AS MATHEMATICS 7356/1

Paper 1

Mark scheme

June 2021

Version: 1.1 Final Mark Scheme

216a7356/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	108
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$-\frac{1}{x^2}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Writes the correct equation ACF. Condone omission of y =. ISW after correct answer.	1.1b	B1	$y = \frac{1}{x-3}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Deduces correct equation of vertical asymptote	2.2a	B1	$x = 3$
	Recalls correct equation of horizontal asymptote	1.2	B1	$y = 0$
	Subtotal		**2**	

	Question Total		**3**	

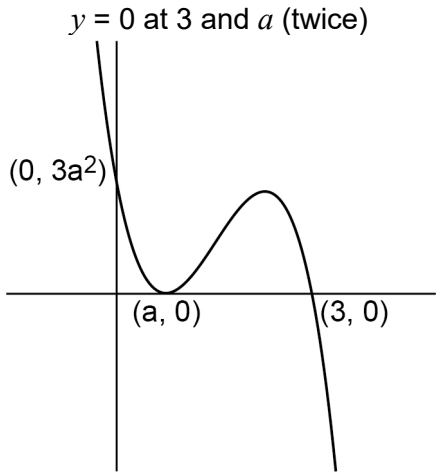
Q	Marking instructions	AO	Marks	Typical solution
4(a)(i)	Uses coordinates of *A* and *B* to find gradient of *AB*	3.1a	M1	Grad *AB* = $\frac{3}{6} = \frac{1}{2}$ = Grad *DC* Equation is $y - 4 = \frac{1}{2}(x - 3)$ $2y = x + 5$
	Obtains correct equation of *CD* (any form)	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)(ii)	Uses perpendicular gradients property.	1.1a	M1	Grad *DA* = $\frac{-1}{\text{Grad AB}} = -2$ Equation is $y + 2 = -2(x - 1)$ $y = -2x$ Intersect at (–1, 2) = *D*
	Obtains correct equation of *AD* using their gradient (any form) Or shows that A to (-1, 2) has required gradient of -2	1.1b	A1F	
	Obtains correct coordinates of *D* Or shows that C to (-1,2) has required gradient of 0.5 Or shows that (-1, 2) lies on both lines	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)(i)	Calculates length of *AB* and *CD*. At least one correct.	1.1a	M1	*AB* = √(36 + 9) = √45 = 3√5 *CD* = √(4 + 16) = √20 = 2√5
	Obtains correct simplified sum	1.1b	A1	*AB* + *CD* = 5√5
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)(ii)	Calculates *AD* and applies trapezium area formula	1.1a	M1	*AD* = √(4 + 16) = √20 = 2√5 $= \frac{1}{2}(5\sqrt{5} \times 2\sqrt{5})$ = 25
	Obtains correct area	1.1b	A1	
	Subtotal		**2**	

	Question Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Draws a correctly orientated cubic graph	1.1b	B1	
	Draws a graph cutting x axis at (3, 0) labelled. Condone omission of 0	1.1b	B1	
	Draws a graph meeting x axis at (a, 0) labelled. Condone omission of 0	2.2a	B1	
	Draws a graph cutting y axis at (0, 3a^2) labelled. Condone omission of 0	1.1b	B1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Deduces that $x < a$ or $a < x < 3$ OE	2.2a	M1	$\{x: x < a\} \cup \{x: a < x < 3\}$
	Expresses answer in set notation. Or $\{x: 0 < x < 3,\ x \neq a\}$	2.5	R1	
	Subtotal		**2**	

	Question Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6	Recalls gradient function for e^{kx}	1.2	B1	Gradient of e^{-2x} is $-2e^{-2x}$ Gradient of line is $-\frac{1}{8}$ $-2e^{-2x} = -\frac{1}{8}$ $e^{2x} = 16$ $2x = \ln 16$ $x = \frac{1}{2}\ln 16 = \ln 4$ $y = e^{-2\ln 4} = e^{-\ln 16} = \frac{1}{e^{\ln 16}}$ $y = \frac{1}{16}$ P is $(\ln 4, \frac{1}{16})$
	Finds gradient of line	1.1b	B1	
	Equates their gradient of line to their gradient of tangent	3.1a	M1	
	Solves their equation for x	1.1a	M1	
	Obtains correct value for x as ln 4	1.1b	A1	
	Substitutes their x value to obtain a value for y in a correct but unsimplified form Or uses gradient = $-2y = -\frac{1}{8}$	1.1a	M1	
	Obtains correct value for y	1.1b	R1	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Explains that a represents the initial population. OE	2.4	E1	a is the population in 2010
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Takes logarithms to base 10 of both sides	1.1a	M1	$P = a(10^{bt})$ $\log_{10}P = \log_{10}(a10^{bt})$
	Completes derivation convincingly AG	2.1	R1	$= \log_{10} a + \log_{10}(10^{bt})$ $= \log_{10} a + bt$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(i)	Completes table correctly, figures seen in table or text or used in part **(c)(ii)**	3.3	B1	

Year	2013	2015
t	3	**5**
P	10 200	12 800
$\log_{10} P$	4.0086	**4.1072**

	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(ii)	Uses data to set up a pair of simultaneous equations	3.1a	M1	$4.0086 = 3b + \log_{10} a$ $4.1072 = 5b + \log_{10} a$
	Solves equations for either b or $\log_{10} a$ correct	1.1b	A1	$b = 0.0493$ $\log_{10} a = 3.8607$
	Converts $\log_{10} a$ to obtain a value of a or uses their b and data to calculate a	1.1a	M1	$a = 7256$
	Obtains both b and a correct. AWRT 0.049 and AWFW 7200 to 7300	1.1b	A1	

	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(iii)	Substitutes their values of a and b into model and t = 14	3.4	M1	$7256 \times 10^{(14 \times 0.0493)}$ = 35555
	Calculates correct value of population AWFW 35500 to 35600 FT provided >12800	1.1b	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)(i)	States an appropriate assumption about the model.	3.5b	E1	The value of constant b does not change after 2020
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)(ii)	Makes appropriate comment about limited data, or length of extrapolation, changing food supply, disease or equivalent specific factor.	3.5a	E1	Not very reliable, because it is only based on data from two years
	Subtotal		**1**	

	Question Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ identity	1.2	M1	$3\sin\theta\tan\theta = 5\cos\theta - 2$ $3\sin\theta\frac{\sin\theta}{\cos\theta} = 5\cos\theta - 2$ $3\sin^2\theta = 5\cos^2\theta - 2\cos\theta$ $3(1 - \cos^2\theta) = 5\cos^2\theta - 2\cos\theta$ $8\cos^2\theta - 2\cos\theta - 3 = 0$ $(4\cos\theta - 3)(2\cos\theta + 1) = 0$
	Uses $\sin^2\theta + \cos^2\theta = 1$ identity	1.2	M1	
	Manipulates to obtain the given equation	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Obtains any two solutions (AWRT)	1.1a	M1	$\theta = \pm 41°$ and $\pm 120°$
	Obtains all four solutions (AWRT)	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Deduces that the required solutions are double their previous solutions PI by ±83°or ±82° or ±240°	2.2a	M1	$\frac{1}{2}\theta = \pm 41.4$ and $\pm 120°$ $\theta = \pm 83°$
	Obtains ±83° or AWRT ±83° and no further solutions.	1.1b	A1	
	Subtotal		**2**	

	Question Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9	Expresses all terms as powers of x at least 2 correct. PI	1.1a	M1	$y = ax^{-0.5} + bx^2 + cx^{-3}$ $\frac{dy}{dx} = -\frac{1}{2}ax^{-1.5} + 2bx - 3cx^{-4}$ $\frac{d^2y}{dx^2} = \frac{3}{4}ax^{-2.5} + 2b + 12cx^{-5}$ As a, b, c and x are all > 0, all terms must be positive so $\frac{d^2y}{dx^2}$ is positive so turning point is a minimum
	Differentiates at least one of their negative powers correctly	1.1a	M1	
	Obtains completely correct differential	1.1b	A1	
	Differentiates again, powers and signs correct	1.1a	M1	
	Deduces that $\frac{d^2y}{dx^2}$ is positive	2.2a	A1F	
	Explains that positive second differential shows that turning point is a minimum	2.4	E1F	
	Shows completely correct mathematics throughout, including coefficients of $\frac{d^2y}{dx^2}$ must refer to a, b, c and x being > 0	2.1	R1	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
10	Circles correct answer	1.1b	B1	$\begin{bmatrix}-4\\-7\end{bmatrix}$ N
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
11	Ticks correct box	2.2a	B1	Jackie and Tom are both right
	Total		**1**	

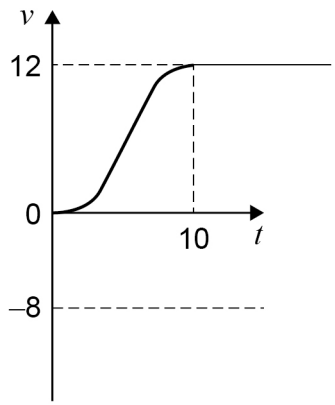
Q	Marking instructions	AO	Marks	Typical solution
12(a)	Uses Pythagoras to show given magnitude AG	1.1b	B1	$\sqrt{8^2 + 6^2} = 10$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Uses appropriate suvat equation with given values to find correct speed value condone missing units	1.1b	B1	$u = 0,\ a = 10,\ t = 3$ so $v = 0 + (3)(10) = 30$ m s^{-1}
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Uses F = ma, using magnitude of force with magnitude of acceleration or in vector form PI by correct answer	3.4	M1	$\sqrt{2^2 + 1.5^2} = 2.5$ 2.5 = m × 10 $m = 0.25$ kg
	Finds correct mass condone missing units	1.1b	A1	
	Subtotal		**2**	

	Question Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Finds any area either above or below the axis	3.1b	M1	Area above = $\left(\frac{1}{2}\right)(12)(10+32)$ = 252 Area below = $\left(\frac{1}{2}\right)(13)(8)$= 52 r = 252 – 52 = 200 m
	Obtains correct values of area above or area below the axis PI	1.1b	A1	
	Obtains correct displacement condone missing units	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Sketches a curve at the end t = 0 correctly	3.5c	B1	
	Sketches a curve at the end t = 10 correctly	3.5c	B1	
	Subtotal		**2**	

	Question Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Integrates given expression to find v with at least one term correct	3.4	M1	$v = \int a\,dt$ $= 3t + 0.1t^2 + c$ When t = 2, $v = k$ $k = 6 + 0.4 + c$ $c = k - 6.4$ $v = 3t + 0.1t^2 + k - 6.4$ Since $v = c$ when $t = 0$ the initial velocity is $k - 6.4$
	Obtains an expression for v with both terms correct condone omission of $+ c$	1.1b	A1	
	Substitutes t = 2 and $v = k$ into their integrated expression (must include constant of integration)	1.1a	M1	
	Completes rigorous argument with no slips to obtain $v = k - 6.4$ when $t = 0$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Forms equation using k, $k - 6.4$, and 0.2 or 5	3.1b	M1	$0.2k = k - 6.4$
	Obtains $k = 8$	1.1b	A1	$k = 8$
	Subtotal		**2**	

	Question Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	States either friction or air resistance or both but no incorrect forces.	3.3	B1	Friction
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Uses $F = ma$ to form a three term equation modelling the motion of P	3.3	M1	$10 - T = 1 \times a$ $T - 5 = 4 \times a$ $5 = 5a$ $a = 1$
	Uses $F = ma$ to form a three term equation modelling the motion of B	3.3	M1	
	Obtains two correct equations	1.1b	A1	
	Completes a rigorous argument to show that $a = 1$	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Uses appropriate suvat equation with given values to find v or v^2	3.3	M1	$v^2 = 0 + 2 \times 1 \times 0.2 = 0.4$
	Uses $F = ma$ to find acceleration of B after string breaks	3.4	M1	$-5 = 4a$ $a = -1.25$
	Obtains $a = -1.25$	1.1b	A1	$0^2 = 0.4 + (2)(-1.25)s$
	Uses appropriate suvat equation to obtain 0.16m. Must include units.	1.1b	A1	$s = 0.16$ m $s = 0.2$ m, to 1 sf
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(d)	States a valid assumption Accept: • No air resistance to motion of P • string remains parallel to table • B does not reach end of table before it stops • P does not hit floor before string breaks	3.5b	E1	String is inextensible
	Subtotal		**1**	

	Question Total		**10**	