Question	Scheme	Marks	AOs
2	$\frac{9^{x-1}}{3^{y+2}}=81\Rightarrow\frac{3^{2x-2}}{3^{y+2}}=3^4$ or $\frac{9^{x-1}}{3^{y+2}}=81\Rightarrow\frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}}=9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4\Rightarrow y=$ or $\Rightarrow x-1-\frac{1}{2}y-1=2\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\frac{9^{x-1}}{3^{y+2}}\right)=\log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3\left(9^{x-1}\right)-(y+2)\log_3\left(3^{y+2}\right)=4$ $\Rightarrow 2(x-1)-y-2=4\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
			(3 marks)

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1}=3^{2x-1}$ and $9^{x-1}=3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x. Condone slips in their rearrangement.

A1: $y=2x-8$

Question	Scheme		Marks	AOs
2(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow (b^2+9)(b^2-2) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
			(8 marks)	

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $(C \neq 0)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

Eg. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $(b^2+m)(b^2+n) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

Question	Scheme	Marks	AOs
3 (i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2} \Rightarrow x=\ldots$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^{c}$ or $4^{dx+e}=4^{f}$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}} \Rightarrow 2\left(3x-2\right)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

Question	Scheme		Marks	AOs
12(a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	**Way 1:** $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	**Way 2:** $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer	$x=\dfrac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	
			(4 marks)	

Notes:

(a)

B1: Lists error in line 2 (as above)

B1: Lists error in line 4 (as above)

(b)

M1: Correct work with powers reaching this equation

A1: Correct answer here – there are many exact equivalents

Question Number	Scheme	Marks
2 (a)	$\frac{1}{3}x^2$	B1
		(1)
(b)	$\left(\frac{x}{\sqrt{2}}\right)^{-2} = \frac{2}{x^2}$	B1
		(1)
(c)	$\sqrt{3}(x) \div \sqrt{\frac{48}{x^4}} = \frac{\sqrt{3}}{\sqrt{48}} \times x\sqrt{x^4} = \frac{1}{4}x^3$	M1A1
		(2) **(4 marks)**

(a)

B1 $\frac{1}{3}x^2$ Accept exact alternatives like $\frac{x^2}{3}$ and $0.\dot{3}x^2$ but not expressions such as $0.33x^2$

(b)

B1 $\frac{2}{x^2}$ Accept exact alternatives such as $2\times x^{-2}$ or $2\times\frac{1}{x^2}$ (All forms must have a '2')

(c)

M1 Either the correct coefficient (accept $\frac{1}{4}$ or 0.25) or the correct power of x (accept x^3 or $\frac{1}{x^{-3}}$)

A1 Only accept $\frac{1}{4}x^3$ or simplified equivalents such as $0.25\times x^3$ Do NOT accept $\frac{1}{4x^{-3}}$ for this mark.

Question	Scheme	Marks
3 (i)	$\sqrt{45}-\frac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}$	
	$=\sqrt{9}\sqrt{5}-\frac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}+\sqrt{6}\sqrt{6}\sqrt{5}\quad=3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$	M1
	$=5\sqrt{5}$	A1*
		[2]
(ii)	$\text{LHS}=\frac{17\sqrt{2}(\sqrt{2}-6)}{(\sqrt{2}+6)(\sqrt{2}-6)}$	M1
	$=\frac{17\times2-17\times6\sqrt{2}}{2-36}$ oe	A1
	$=\frac{34-102\sqrt{2}}{-34}\quad=3\sqrt{2}-1*$	A1*
		[3]
		5 marks

Notes

(i)

M1: Shows at least **one term** on LHS as multiple of $\sqrt{5}$ with a correct intermediate step

Look for $\sqrt{45}=\sqrt{9}\times\sqrt{5}$ *or* $\sqrt{3\times3\times5}=3\sqrt{5}$, or even $45=3\times3\times5$ or 9×5 followed by $\sqrt{45}=3\sqrt{5}$

$\frac{20}{\sqrt{5}}=\frac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}$ or $\frac{20\sqrt{5}}{5}=4\sqrt{5}$ or $\frac{4\times5}{\sqrt{5}}=4\sqrt{5}$

$\sqrt{6}\sqrt{30}=\sqrt{6}\sqrt{6}\sqrt{5}$ or $\sqrt{6}\sqrt{30}=\sqrt{180}=\sqrt{36\times5}=6\sqrt{5}$

or even $180=2\times2\times3\times3\times5$ followed by $\sqrt{180}=6\sqrt{5}$

A1*: All three terms must have the intermediate step with $3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$ followed by $5\sqrt{5}$

Special Case: Score M1 A0 for $\sqrt{45}-\frac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}=3\sqrt{5}-4\sqrt{5}+6\sqrt{5}=5\sqrt{5}$ without the intermediate steps

Alternative method:

M1: Multiplies all terms by $\sqrt{5}$ to achieve $\sqrt{45}\times\sqrt{5}-20+\sqrt{5}\sqrt{6}\sqrt{30}=5\sqrt{5}\sqrt{5}$ and simplifies any one of the above terms to 15, -20, 30 or 25 showing the intermediate step

A1: All terms simplified showing the intermediate step (see main scheme on how to apply) followed by 15 – 20 + 30 = 25, and minimal conclusion eg. hence true

(ii)

M1: Multiply numerator and denominator by $\sqrt{2}-6$ or $6-\sqrt{2}$

A1: Multiplies out to a correct (unsimplified) answer. For example allow $=\frac{17\times2-17\times6\sqrt{2}}{2-36}$

A1: The denominator must be simplified so $\frac{34-17\times6\sqrt{2}}{-34}$ or similar such as $\frac{17\times2-102\sqrt{2}}{-34}$ is seen before you see the given answer $3\sqrt{2}-1$. There is no need to 'split' into two separate fractions.

Alternative method:

M1: Alternatively multiplies the rhs by $(\sqrt{2}+6)(3\sqrt{2}-1)$

A1: Correct unsimplified rhs Accept $3\times2-6+18\sqrt{2}-\sqrt{2}$

A1*: Simpifies rhs to $17\sqrt{2}$ **and** gives a minimal conclusion e.g. hence true or hence $\frac{17\sqrt{2}}{(\sqrt{2}+6)}=3\sqrt{2}-1$

Question Number	Scheme	Marks
3 (i)	Either $4^{2x+1}=2^{2(2x+1)}$ and $8^{4x}=2^{3\times 4x}$ or $8^{4x}=4^{\frac{3}{2}\times 4x}$	M1
	$2(2x+1)=12x\Rightarrow x=\frac{1}{4}$	dM1A1
		(3)
(ii)(a)	$3\sqrt{18}-\sqrt{32}=9\sqrt{2}-4\sqrt{2}=5\sqrt{2}$	M1A1
		(2)
(b)	$\sqrt{n}=5\sqrt{2}\Rightarrow n=\left(5\sqrt{2}\right)^2=25\times 2=50$	M1A1
		(2)
		(7 marks)
Alt 3 (i)	Taking logs of both sides and proceeding to $(2x+1)\log 4=4x\log 8$	M1
	$\Rightarrow x=\frac{\log 4}{4\log 8-2\log 4}$	
	$\Rightarrow x=\frac{\log 4}{\log 256}=\frac{1}{4}$	dM1A1
		(3)

(i)

M1 Writes both sides as powers of 2 or equivalent Eg $2^{2(2x+1)}=2^{3\times 4x}$

Alternatively writes both sides as powers of 4 or 8 or 64. Eg $8^{4x}=4^{\frac{3}{2}\times 4x}$

Note that expressions such as $2^{2+(2x+1)}=2^{3+4x}$ would be M0

Condone poor (or missing) brackets $2^{2\times 2x+1}=2^3$ but not incorrect index work eg $4^{2x+1}=8^{\frac{1}{2}(2x+1)}$

It is possible to use logs. most commonly with base 2 or 4. Using logs it is for reaching a linear form of the equation, again condoning poor bracketing .

$4^{2x+1}=8^{4x}\Rightarrow \log 4^{2x+1}=\log 8^{4x}\Rightarrow (2x+1)\log 4=4x\log 8$

dM1 Dependent upon the previous M. It is for equating the indices and proceeding to $x=..$

Condone sign/bracketing errors when manipulating the equation but not processing errors

If logs are used they must be evaluated without a calculator. Lengthy decimals would be evidence of this and would be dM0

$(2x+1)\log_2 4=4x\log_2 8\Rightarrow (2x+1)\times 2=4x\times 3\Rightarrow x=..$

$4^{2x+1}=8^{4x}\Rightarrow 2x+1=4x\log_4 8\Rightarrow 2x+1=\frac{3}{2}\times 4x\Rightarrow x=..$ is fine

A1 $x=\frac{1}{4}$ or equivalent

(ii)(a) Mark part (ii) as one complete question. Marks in (a) can be gained from (b)

M1 Writes either $\sqrt{18}=3\sqrt{2}$ or $3\sqrt{18}=9\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$

If the candidate writes $3\sqrt{18}-\sqrt{32}=k\sqrt{2}$ it can be scored for $\frac{3\sqrt{18}}{\sqrt{2}}=9$ or $\frac{\sqrt{32}}{\sqrt{2}}=4$

A1 $5\sqrt{2}$ or states $k=5$

The answer without working (the M1) would be 0 marks

(ii)(b)

M1 Moves from $\sqrt{n}=k\sqrt{2}$ to $n=2k^2$

Also accept for this mark $\sqrt{n}=\sqrt{50}$ or indeed $\sqrt{50}$ on its own

A1 $(n=)50$

Question Number	Scheme	Marks
2(i)	**Way 1:** $\frac{49}{\sqrt{7}} = \frac{7^2}{7^{\frac{1}{2}}} = 7^{2-\frac{1}{2}}$ **Way 2:** $7\sqrt{7} = 7^{1+\frac{1}{2}}$ **Way 3:** $7^a = \frac{49}{\sqrt{7}} \Rightarrow a = \frac{\log\frac{49}{\sqrt{7}}}{\log 7}$! or $7^a = \frac{49}{\sqrt{7}} \Rightarrow a = \log_7 \frac{49}{\sqrt{7}}$	M1
	!! $(a =)1\frac{1}{2}$ (oe) or see answer $= 7^{1\frac{1}{2}}$	A1
	!	[2]
(ii)	**Way 1:** $\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$! **Way 2:** $(15\sqrt{2}+20)(\sqrt{18}-4)$	M1
	Way 1: $= \frac{\ldots}{2}$! **Way 2:** $= 15\sqrt{36} - 60\sqrt{2} + 20\sqrt{18} - 80$!	B1
	Way 1: $\frac{10}{\sqrt{18}-4} = 5\left(3\sqrt{2}+4\right) = 15\sqrt{2}+20$ * ! **Way 2:** $= 90 - 60\sqrt{2} + 60\sqrt{2} - 80$ $= 10$ so $\frac{10}{\sqrt{18}-4} = 15\sqrt{2}+20$ *	A1cso
	!	[3]
		5 marks

Notes

(i)

Way 1:	Way 2:	Way 3:
M1: Subtracts **their** powers of 7	**M1:** Cancels fraction to $7\sqrt{7}$ and adds **their** powers of 7	**M1:** Correct use of logs to obtain a correct expression for a

A1: cao (answer only is 2 marks)
Do not allow work with inexact decimals for this mark e.g.
$49 \times 7^{-\frac{1}{2}} = 18.52 \Rightarrow \log 18.52 = 1.4999... \Rightarrow a = 1.5$ scores M1A0

(ii)

Way 1:

M1: Multiply numerator and denominator by $\sqrt{18}+4$ or equivalent. The statement $\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$ is sufficient but do not allow $\frac{10(\sqrt{18}+4)}{\sqrt{18}-4(\sqrt{18}+4)}$ unless missing brackets are implied by subsequent work.
B1: Correctly obtains ±2 in the denominator (**Must follow M1 – i.e. treat as A1**). May be implied by e.g. $\frac{10(\sqrt{18}+4)}{18-16} = 5\left(\sqrt{18}+4\right)$
A1: Correct result with no errors seen and $\sqrt{18} = 3\sqrt{2}$ **used before their final answer.**
Note that for **Way 1**, correct work leading to $5\sqrt{18}+20$ followed by ! $15\sqrt{2}+20$ with no intermediate step would lose the final mark

Way 2:

M1: Attempts to expand $(15\sqrt{2}+20)(\sqrt{18}-4)$ to obtain at least 3 (not necessarily correct) terms
B1: All 4 terms correct (**Must follow M1 – i.e. treat as A1**)
A1: Obtains 10 with no errors and $\sqrt{18} = 3\sqrt{2}$ seen or implied by e.g. $20\sqrt{18} = 60\sqrt{2}$ **and** conclusion that states the given answer i.e. not just 10 = 10

Question Number	Scheme	Marks
3.(a)	$\frac{1}{3}x$ as the final answer.	B1
(b)	$81x^{-3}$ as the final answer	B1
(c)	$x^{\frac{3}{2}}$ as the final answer	B1
		[3] **(3 marks)**

In all parts of this question candidates do not have to explicitly state the values of k and n. Award the mark(s) as above.
If they go on to give incorrect values of k and n you may isw, but do not isw on incorrect index work Eg. $\frac{1}{3}x = x^{-3}$.
If candidates make two different attempts and give two (or more) different answers then please put these in review

(a)

B1 $\frac{1}{3}x$ but accept equivalent such as $\frac{x}{3}$, $\frac{1}{3}\times x$, $\frac{1}{3}x^1$ or $3^{-1}x$ etc.

(b)

B1 $81x^{-3}$. Accept exact equivalents such as $81\times x^{-3}$

Do not accept $\frac{81}{x^3}$ as the final answer unless it is preceded by $81x^{-3}$ as it is not in the form required by the question.

(c)

B1 $x^{\frac{3}{2}}$ but accept exact equivalents such as $1\times x^{1.5}$

Do not accept $x\,x^{\frac{1}{2}}$ or $x\sqrt{x}$ as the final answer unless preceded by $x^{\frac{3}{2}}$ as they are not in the form required by the question.

January 2015
International A Level WMA01/01 Core Mathematics C12
Mark Scheme

Question Number	Scheme	Marks
1.	(a) x^2	B1 **[1]**
	(b) $\frac{1}{4}x^4$ or $\frac{1}{2^2}x^4$ or $0.25x^4$	B1, B1 **[2]**
		3 marks
	Notes	

(a) **B1**: This answer only

(b) **B1:** For $\frac{1}{4}x^k$ **as final answer**, k can even be 0. Also accept $\frac{1}{2^2}$ for B1 but 2^{-2} is not simplified and is B0

B1: for x to power 4 (independent mark) so kx^4 with k a constant (could even be 1) **as final answer**

n.b. Can score B0B1 or B1B0 or B0B0 or B1B1

Mark the final answer on this question

Also note : Candidates who misread question as $\sqrt{2x^3} \div \sqrt{\frac{32}{x^2}}$ should get $\frac{1}{4}x^{\frac{5}{2}}$ This is awarded B1B0

Special case: The answer $\left(\frac{1}{\sqrt{2}}x\right)^4$ is awarded B0 B1 as x may be in a bracket with power 4 outside.

Question Number	Scheme	Marks
2	$\sqrt{27} = 3\sqrt{3},\ \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$	
	$x\sqrt{27} + 21 = \dfrac{6x}{\sqrt{3}} \Rightarrow 3\sqrt{3}x + 21 = 2\sqrt{3}x$	M1 A1
	$\Rightarrow \sqrt{3}x = -21$	
	$\Rightarrow x = -\dfrac{21}{\sqrt{3}} \Rightarrow x = -7\sqrt{3}$	M1 A1
		(4 marks)

M1 Simplify either $\sqrt{27} = 3\sqrt{3}$ or $\dfrac{6}{\sqrt{3}} = 2\sqrt{3} = \left(\dfrac{6\sqrt{3}}{3}\right)$

A1 Uses both $\sqrt{27} = 3\sqrt{3}$ and $\dfrac{6}{\sqrt{3}} = 2\sqrt{3}$ to rewrite equation in a form equivalent to $3\sqrt{3}x + 21 = 2\sqrt{3}x$

M1 Collects x terms on one side of the equation, simplifies and divides reaching $x=\ldots$

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
Alt 2	$x\sqrt{27} + 21 = \dfrac{6x}{\sqrt{3}} \quad \left(\times\sqrt{3}\right) \Rightarrow \sqrt{3}\sqrt{27}x + 21\sqrt{3} = 6x$	M1
	$\Rightarrow 9x + 21\sqrt{3} = 6x$	A1
	$\Rightarrow 3x = -21\sqrt{3} \Rightarrow x = -7\sqrt{3}$	M1A1
		(4 marks)

M1 Multiply equation by $\sqrt{3}$, seen in at least two terms.

A1 $9x + 21\sqrt{3} = 6x$ or equivalent but the $\sqrt{81}$ must have been dealt with

M1 Collects terms in x, and proceeds to $x=..$

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
4. (i)	$\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{2}+\sqrt{6}\right)}$	M1
	$\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{2}+\sqrt{6}\right)=8-6=2$	B1
	$\sqrt{6}=\sqrt{2}\sqrt{3}$ used in numerator - may be implied by a **correct** factorisation of numerator	B1
	Concludes $\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{2}=2\sqrt{2}\left(2+\sqrt{3}\right)$ *	A1 *
		[4]
(ii)	1st two terms $\sqrt{27}=3\sqrt{3}$ **and** $\sqrt{21}\times\sqrt{7}=7\sqrt{3}$	B1
	3rd term See $2\sqrt{3}$ or $\dfrac{6\sqrt{3}}{3}$	B1
	$3\sqrt{3}+7\sqrt{3}-2\sqrt{3}=8\sqrt{3}$ or $3\sqrt{3}+7\sqrt{3}-\dfrac{6\sqrt{3}}{3}=8\sqrt{3}$ *	B1 *
		[3]
Alternative for (i)	Assume result and multiply both sides by $\left(2\sqrt{2}-\sqrt{6}\right)$	M1
	$\left(2\sqrt{2}-\sqrt{6}\right)\left(4\sqrt{2}+2\sqrt{6}\right)=16-12=4$	B1 B1
	So LHS = RHS and result is true	A1 **[4]**
Alternative for (ii)	$\dfrac{\sqrt{81}+\sqrt{21\times7\times3}-6}{\sqrt{3}}$ *Or* $\sqrt{81}+\sqrt{21\times7\times3}-6=8\sqrt{3}\sqrt{3}$	B1
	$\dfrac{9+21-6}{\sqrt{3}}$ $9+21-6=$	B1
	$\dfrac{24}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=8\sqrt{3}$ $9+21-6=24$ **so equation is true**	B1 **[3]** **(7 marks)**
	Notes	

(i) M1: Multiplies numerator and denominator by $\pm\left(2\sqrt{2}+\sqrt{6}\right)$

B1: correct treatment of denominator to give 2 (may be implied by answer obtained with no errors seen)

B1: Splits $\sqrt{6}=\sqrt{2}\sqrt{3}$ - may be implied , but **B0 for** $2\sqrt{6}=2\sqrt{2}(2\sqrt{3}...)$ **A1** cao reaches result and no errors should be seen

N.B. $\dfrac{4\left(2\sqrt{2}+\sqrt{6}\right)}{2}=2\sqrt{2}\left(2+\sqrt{3}\right)$ may be awarded B1 A1 as there is an implication that $\sqrt{6}=\sqrt{2}\sqrt{3}$

(ii) B1: expresses both of first two terms as multiple of root 3 correctly

B1: rationalises denominator in second term -may not see working

B1: has used $3\sqrt{3}+7\sqrt{3}-2\sqrt{3}=8\sqrt{3}$ N.B. $3\sqrt{3}+7\sqrt{3}-\dfrac{6}{\sqrt{3}}=8\sqrt{3}$ is B1B0B0

(i) Alternative	**M1:** Assume result and multiply both sides by $\left(2\sqrt{2}-\sqrt{6}\right)$ **2nd B1**: Uses $\sqrt{2}\sqrt{3}=\sqrt{6}$ **1st B1**: Multiplies out these two brackets to give 4 **A1**: conclusion
(ii) Alternatives	**B1:** Uses common denominator **or** multiplies **both sides** by root 3 and obtains correct unsimplified equation **B1: LHS** numerator correctly simplified or just see 9 + 21 – 6 B1: In the first alternative must see multiplication of numerator and denominator by $\sqrt{3}$ to give $8\sqrt{3}$ In the second need statement LHS = RHS and so true

Question Number	Scheme	Notes	Marks
3.(a)	$\sqrt{50}-\sqrt{18}=5\sqrt{2}-3\sqrt{2}$	$\sqrt{50}=5\sqrt{2}$ or $\sqrt{18}=3\sqrt{2}$ **and** the other term in the form $k\sqrt{2}$. This mark may be implied by the correct answer $2\sqrt{2}$	M1
	$=2\sqrt{2}$	Or $a=2$	A1
			[2]
(b) **WAY 1**	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{12\sqrt{6}}{4}$	Rationalises the denominator by a correct method e.g. multiplies numerator and denominator by $k\sqrt{2}$ to obtain a multiple of $\sqrt{6}$. Note that multiplying numerator and denominator by $2\sqrt{2}$ or $-2\sqrt{2}$ is quite common and is acceptable for this mark. May be implied by a correct answer. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 2	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}\times\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{50}+\sqrt{18}}$ **or** $\dfrac{12\sqrt{3}}{5\sqrt{2}-3\sqrt{2}}\times\dfrac{5\sqrt{2}+3\sqrt{2}}{5\sqrt{2}+3\sqrt{2}}$	For rationalising the denominator by a correct method i.e. multiplying numerator and denominator by $k\left(\sqrt{50}+\sqrt{18}\right)$	M1
	$\dfrac{60\sqrt{6}+36\sqrt{6}}{50-18}$	For replacing numerator by $\alpha\sqrt{6}+\beta\sqrt{6}$. **This is dependent on the first M1 and there is no need to consider the denominator for this mark.**	dM1
	$=3\sqrt{6}$ or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 3	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\dfrac{12\sqrt{3}}{2\sqrt{2}}=\dfrac{6\sqrt{3}}{\sqrt{2}}=\dfrac{\sqrt{108}}{\sqrt{2}}=\sqrt{54}=\sqrt{9}\sqrt{6}$	Cancels to obtain a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3$, $c=6$	Cao and cso	A1
			[3]
(b) WAY 4	$\dfrac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\dfrac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$\left(\dfrac{12\sqrt{3}}{"2"\sqrt{2}}\right)^2=\dfrac{432}{8}$		
	$\sqrt{54}=\sqrt{9}\sqrt{6}$	Obtains a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3$, $c=6$	Cao and cso (do not allow $\pm3\sqrt{6}$)	A1
			5 marks

Question Number	Scheme		Marks
1.(a)	20	Sight of 20. (4×5 is not sufficient)	B1
			(1)
(b)	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$	Multiplies top and bottom by a **correct** expression. This statement is sufficient. NB $2\sqrt{5}+3\sqrt{2} \equiv \sqrt{20}+\sqrt{18}$	M1
	(Allow to multiply top and bottom by $k(2\sqrt{5}+3\sqrt{2})$)		
	$=\frac{...}{2}$	Obtains a denominator of 2 or sight of $(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})=2$ with no errors seen in this expansion. May be implied by $\frac{...}{2k}$	A1
	Note that M0A1 is not possible. The 2 must come from a correct method.		
	Note that if M1 is scored there is no need to consider the numerator. **e.g.** $\frac{2(MR?)}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{...}{2}$ scores M1A1		
	Numerator = $\sqrt{2}(2\sqrt{5}\pm3\sqrt{2}) = 2\sqrt{10}\pm6$	An attempt to multiply the numerator by $\pm(2\sqrt{5}\pm3\sqrt{2})$ and obtain an expression of the form $p+q\sqrt{10}$ where p and q are integers. This may be implied by e.g. $2\sqrt{10}+3\sqrt{4}$ or by their final answer.	M1
	(Allow attempt to multiply the numerator by $k(2\sqrt{5}\pm3\sqrt{2})$)		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{2\sqrt{10}+6}{2} = 3+\sqrt{10}$	Cso. For the answer as written or $\sqrt{10}+3$ or a statement that a = 3 and b = 10. Score when first seen and ignore any subsequent attempt to 'simplify'. Allow $1\sqrt{10}$ for $\sqrt{10}$	A1
			(4)
			(5 marks)
	Alternative for (b)		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{1}{\sqrt{10}-3}$ or $\frac{2}{2\sqrt{10}-6}$	M1: Divides or multiplies top and bottom by $\sqrt{2}$ A1: $\frac{k}{k(\sqrt{10}-3)}$	M1A1
	$=\frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$	M1: Multiplies top and bottom by $\sqrt{10}+3$	M1
	$=3+\sqrt{10}$		A1
2.	$y-2x-4=0,\ \ 4x^2+y^2+20x=0$		