13. The curve C has parametric equations

$$x = \sin 2\theta \qquad y = \operatorname{cosec}^3 \theta \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

4.

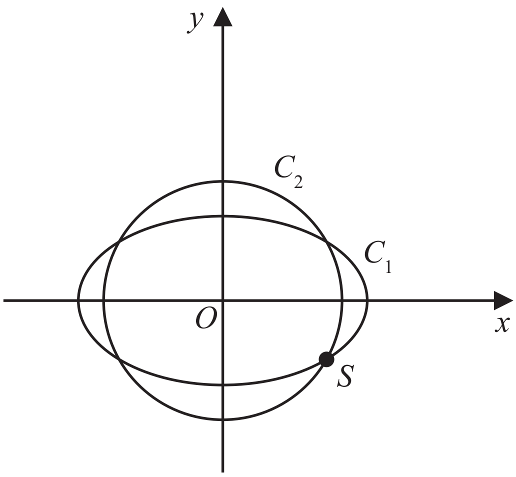

Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

Leave blank

11. The curve C has parametric equations

$$x = 10\cos 2t, \quad y = 6\sin t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

The point A with coordinates (5, 3) lies on C.

(a) Find the value of t at the point A.

(1)

(b) Show that an equation of the normal to C at A is

$$3y = 10x - 41$$

(6)

The normal to C at A cuts C again at the point B.

(c) Find the exact coordinates of B.

(8)

Leave blank

9.

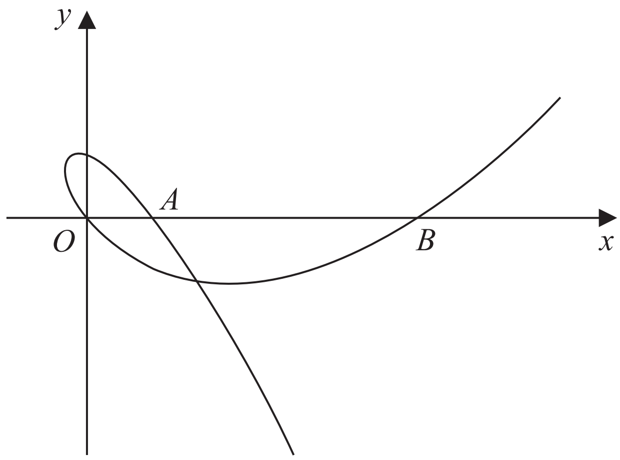

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t, \qquad y = t^3 - 9t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is $9x - 4y - 135 = 0$

(5)

The tangent to the curve at B cuts the curve again at the point X.

(c) Find the coordinates of X.

(5)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

P 4 4 9 7 0 A 0 2 6 4 4

Leave blank

13. A curve C has parametric equations

$$x = 6\cos 2t, \qquad y = 2\sin t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(a) Show that $\frac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ.

(4)

(b) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are simplified surds.

(6)

The cartesian equation for the curve C can be written in the form

$$x = f(y), \qquad -k < y < k$$

where $f(y)$ is a polynomial in y and k is a constant.

(c) Find $f(y)$.

(3)

(d) State the value of k.

(1)

Leave blank

13.

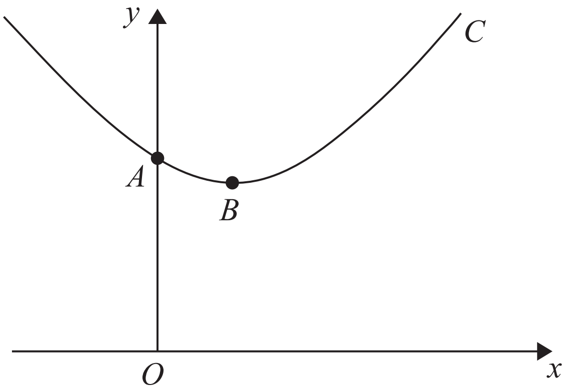

Figure 4

The curve C shown in Figure 4 has parametric equations

$$x = 1 + \sqrt{3}\tan\theta, \qquad y = 5\sec\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

The curve C crosses the y-axis at A and has a minimum turning point at B, as shown in Figure 4.

(a) Find the exact coordinates of A.

(3)

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \sin\theta$, giving the exact value of the constant λ.

(4)

(c) Find the coordinates of B.

(2)

(d) Show that the cartesian equation for the curve C can be written in the form

$$y = k\sqrt{(x^2 - 2x + 4)}$$

where k is a simplified surd to be found.

(3)

Leave blank

10.

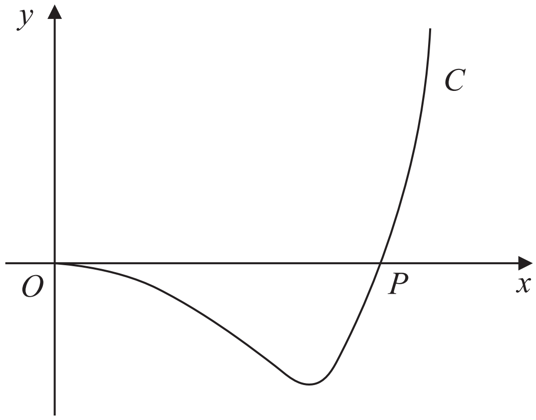

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \frac{20t}{2t+1} \qquad y = t(t-4), \quad t > 0$$

The curve cuts the x-axis at the point P.

(a) Find the x coordinate of P.

(2)

(b) Show that $\frac{dy}{dx} = \frac{(t-A)(2t+1)^2}{B}$ where A and B are constants to be found.

(5)

(c) (i) Make t the subject of the formula

$$x = \frac{20t}{2t+1}$$

(ii) Hence find a cartesian equation of the curve C. Write your answer in the form

$$y = \mathrm{f}(x), \qquad 0 < x < k$$

where $\mathrm{f}(x)$ is a single fraction and k is a constant to be found.

(6)

Leave blank

1. The curve C has parametric equations

$$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \qquad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \qquad x > -4$$

where a and b are integers to be determined.

(3)

Leave blank

5.

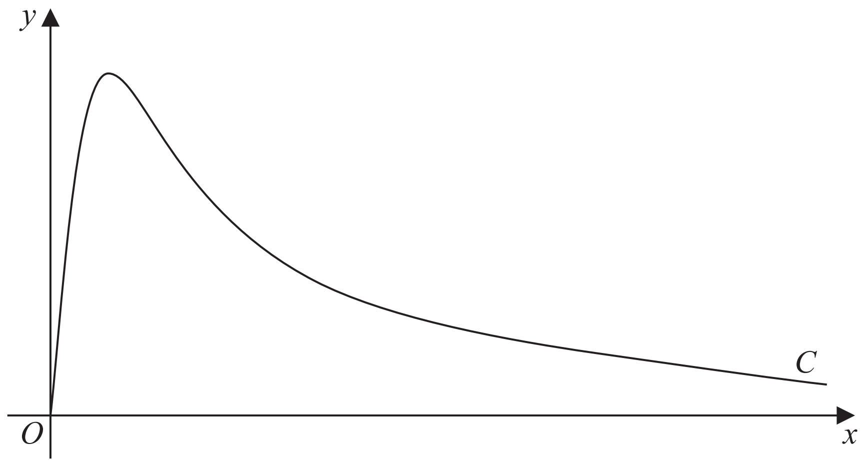

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q.

(2)

Leave blank

5. A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

Leave blank

8.

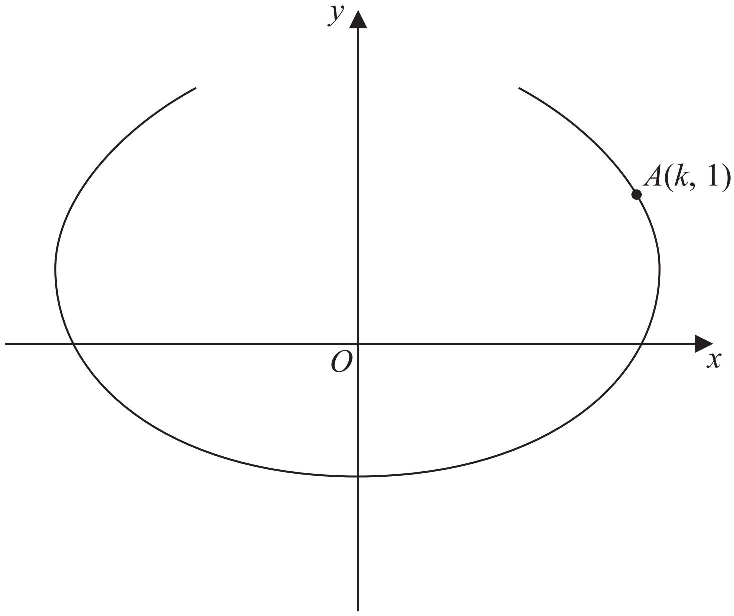

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4\sin t,\ \ y = 1 - 2\cos t, \qquad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

! [*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

Leave blank

5.

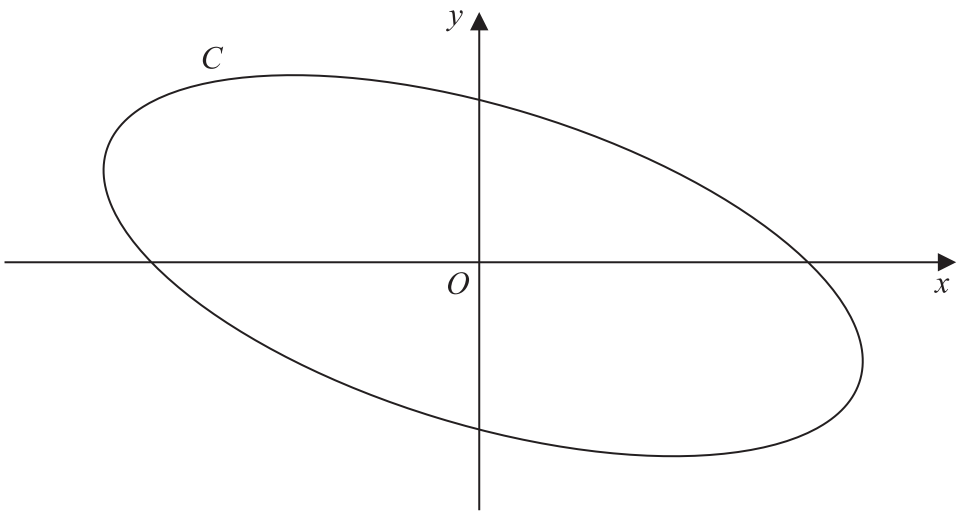

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

Leave blank

4. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\dfrac{dy}{dx}$ at the point where $t = \dfrac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = f(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(3)

(c) Write down the range of $f(x)$.

(2)

Leave blank

6.

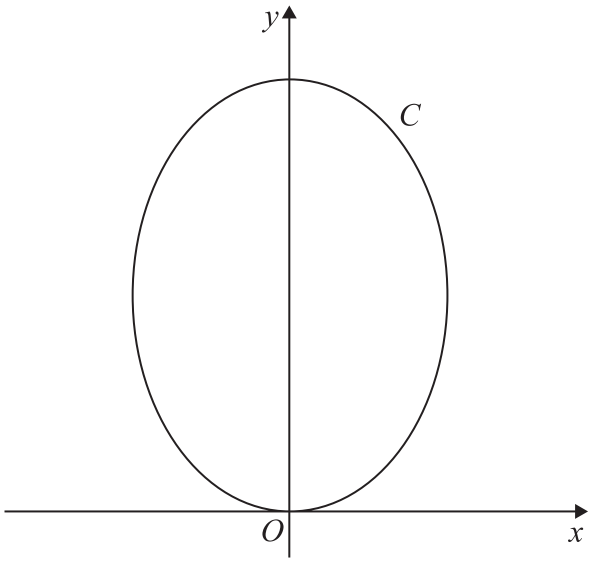

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4\cos^2 t, \quad 0 \leqslant t \leqslant \pi$$

(a) Show that $\frac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(c) Find a cartesian equation of C.

(3)

Leave blank

5.

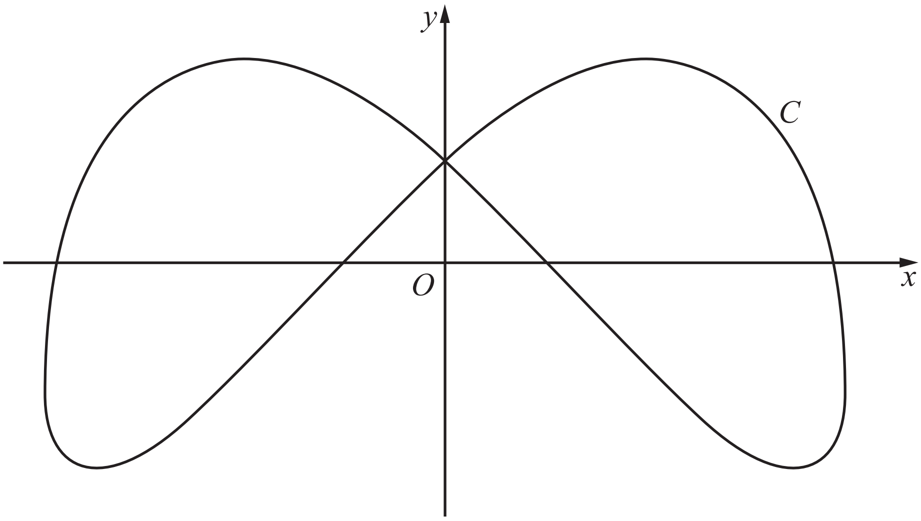

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)