **(1)** **(6 marks)**

(a)

M1 Proceeds as far as $x \leqslant ...$ after firstly multiplying out brackets oe. Condone $x<...$ for this mark
Minimum expectation is that you see $4x-8...2x+1 \Rightarrow x...c$ where ... is $\leqslant$ or <

A1 $x \leqslant 4.5$ or equivalent in set notation such as $\{x : x \leqslant 4.5\}$ $x \in (-\infty, 4.5]$. Accept just $(-\infty, 4.5]$

(b)

B1 Critical values are 1.5, -5

M1 Chooses the outside values of their critical values. You may well see candidates multiply out the brackets and factorise incorrectly. They can score this mark for choosing the 'outsides'
Do not allow this mark from just a diagram. You must see the inequalities.
Accept for the method mark $x \leqslant -5, x \geqslant 1.5$

A1 Accept any of '$x<-5, x>1.5$' '$x<-5$ or $x>1.5$' '$\{x : x<-5 \cup x>1.5\}$'
'$\{x : -\infty < x < -5 \cup 1.5 < x < \infty\}$' or their exact equivalents

Do not accept on its own (without seeing any of the above)
'$x<-5$ and $x>1.5$" '$-5>x>1.5$' '$\{x : x<-5 \cap x>1.5\}$'

(c)

B1 cao Accept any of
'$x<-5, 1.5<x \leqslant 4.5$' '$x<-5$ or $1.5<x \leqslant 4.5$' '$\{x : x<-5 \cup 1.5<x \leqslant 4.5\}$'
'$x \in -\infty < x < -5 \cup 1.5 < x \leqslant 4.5$' or their exact equivalents.

There must be just two distinct regions represented by just two inequalities.
If a candidate writes $x \leqslant 4.5$ $x<-5, x>1.5$ it is B0

Question Number	Scheme	Marks
13(a)	$3kx^2+(8k+6)x+9k-2=0$ or $3kx^2+8kx+6x+9k-2=0$	B1
	Uses b^2-4ac with $a=3k$, $b=8k\pm 6$ and $c=9k\pm 2$	M1
	$-44k^2+120k+36<0$ or!!! $36<44k^2-120k$ o.e. **Reached with no errors**	A1
	$11k^2-30k-9>0$ *	A1*
		[4]
(b)	Attempts to solve $11k^2-30k-9=0$ to give $k=$	M1
	$\Rightarrow$ Critical values, $k=3, -\frac{3}{11}$	A1
	$k>3$ (or) $k<-\frac{3}{11}$	M1 A1cao
		[4]
		8 marks
	Notes	
(a)	**B1:** Multiplies by k and collects terms to one side in any order. Allow the x terms not to be combined and the '= 0' may be implied by use of a **correct** discriminant. **M1:** Attempts b^2-4ac with $a=3k$, $b=8k\pm 6$ and $c=9k\pm 2$ or uses quadratic formula with b^2-4ac seen to solve their equation or uses $b^2=4ac$ or e.g. $b^2<4ac$. **There must be no *x*'s.** **A1: Obtains a correct three term quadratic inequality** that is not the printed answer with **no errors seen.** **A1:** Correct answer with **no errors**	
(b)	**M1:** Uses factorisation, formula, or completion of square method to find **two values** for k or finds two **correct** answers with no obvious method for <u>**the given**</u> three term quadratic **A1:** Obtains $k=3, -\frac{3}{11}$ accept awrt - 0.272 **M1: Chooses outside region (**$k<$ Their Lower Limit $\quad k>$ Their Upper Limit) for a 3 term quadratic inequality. Do not award simply for diagram or table. **A1:** $k>3$ (or) $k<-\frac{3}{11}$ must be exact here but allow $-0.\dot{2}\dot{7}$ for $-\frac{3}{11}$. Allow other notation such as $\left(-\infty,-\frac{3}{11}\right)\cup(3,\infty)$ $k>3$ **and** $k<-\frac{3}{11}$ and $-\frac{3}{11}>k>3$ score M1A0 ISW if possible e.g. $k>3$, $k<-\frac{3}{11}$ followed by $-\frac{3}{11}>k>3$ can score M1A1 $k>3$, $k>-\frac{3}{11}$ followed by $k>3$ (or) $k<-\frac{3}{11}$ can score M1A1 Allow (b) to be solved in terms of x for the first 3 marks but the final A mark needs the regions in terms of k. **Fully correct answer with no working scores full marks.** **Answers that are otherwise correct but use $\le, \ge$ lose final mark.**	

Question Number	Scheme	Marks
9(a)	Uses $b^2-4ac=(6k+4)^2-4\times1\times3$	B1
	Multiplies out and uses $b^2-4ac<0$	M1
	to give $36k^2+48k+16-12<0$ so $9k^2+12k+1<0$ *	A1 *
		[3]
(b)	Solves quadratic by formula or completion of square to give $k=$	M1
	Obtains $k=\dfrac{-12\pm\sqrt{108}}{18}$ or accept $k=$ awrt $-1.24, -0.09$	A1
	Chooses region between two values and deduces $\dfrac{-2-\sqrt{3}}{3}<k<\dfrac{-2+\sqrt{3}}{3}$	M1 A1**cao**
		[4]
		7 marks

Question Number	Scheme	Marks
13.	$y = 3x^2 - 4x + 2$	
(a)	$\frac{dy}{dx} = 6x - 4 + \{0\}$	M1A1
	At (1, 1) gradient of curve is 2 and so gradient of normal is $-\frac{1}{2}$	M1
	$\therefore (y-1) = -\frac{1}{2}(x-1)$ and so $x + 2y - 3 = 0$*	M1 A1*
		[5]
(b)	Eliminate x or y to give $2(3x^2 - 4x + 2) + x - 3 = 0$ or $y = 3(3-2y)^2 - 4(3-2y) + 2$	M1
	Solve three term quadratic e.g $6x^2 - 7x + 1 = 0$ or $12y^2 - 29y + 17 = 0$ to give $x =$ or $y =$	M1
	$x = \frac{1}{6}$ **or** $y = 1\frac{5}{12}$	A1
	Both $x = \frac{1}{6}$ and $y = 1\frac{5}{12}$ i.e. $(\frac{1}{6}, 1\frac{5}{12})$ *or* (0.17,1.42) { Ignore (1 , 1) listed as well }	A1
		[4]
(c)	When this line meets the curve $2(3x^2 - 4x + 2) + kx - 3 = 0$	M1
	So $6x^2 + (k-8)x + 1 = 0$	dM1
	Uses condition for equal roots "$b^2 = 4ac$" on their three term quadratic to get expression in k	ddM1
	So obtain $(k-8)^2 = 24$ i.e. $k^2 - 16k + 40 = 0$ *	A1 *
	If they use **gradient of tangent** to do part (c) see the end of the notes below*.	**[4]**
(d)	Solve the given quadratic or their quadratic by formula or completion of the square to give	M1A1
	$k = 8 \pm \sqrt{24}$ or $8 \pm 2\sqrt{6}$ or $\frac{16 \pm \sqrt{96}}{2}$	**[2]**
		15 marks
	Notes	

Question Number	Scheme	Marks
10(a)	$kx^2+4x+k=2 \Rightarrow kx^2+4x+k-2=0$	
	Attempts to calculate b^2-4ac with $a=k, b=4$ and $c=k\pm 2$	M1
	$b^2-4ac=4^2-4\times k\times(k-2)$	A1
	Sets their $b^2-4ac>0 \Rightarrow 16-4k(k-2)>0$	dM1
	$4k^2-8k-16<0$	
	$k^2-2k-4<0$	A1*
		(4)
(b)	Solves $(k-1)^2-5=0 \Rightarrow k=1\pm\sqrt{5}$	M1
	'Insides' $1-\sqrt{5}<k<1+\sqrt{5}$	M1A1
		(3)
		(7 marks)

(a) Note that this is M1M1A1A1 on e pen. Mark in the order seen below as M1A1M1A1.

M1 Attempts to calculate b^2-4ac with $a=k, b=4$ and $c=k\pm 2$. Condone poor/incomplete bracketing.

Alternatively they may set $b^2 ... 4ac$ with ... being $=,>,<,\leqslant$ or $\geqslant$ with $a=k, b=4$ and $c=k\pm 2$.

A1 Correct (unsimplified) b^2-4ac. Accept $4^2-4k(k-2)$ oe. The bracketing must be correct.

In the alternative it is for $4^2 ... 4k(k-2)$ with ... being $=,>,<,\leqslant$ or $\geqslant$ The bracketing must be correct

dM1 Sets their $b^2-4ac>0$ with $a=k, b=4$ and $c=k\pm 2 \Rightarrow 4^2-4k(k\pm 2)>0$

In the alternative it is for just $b^2>4ac$ with $a=k, b=4$ and $c=k\pm 2 \Rightarrow 4^2>4k(k\pm 2)$

A1* Proceeds correctly to the given answer $k^2-2k-4<0$. You should expect to see the terms moved over to the other side of the inequality and a division of 4. Alternatively a division of -4 should be proceeded by the reversal of the inequality. There is no requirement for these steps to be explained but you need to check for sign errors which would be A0.

Special case: If they start with $a=kx^2, b=4x$ and $c=k\pm 2$ and proceed correctly they will score a maximum of M1A1dM1A0*. Treat candidates who start with $\sqrt{b^2-4ac}$ in a similar way.

(b)

M1 For an attempt by either the formula or completing the square to solve 3TQ=0. Do not accept factorisation. If the formula is quoted it must be correct. If it is not quoted only accept expressions of the form $\frac{-(-2)\pm\sqrt{(-2)^2-4\times 1\times -4}}{2\times 1}$ with or without either bracket. Accept awrt 3.24, -1.24 from a GC.

If completing the square is attempted accept $k^2-2k-4=0 \Rightarrow (k\pm 1)^2\pm 1\pm 4=0 \Rightarrow k=...$

M1 Chooses the inside values to their solution to the 3TQ=0. This is not dependent upon the previous M so for this mark you can accept the inside region from their roots obtained factorisation. If α is the smaller root and β is the larger root look for $\alpha<k<\beta$ **or** $k>\alpha$ and $k<\beta$ **or** $k>\alpha$ or $k<\beta$ **or** (α,β)

A1 $1-\sqrt{5}<k<1+\sqrt{5}$. Accept also $k>1-\sqrt{5}$ and $k<1+\sqrt{5}$, $k>1-\sqrt{5}$ $k<1+\sqrt{5}$, $\left(1-\sqrt{5}, 1+\sqrt{5}\right)$

Also allow $k>1-\sqrt{5}$ $k<1+\sqrt{5}$ with a comma between.

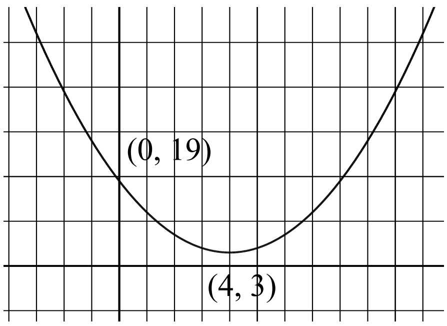
Accept exact equivalents like $\frac{2-\sqrt{20}}{2}<k<\frac{2+\sqrt{20}}{2}$ or $1-\frac{\sqrt{20}}{2}<k<1+\frac{\sqrt{20}}{2}$

Do not accept $k>1-\sqrt{5}$ or $k<1+\sqrt{5}$, $\left[1-\sqrt{5}, 1+\sqrt{5}\right]$ or decimals

Do not accept x in place of k for the final mark.

Special case for candidate who achieves $1-\sqrt{5}\leq k\leq 1+\sqrt{5}$ or $\left[1-\sqrt{5}, 1+\sqrt{5}\right]$ score M1M1A0

Question Number	Scheme	Marks
8.	$kx^2 + 8x + 2(k+7) = 0$	
	Uses $b^2 - 4ac$ with $a = k$, $b = 8$ and attempt at $c = 2(k + 7)$	M1
	$b^2 - 4ac = 64 - 56k - 8k^2$ *or* $64 = 56k + 8k^2$ *o.e.*	A1
	Attempts to solve "$k^2 + 7k - 8 = 0$" to give $k =$	dM1
	$\Rightarrow$ Critical values, $k = 1, -8.$	A1cso
	Uses $b^2 - 4ac < 0$ or $b^2 < 4ac$ or $4ac - b^2 > 0$	M1
	$k^2 + 7k - 8 > 0$ gives $k > 1$ (or) $k < -8$	M1 A1
		[7]
		7 marks
	Notes	
	M1: Attempts $b^2 - 4ac$ for $a = k$, $b = 8$ and $c = 2(k+7)$ or attempt at c from quadratic = 0 (may omit bracket or make sign slip or lose the 2, so $2k + 7$ or $k + 7$ for example) or uses quadratic formula to solve equation or uses on two sides of an equation or inequation **A1: Correct three term quadratic** expression for $b^2 - 4ac$ - (may be under root sign) **dM1:** Uses factorisation, formula, or completion of square method to find two values for k, or finds two **correct** answers with no obvious method for **their** three term quadratic **A1:** Obtains 1 and -8 **M1:** states $b^2 - 4ac < 0$ *or* $b^2 < 4ac$ anywhere (may be implied by the following work) **M1: Chooses outside region (** $k <$ Their Lower Limit $k >$ Their Upper Limit) for appropriate 3 term quadratic inequality . Do not award simply for diagram or table. **A1:** $k > 1$ or $k < -8$ - allow anything which clearly indicates these regions e.g. $(-\infty, -8)$ or $(1, \infty)$ $k > 1$, $k < -8$ is A1 but $k > 1$ and $k < -8$ is A0 but $x > 1$, $x < -8$ is A0 (only lose 1 mark for using x instead of k) and $k \geq 1$ (or) $k \leq -8$ is A0 Also $1 < k < -8$ is M1 A0 N.B. Lack of working: If there is no mention of $b^2 - 4ac < 0$ or $b^2 < 4ac$ then just the correct answer $k > 1$, $k < -8$ can imply the last M1M1A1 $k \geq 1$, $k \leq -8$ can imply M0M1A0 $k > 1$, $k < -8$ can imply M1M1A0 Anything else needs to apply scheme	

Question Number	Scheme		Marks
5.(a)	$\mathrm{f}(x) = (x-4)^2 + 3$	M1: $\mathrm{f}(x) = (x \pm 4)^2 \pm \alpha,\ \alpha \neq 0$ (where α is a single number or a numerical expression $\neq 0$)	M1A1
		A1: Allow $(x +^- 4)^2 + 3$ and ignore any spurious "= 0"	
	Allow *a* = -4, *b* = 3 to score both marks		
			(2)
(b)		B1: U shape anywhere even with no axes. Do not allow a "V" shape i.e. with an obvious vertex.	B1
		B1: *P*(0, 19). Allow (0, 19) or just 19 marked in the correct place as long as the curve (or straight line) passes through or touches here and allow (19, 0) as long as it is marked in the correct place. Correct coordinates may be seen in the body of the script as long as the curve (or straight line) passes through or touches here. If there is any ambiguity, the sketch has precedence. **(There must be a sketch to score this mark)**	B1
		B1: *Q*(4, 3). Correct coordinates that can be scored without a sketch but if a sketch is drawn then it must have a minimum in the first quadrant and no other turning points. May be seen in the body of the script. If there is any ambiguity, the sketch has precedence. Allow this mark if 4 is clearly marked on the *x*-axis below the minimum and 3 is marked clearly on the *y*-axis and corresponds to the minimum,	B1
			(3)

(c)	$PQ^2 = (0-4)^2 + (19-3)^2$	**Correct** use of Pythagoras' Theorem on 2 points of the form (0, p) and (q, r) where $q \neq 0$ and $p \neq r$ with p, q and r numeric.	M1
	$PQ = \sqrt{4^2 + 16^2}$	Correct un-simplified numerical expression for PQ including the square root. **<u>This must come from a correct *P* and *Q*</u>**. Allow e.g $PQ = \sqrt{(0-4)^2 + (19-3)^2}$. Allow $\pm\sqrt{(0-4)^2 + (19-3)^2}$	A1
	$PQ = 4\sqrt{17}$	Cao and cso i.e. **<u>This must come from a correct *P* and *Q*</u>**.	A1
	Note that it is possible to obtain the correct value for PQ from (-4,3) and (0, 19) and e.g. (0, 13) and (4, -3) but the A marks in (c) can only be awarded for the correct P and Q.		
			(3)
			(8 marks)

Question Number	Scheme		Marks
6.(a)	Replaces 2^{2x+1} with $2^{2x}\times 2$ or states $2^{2x+1}=2^{2x}\times 2$ or states $\left(2^{x}\right)^{2}=2^{2x}$	Uses the addition **or** power law of indices on 2^{2x} or 2^{2x+1}. E.g. $2^{x}\times 2^{x}=2^{2x}$ or $\left(2^{x}\right)^{2}=2^{2x}$ or $2^{2x+1}=2\times 2^{2x}$ or $2^{x+0.5}=2^{x}\times\sqrt{2}$ or $2^{2x+1}=\left(2^{x+0.5}\right)^{2}$.	M1
	$2^{2x+1}-17\times 2^{x}+8=0$ $\Rightarrow 2y^{2}-17y+8=0$*	Cso. Complete proof that includes explicit statements for the addition **and** power law of indices on 2^{2x+1} with no errors. The equation needs to be as printed including the "= 0". If they work backwards, they do not need to write down the printed answer first but must end with the version in 2^{x} including '= 0'.	A1*
	The following are examples of acceptable proofs.		
	$2^{2x+1}=\left(2^{x+0.5}\right)^{2}=\left(2^{x}\sqrt{2}\right)^{2}=\left(y\sqrt{2}\right)^{2}=2y^{2}$ $\Rightarrow 2^{2x+1}-17\left(2^{x}\right)+8=2y^{2}-17y+8=0$		
	$2y^{2}=2\times 2^{x}\times 2^{x}=2^{2x+1}$ $\Rightarrow 2^{2x+1}-17\left(2^{x}\right)+8=2y^{2}-17y+8=0$		
	$2y^{2}-17y+8=0\Rightarrow 2\left(2^{x}\right)^{2}-17\left(2^{x}\right)+8=0$ $\Rightarrow 2\times 2^{2x}-17\left(2^{x}\right)+8=0\Rightarrow 2^{2x+1}-17\left(2^{x}\right)+8=0$		
	$2^{2x+1}=2\times 2^{2x}\Rightarrow 2\times 2^{2x}-17\left(2^{x}\right)+8=0$ $\Rightarrow 2y^{2}-17y+8=0$ Scores **M1A0** as $2^{2x}=\left(2^{x}\right)^{2}$ has not been shown explicitly		
	Special Case: $2^{2x+1}=2^{1}\times\left(2^{x}\right)^{2}$ or $2^{2x+1}=\left(2^{x}\right)^{2}\times 2^{1}$ **With or without the multiplication signs and with no subsequent explicit evidence of the power law scores M1A0**		
	Example of insufficient working: $2^{2x+1}=2\left(2^{x}\right)^{2}=2y^{2}$ **scores no marks as neither rule has been shown explicitly.**		
			(2)

<table>
<tr><td rowspan="5">(b)</td><td colspan="2">$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=...$
or
$2(2^x)^2-17(2^x)+8=0\Rightarrow(2(2^x)-1)((2^x)-8)(=0)\Rightarrow 2^x=...$
Solves the given quadratic either in terms of y or in terms of 2^x
See General Principles for solving a 3 term quadratic
Note that completing the square on e.g. $y^2-\frac{17}{2}y+4=0$ requires
$\left(y\pm\frac{17}{4}\right)^2\pm q\pm 4=0\Rightarrow y=...$</td><td>M1</td></tr>
<tr><td>$(y=)\frac{1}{2},8$ or $(2^x=)\frac{1}{2},8$</td><td>Correct values</td><td>A1</td></tr>
<tr><td rowspan="2">$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$</td><td>M1: Either finds one correct value of x for their 2^x or obtains a correct numerical expression in terms of logs e.g. for $k>0$
$2^x=k\Rightarrow x=\log_2 k$ or $\frac{\log k}{\log 2}$</td><td rowspan="2">M1 A1</td></tr>
<tr><td>A1: $x=-1,3$ only. Must be values of x.</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td></td><td colspan="2"></td><td>(6 marks)</td></tr>
</table>

Question Number	Scheme	Notes	Marks
8.(a)	$2px^2-6px+4p\,"="\,3x-7$ or $y=2p\left(\frac{y+7}{3}\right)^2-6p\left(\frac{y+7}{3}\right)+4p$	**Either:** Compares the given quadratic expression with the given linear expression using <, >, = , ≠ (May be implied) **or** Rearranges $y = 3x - 7$ to make x the subject and substitutes into the given quadratic	M1
	Examples $2px^2-6px+4p-3x+7(=0)$, $-2px^2+6px-4p+3x-7(=0)$ $2p\left(\frac{y+7}{3}\right)^2-6p\left(\frac{y+7}{3}\right)+4p-y(=0)$, $2py^2+(10p-9)y+8p(=0)$ $y=2px^2-6px+4p-3x+7$		dM1
	Moves all the terms to one side allowing sign errors only. Ignore > 0, < 0, = 0 etc. **The terms do not need to be collected. Dependent on the first method mark.**		
	E.g. $b^2-4ac=(-6p-3)^2-4(2p)(4p+7)$ $b^2-4ac=(10p-9)^2-4(2p)(8p)$	Attempts to use b^2-4ac with their a, b and c where $a=\pm 2p$, $b=\pm(-6p\pm 3)$ and $c=\pm(4p\pm 7)$ or for the quadratic in y, $a=\pm 2p$, $b=\pm(10p\pm 9)$ and $c=\pm 8p$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac . There must be no x's or y's. **Dependent on both method marks.**	ddM1
	$4p^2-20p+9<0$ *	Obtains printed answer with **no errors** seen (Allow $0>4p^2-20p+9$) **but this** < 0 must been seen at some stage before the last line.	A1*
			[4]
(b)	$(2p-9)(2p-1)=0\Rightarrow p=...$ to obtain $p=$	Attempt to solve the **given** quadratic to find 2 values for p. See general guidance.	M1
	$p=\frac{9}{2},\quad \frac{1}{2}$	Both correct. May be implied by e.g. $p<\frac{9}{2}$, $p<\frac{1}{2}$. Allow equivalent values e.g. 4.5, $\frac{36}{8}$, 0.5 etc. If they use the quadratic formula allow $\frac{20\pm 16}{8}$ for this mark but not $\sqrt{256}$ for 16 and allow e.g. $\frac{5}{2}\pm 2$ if they complete the square.	A1
	$\frac{1}{2}<p<4\frac{1}{2}$ Allow equivalent values e.g. $\frac{36}{8}$ for $4\frac{1}{2}$	M1: Chooses 'inside' region i.e. Lower Limit < p < Upper Limit or e.g. Lower Limit ≤ p ≤ Upper Limit A1: Allow $p\in(\frac{1}{2},4\frac{1}{2})$ or just $(\frac{1}{2},4\frac{1}{2})$ and allow $p>\frac{1}{2}$ **and** $p<4\frac{1}{2}$ and $4\frac{1}{2}>p>\frac{1}{2}$ but $p>\frac{1}{2}$, $p<4\frac{1}{2}$ scores M1A0 $\frac{1}{2}>p>4\frac{1}{2}$ scores M0A0	M1A1
	Allow working in terms of *x* in (b) but the answer must be in terms of *p* for the final A mark.		**[4]**
			8 marks