Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow t rather than T and A rather than a.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$

$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \ *$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for b (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of a.

E.g. substitutes one of the given points and their value for b into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of a. Allow slips in rearranging their equation but must be correct log work to find a.

Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of a.

E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation

Question	Scheme	Marks	AOs
8(a)	$A = 1000$	B1	3.4
	$2000 = 1000\mathrm{e}^{5k}$ or $\mathrm{e}^{5k} = 2$	M1	1.1b
	$\mathrm{e}^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \ldots$	M1	2.1
	$N = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000\mathrm{e}^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{\mathrm{d}N}{\mathrm{d}t} = 1000\times\left(\frac{1}{5}\ln 2\right)\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{\mathrm{d}N}{\mathrm{d}t} = 1000\times 0.139\mathrm{e}^{0.139t}$ $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000\times\left(\frac{1}{5}\ln 2\right)\mathrm{e}^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000\times 0.139\mathrm{e}^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500\mathrm{e}^{1.4\times\left(\frac{1}{5}\ln 2\right)T} = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)T}$ or $500\mathrm{e}^{1.4\times\text{"}0.139\text{"}t} = 1000\mathrm{e}^{\text{"}0.139\text{"}t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4\times\text{"}0.339\text{"}T = \ln 2 + \text{"}0.339\text{"}t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000\mathrm{e}^{\ldots t}$

M1: Uses the model to set up a correct equation in *k*. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $a\mathrm{e}^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000\mathrm{e}^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000\times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates $\alpha\mathrm{e}^{kt}$ to $\beta\mathrm{e}^{kt}$ and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for *k,* but also allow in terms of *k*

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

....

Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000\mathrm{e}^{0.139t}$, and then writes at $t = 8$ $\frac{\mathrm{d}N}{\mathrm{d}t} =$ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500\mathrm{e}^{1.4\times\text{"}0.139\text{"}t} = 1000\mathrm{e}^{\text{"}0.139\text{"}t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0

....

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\quad 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$		
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

Question	Scheme	Marks	AOs
9(a)	$t=0,\ \theta=18 \Rightarrow 18=A-B$ **or** $t=10,\ \theta=44 \Rightarrow 44=A-B\mathrm{e}^{-0.7}$	M1	3.1b
	$t=0,\ \theta=18 \Rightarrow 18=A-B$ **and** $t=10,\ \theta=44 \Rightarrow 44=A-B\mathrm{e}^{-0.7}$ **and** $\Rightarrow A=\ldots,\ B=\ldots$	M1	3.1a
	At least one of: $A = 69.6$, $B = 51.6$ but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta=69.6-51.6\mathrm{e}^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Question Number	Scheme	Marks
6(a)	$25\mathrm{e}$ or equivalent decimal - Accept awrt 68	B1
		(1)
(b)	$50 = 25\mathrm{e}^{1-10k}$	M1
	Way 1 $\mathrm{e}^{1-10k} = 2$ $\Rightarrow 1 - 10k = \ln 2$ $\Rightarrow k = \dfrac{\ln \mathrm{e} - \ln 2}{10}$; Way 2 $\mathrm{e}^{-10k} = 2/\mathrm{e}$ $-10k = \ln(2/\mathrm{e})$ $k = \dfrac{-\ln\left(\frac{2}{\mathrm{e}}\right)}{10}$; Way 3 $\mathrm{e}^{10k} = \mathrm{e}/2$ $10k = \ln(\mathrm{e}/2)$ No intermediate step needed	A1 M1
	$\Rightarrow k = \dfrac{\ln\left(\frac{1}{2}\mathrm{e}\right)}{10}$ *	A1*
		(4)
(c)	Uses $m = 20$ and their numerical k so $20 = 25\mathrm{e}^{1-'k't} \Rightarrow \mathrm{e}^{1-'k't} = 0.8$ o.e.	M1
	$\Rightarrow t = \dfrac{1 - \ln 0.8}{'k'}$	dM1
	$\Rightarrow t =$ awrt 40 (years)	A1
		(3)
		(8 marks)

Question Number	Scheme	Marks
6(a)	320 $(°C)$	B1 [1]
(b)	$\text{T}=180 \Rightarrow 300\text{e}^{-0.04t}=160, \Rightarrow \text{e}^{-0.04t}=\frac{160}{300}(\text{awrt } 0.53)$	M1, A1
	$t=\frac{1}{-0.04}\ln\left(\frac{160}{300}\right)$ *or* $\frac{1}{0.04}\ln\left(\frac{300}{160}\right)$	dM1
	15.7 (minutes) cao	A1cso [4]
(c)	$\frac{\text{d}T}{\text{d}t}=(-0.04)\times 300\text{e}^{-0.04t} = (-0.04)\times(T-20)$	M1 A1
	$=\frac{20-T}{25}$ *	A1* [3]
		(8 marks)
Alt **(b)**	Puts $T = 180$ so $180 = 300\text{e}^{-0.04t} + 20$ and $300\text{e}^{-0.04t}=160$	M1
	$\ln 300 - 0.04t = \ln 160 \Rightarrow t = ..,\ \frac{\ln 300-\ln 160}{0.04}$	dM1, A1
	15.7 (minutes) cao	A1cso [4]

(a)

B1 320 cao - do not need ° C

(b)

M1 Substitutes T= 180 and proceeds to a form $Ae^{-0.04t}=B$ or $Ce^{0.04t}=D$

Condone slips on the power for this mark. For example condone $Ae^{-0.4t}=B$

A1 For $e^{-0.04t}=\frac{160}{300}$ or $e^{0.04t}=\frac{300}{160}$ or exact equivalent such as $e^{-0.04t}=\frac{8}{15}$

Accept decimals here $e^{-0.04t}=0.53..$ or $e^{0.04t}=1.875$

dM1 Dependent upon having scored the first M1, it is for moving from $\text{e}^{kt}=c, c>0 \Rightarrow t=\frac{\ln c}{k}$

A1 15.7 correct answer and correct solution only. Do not accept awrt

(c)

M1 Differentiates to give $\frac{\text{d}T}{\text{d}t}=k\text{e}^{-0.04t}$. Condone $\frac{\text{d}T}{\text{d}t}=k\text{e}^{-0.4t}$ following $\text{T}=300\text{e}^{-0.4t}+20$

This can be achieved from $\text{T}=300\text{e}^{-0.04t}+20 \Rightarrow t=\frac{1}{-0.04}\ln\left(\frac{T-20}{300}\right) \Rightarrow \frac{\text{d}t}{\text{d}T}=\frac{k}{(T-20)}$ for M1

A1 Correct derivative and correctly eliminates t to achieve $\frac{\text{d}T}{\text{d}t}=(-0.04)\times(T-20)$ oe

If candidate changes the subject it is for $\frac{\text{d}t}{\text{d}T}=\frac{-25}{(T-20)}$ oe

Alternatively obtains the correct derivative, substitutes T in $\frac{\text{d}T}{\text{d}t}=\frac{20-T}{25} \rightarrow \frac{\text{d}T}{\text{d}t}=-12\text{e}^{-0.04t}$ and compares. To score the A1* under this method there must be a statement.

A1* Obtains printed answer correctly – no errors

Question Number	Scheme		Marks
9.(a)	$t=0 \Rightarrow P=\dfrac{9000}{3+7}=900$	M1: Sets $t=0$, may be implied by $e^0=1$ or may be implied by $\dfrac{9000}{3+7}$ or by a correct answer of 900.	M1A1
		A1: 900	
			(2)
(b)	$t\to\infty \quad P\to\dfrac{9000}{3}=3000$	Sight of 3000	B1
			(1)
(c)	$t=4,\ P=2500 \Rightarrow 2500=\dfrac{9000e^{4k}}{3e^{4k}+7}$	Correct equation with $t=4$ and $P=2500$	B1
	$e^{4k}=\dfrac{17500}{1500}=(\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k}=\dfrac{1500}{17500}=(\text{awrt } 0.857)$	M1: Rearranges the equation to make $e^{\pm 4k}$ the subject. They need to multiply by the $3e^{4k}+7$ term, and collect terms in e^{4k} or e^{-4k} reaching $e^{\pm 4k}=C$ where C is a constant.	M1A1
		A1: Achieves intermediate answer of $e^{4k}=\dfrac{17500}{1500}=(\text{awrt } 11.7 \text{ or } 11.6)$ or $e^{-4k}=\dfrac{1500}{17500}=(\text{awrt } 0.857)$	
	$k=\dfrac{1}{4}\ln\left(\dfrac{35}{3}\right)$ **or** awrt 0.614	**d**M1: Proceeds from $e^{\pm 4k}=C,\ \ C>0$ by correctly taking ln's and then making k the subject of the formula. Award for e.g. $e^{4k}=C \Rightarrow 4k=\ln(C) \Rightarrow k=\dfrac{\ln(C)}{4}$	**d**M1A1
		A1: cao: Awrt 0.614 or the correct exact answer (or equivalent)	
			(5)

	Alternative correct work in (c):		
	$t=4,\ P=2500 \Rightarrow 2500=\dfrac{9000e^{4k}}{3e^{4k}+7}$	Correct equation with $t=4$ and $P=2500$	B1
	$7500e^{4k}+17500=9000e^{4k}$		
	$1500e^{4k}=17500$		
	$\ln 1500+\ln e^{4k}=\ln 17500$	M1: Takes ln's correctly	M1A1
		A1: Correct equation	
	$\ln e^{4k}=\ln 17500-\ln 1500$		
	$4k=\ln 17500-\ln 1500$		
	$k=\dfrac{\ln 17500-\ln 1500}{4}$	Makes k the subject	M1A1
	$k=\dfrac{1}{4}\ln\left(\dfrac{35}{3}\right)$ **or** awrt 0.614	cao: Awrt 0.614 or the correct exact answer (or equivalent)	

(d)	$\frac{dP}{dt} = \frac{(3e^{kt}+7)\times 9000ke^{kt} - 9000e^{kt}\times 3ke^{kt}}{(3e^{kt}+7)^2}\left(=\frac{63000ke^{kt}}{(3e^{kt}+7)^2}\right)$ Differentiates using the quotient rule to achieve $\frac{dP}{dt} = \frac{(3e^{kt}+7)\times Pe^{kt} - 9000e^{kt}\times Qe^{kt}}{(3e^{kt}+7)^2}$ **or** $\frac{dP}{dt} = 9000ke^{kt}\left(3e^{kt}+7\right)^{-1} - 9000e^{kt}\left(3e^{kt}+7\right)^{-2}\times 3ke^{kt}$ Differentiates using the product rule to achieve $\frac{dP}{dt} = Pe^{kt}\left(3e^{kt}+7\right)^{-1} - 9000e^{kt}\left(3e^{kt}+7\right)^{-2}\times Qe^{kt}$ **or** $\frac{dP}{dt} = 63000ke^{-kt}\left(3+7e^{-kt}\right)^{-2}$ Differentiates using the chain rule on $P = 9000\left(3+7e^{-kt}\right)^{-1}$ to achieve $\frac{dP}{dt} = \pm De^{-kt}\left(3+7e^{-kt}\right)^{-2}$ **Watch for** $e^{kt} \to kte^{kt}$ **which is M0**		M1
	Sub t =10 and $k = 0.614 \Rightarrow \frac{dP}{dt} = ...$	Substitutes t = 10 and their k to obtain a value for $\frac{dP}{dt}$. If the value for $\frac{dP}{dt}$ is incorrect then the **substitution** of t = 10 must be seen explicitly.	dM1 (A1 on Epen)
	$\frac{dP}{dt} = 9$	Awrt 9 (NB $\frac{dP}{dt} = 9.1694...$)	A1
			(3)
			(11 marks)

Qu	Scheme	Marks
10.(a)	When $t = 0$ $N = 15$	B1 **(1)**
(b)	Puts $t = 10$ so $N =$ 56.6 (accept 56 or 57)	M1A1 **(2)**
(c)	$82 = \dfrac{300}{3+17e^{-0.2t}} \Rightarrow e^{-0.2t} = \dfrac{54}{1394} = \text{awrt } 0.039$	M1 A1
	$-0.2t = \ln\left(\dfrac{54}{1394}\right) \Rightarrow t =$	dM1
	$t = \text{awrt } 16.3$	A1 **(4)**
(d)	$\dfrac{dN}{dt} = (-0.2)\times 300\times(-1)\times 17e^{-0.2t}(3+17e^{-0.2t})^{-2}$	M1 A1
	=4.38 so 4 insects per week	A1 cso **(3)**
		(10 marks)

(a)
B1: 15 cao
(b)
M1: Substitutes $t = 10$ into the correct formula. Sight of $N = \dfrac{300}{3+17e^{-0.2\times 10}}$ is fine
A1: Accept 56 or 57 or awrt 56.6. These values would imply the M.
(c)
M1: Substitutes 82 and proceeds to obtain $e^{\pm 0.2t} = C$ Condone slips on the power
A1: For $e^{-0.2t} = \dfrac{27}{697}$ oe $e^{0.2t} = \dfrac{697}{27}$ oe Accept decimals Eg $e^{-0.2t} = \text{awrt } 0.039$ or $e^{0.2t} = \text{awrt } 25.8$
dM1: Dependent upon previous M, scored for taking ln's (of a positive value) and proceeding to $t =$
A1: awrt 16.3 Accept 16 (weeks), 16.25 (weeks), 16 weeks 2 days or 17 weeks following correct log work and acceptable accuracy. Accept $t = 5\ln\left(\dfrac{1394}{54}\right)$ *oe* for this mark

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It is possible to answer this by taking ln's at the point $1394e^{-0.2t} = 54$
M1A1 $\ln(1394) - 0.2t = \ln(54)$ dM1 A1 As scheme

(d)
M1: Differentiates to give a form equivalent to $\dfrac{dN}{dt} = ke^{-0.2t}(3+17e^{-0.2t})^{-2}$ (may use quotient rule)
A1: Correct derivative which may be unsimplified $\dfrac{dN}{dt} = 1020e^{-0.2t}(3+17e^{-0.2t})^{-2}$
A1: Obtains awrt 4 **following a correct derivative**. This is cso

Question Number	Scheme	Marks
13.(a)	$t=0 \Rightarrow (P=)200-\frac{160}{15+1}=190 \Rightarrow 190\ 000$	M1A1
		(2)
(b)	$e^{kt} \rightarrow ae^{kt}$	M1
	$\frac{dP}{dt}=-\frac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}$	M1A1
		(3)
(c)	Sets $\pm\frac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}=0 \Rightarrow e^{0.8t}=45$	M1A1
	$\Rightarrow T=\frac{\ln 45}{0.8}=4.76$	M1A1
		(4)
		(9 marks)

(a)

M1: Sets $t = 0$ in the top and bottom of the fraction, giving $e^0=1$. Award if candidate attempts $200-\frac{160}{15+1}$ but not $\frac{200-160}{15+1}$ This can be awarded for a correct answer.

A1: Correct answer only. Accept 190 000 or (P =) 190 (ants).

The answer is an integer so do **not** allow awrt 190 or awrt 190 000 i.e. there should be no decimals.

(b)

M1: For showing that $e^{kt} \rightarrow ae^{kt}$ where a is a constant. This may be embedded within the product or quotient rule or their attempt to differentiate.

M1: For applying the quotient rule to obtain $\frac{dP}{dt}=\pm\frac{(15+e^{0.8t})\times pe^{0.6t}-qe^{0.8t}\times e^{0.6t}}{(15+e^{0.8t})^2}$ or applying the product rule to obtain $\frac{dP}{dt}=\pm\left[Ae^{0.6t}(15+e^{0.8t})^{-2}\times Be^{0.8t}+(15+e^{0.8t})^{-1}\times Ce^{0.6t}\right]$

Allow invisible brackets for this mark but not for the A mark below.

A1: A correct un-simplified or simplified $\frac{dP}{dt}$

Note $\frac{dP}{dt}=-\frac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}$ or $-160e^{0.6t}(15+e^{0.8t})^{-2}\times 0.8e^{0.8t}+(15+e^{0.8t})^{-1}\times 128e^{0.6t}$

(c) **Allow recovery here if the signs are reversed.**

M1: Sets their $\frac{dP}{dt}=0$ to obtain $pe^{0.8t}=q$ or $Ae^{0.6t}=Be^{1.4t}$ or equivalent.

A1: $e^{0.8t}=45$ or $1440e^{0.6t}=32e^{1.4t}$ or equivalent correct equation.

M1: Having set their $\frac{dP}{dt}=0$ and obtained either $Ae^{\pm kt}=B$ (k may be incorrect) or $Ce^{\pm\alpha t}=De^{\pm\beta t}$ where $k, \alpha, \beta \neq 0$ it is awarded for the correct order of operations, taking ln's leading to $t=..$
It cannot be awarded from impossible equations Eg $e^{0.8t}=-45$

A1: $T=\frac{\ln 45}{0.8}$ or equivalent or awrt $=4.76$

Question Number	Scheme	Marks
3(a)	3500	B1
		(1)
(b)	$3500(1.035)^t > 10000 \Rightarrow (1.035)^t > \frac{20}{7}$ (awrt 2.86)	M1A1
	$\Rightarrow t > \frac{\log\frac{20}{7}}{\log 1.035} = 30.516 =$ 30hrs 31 mins or 30 hrs 32 mins	M1A1
		(4)
(c)	$\frac{dN}{dt} = 3500(1.035)^t \ln 1.035 \Rightarrow \left.\frac{dN}{dt}\right\|_{t=8} = 3500(1.035)^8 \ln 1.035 = \text{awrt } 159$	B1M1A1
		(3)
		(8 marks)

(a)
B1: 3500
(b)
M1: For substituting $N = 10000$ and proceeding to $(1.035)^t ... A$ where ... is $>, \geqslant, =, <$ or $\leqslant$
A1: $(1.035)^t ... \frac{20}{7}$ where ... is $>, \geqslant, =, <$ or $\leqslant$ Accept awrt 2.86 for $\frac{20}{7}$ or equivalent e.g. $\frac{10000}{3500}, \frac{100}{35}$
M1: Proceeds correctly to find a value for t.
Accept expressions such as $t ... \frac{\log\frac{20}{7}}{\log 1.035}$, $t ... \frac{\ln\frac{20}{7}}{\ln 1.035}$ or $t ... \log_{1.035}\frac{20}{7}$ or awrt 30.5 as evidence
A1: 30hrs 31 mins or 30hrs 32 mins (**Not** 1831 minutes)
Attempts and Trial and Improvement should be sent to review.

(c)
B1: For $\frac{dN}{dt} = 3500(1.035)^t \ln 1.035$ or $\frac{dN}{dt} = 3500e^{t\ln 1.035}\ln 1.035$ (Allow $\frac{dN}{dt} = N\ln 1.035$)
M1: For substituting $t = 8$ into their $\frac{dN}{dt}$ which is a function of t but which is **not** the original function.
A1: awrt 159 (Award as soon as a correct answer is seen and isw)

Question Number	Scheme	Marks
8 (a)	$P_0 = \frac{100}{1+3} + 40 = 65$	B1
		(1)
	$\frac{d}{dt}e^{kt} = Ce^{kt}$	M1
(b)	$\frac{dP}{dt} = \frac{\left(1+3e^{-0.9t}\right)\times -10e^{-0.1t} - 100e^{-0.1t}\times -2.7e^{-0.9t}}{\left(1+3e^{-0.9t}\right)^2}$	M1 A1
		(3)
(c)(i)	At maximum $-10e^{-0.1t} - 30e^{-0.1t}\times e^{-0.9t} + 270e^{-0.1t}\times e^{-0.9t} = 0$	
	$e^{-0.1t}\left(-10+240e^{-0.9t}\right) = 0$	
	$e^{-0.9t} = \frac{10}{240}$ oe $e^{0.9t} = 24$	M1
	$-0.9t = \ln\left(\frac{1}{24}\right) \Rightarrow t = \frac{10}{9}\ln(24) = 3.53$	M1, A1
(c) (ii)	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

(a)

B1 $\left(P_0 =\right) 65$

(b)

M1 For sight of $\frac{d}{dt}e^{kt} = Ce^{kt}$ (Allow C =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\frac{(1+3e^{-0.9t})\times pe^{-0.1t} - qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$

$$\frac{(1+3e^{-0.9t})\times pe^{-0.1t} + qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$$

Question	Scheme	Marks
9(a)	Subs $D=15$ and $t=4$ $\quad x=15\mathrm{e}^{-0.2\times 4}=6.740\ (mg)$	M1A1
		(2)
(b)	$15\mathrm{e}^{-0.2\times 7}+15\mathrm{e}^{-0.2\times 2}=13.754(mg)$	M1A1*
		(2)
(c)	$15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T+5)}=7.5$ $15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times T}\mathrm{e}^{-1}=7.5$	M1
	$15\mathrm{e}^{-0.2\times T}(1+\mathrm{e}^{-1})=7.5\Rightarrow \mathrm{e}^{-0.2\times T}=\dfrac{7.5}{15(1+\mathrm{e}^{-1})}$	dM1
	$\mathrm{T}=-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$	A1, A1
		(4) **(8 marks)**

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=D\mathrm{e}^{-0.2t}$
It can be implied by sight of $15\mathrm{e}^{-0.8}$, $15\mathrm{e}^{-0.2\times 4}$ or awrt 6.7
Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7
Evidence would be $15\mathrm{e}^{-0.2\times 7}+15\mathrm{e}^{-0.2\times 2}$ or similar expressions such as $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times 2}$
Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**
Alternatively finds the amount after 5 hours, $15\mathrm{e}^{-1}=$ awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $\mathrm{e}^{-0.4}$ to get awrt **13.75**.
Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.

A1* cso so both the expression $15\mathrm{e}^{-0.2\times 7}+15\mathrm{e}^{-0.2\times 2}$ and $13.754(mg)$ are required
Alternatively both the expression $\left(15\mathrm{e}^{-0.2\times 5}+15\right)\times\mathrm{e}^{-0.2\times 2}$ and $13.754(mg)$ are required.
Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing
Eg. accept $15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T\pm 5)}=7.5$ or similar equations $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $\mathrm{e}^{-0.2\times T}=\ldots$
An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $\mathrm{e}^{-0.2\times T}$ Also score for candidates who make $\mathrm{e}^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)$

A1 CSO $\mathrm{T}=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$ Condone t appearing for T throughout this question.

Question Number	Scheme	Marks
4(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \dfrac{\ln 0.2}{-\text{their}'\lambda'}$	M1
	$T = \text{awrt } 93$	A1
		(2)
		(7 marks)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \dfrac{\ln 100 - \ln 50}{40} = \dfrac{\ln 2}{40}$	M1A1
		(4)

Question Number	Scheme	Marks
8.(a)	$P=\frac{800e^0}{1+3e^0}, =\frac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250, \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\frac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\frac{dP}{dt}=\frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\frac{dP}{dt}=\frac{(1+3e)\times 80e-240e^2}{(1+3e)^2}=\frac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub $t = 0$ into P **and** use $e^0 = 1$ in at least one of the two cases. Accept $P=\frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

Question Number	Scheme	Marks
8.(a)	$t=0 \Rightarrow P=\dfrac{8000}{1+7}=1000$ cao	M1A1
		(2)
(b)	$t \to \infty \quad P \to \dfrac{8000}{1}=8000$	B1
		(1)
(c)	$t=3,\ P=2500 \Rightarrow 2500=\dfrac{8000}{1+7e^{-3k}}$	B1
	$e^{-3k}=\dfrac{2.2}{7}=(0.31..)$ oe	M1,A1
	$k=-\dfrac{1}{3}\ln\left(\dfrac{2.2}{7}\right)=$ awrt 0.386	M1A1
		(5)
(d)	Sub t=10 into $P=\dfrac{8000}{1+7e^{-0.386t}} \Rightarrow P=6970$ cao	M1A1
		(2)
(e)	$\dfrac{dP}{dt}=-\dfrac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$	M1,A1
	Sub t=10 $\left.\dfrac{dP}{dt}\right\|_{t=10}=346$	A1
		(3)
		(13 marks)