Paper 2: Core Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$\alpha + \beta + \gamma = 8, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 28, \quad \alpha\beta\gamma = 32$	B1	3.1a
	$\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} = \dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$= \dfrac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha+2)(\beta+2)(\gamma+2) = (\alpha\beta + 2\alpha + 2\beta + 4)(\gamma + 2)$	M1	1.1b
	$= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) + 8$	A1	1.1b
	$= 32 + 2(28) + 4(8) + 8 = 128$	A1	1.1b
		(3)	
	Alternative:		
	$(x-2)^3 - 8(x-2)^2 + 28(x-2) - 32 = 0$	M1	1.1b
	$= \ldots - 8 + \ldots - 32 + \ldots - 56 - 32 = -128$	A1	1.1b
	$\therefore (\alpha+2)(\beta+2)(\gamma+2) = 128$	A1	1.1b
		(3)	
(iii)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$	M1	3.1a
	$= 8^2 - 2(28) = 8$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(i)

B1: Identifies the correct values for all 3 expressions (can score anywhere)

M1: Uses a correct identity

A1ft: Correct value (follow through their 8, 28 and 32)

(ii)

M1: Attempts to expand

A1: Correct expansion

A1: Correct value

Alternative:

M1: Substitutes $x - 2$ for x in the given cubic

A1: Calculates the correct constant term

A1: Changes sign and so obtains the correct value

(iii)

M1: Establishes the correct identity

A1ft: Correct value (follow through their 8, 28 and 32)

Question	Scheme	Marks	AOs
2(a)	$\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ 12 \end{pmatrix} = 18 - 8 + 24$	M1	3.1a
	$d = \dfrac{18 - 8 + 24 - 5}{\sqrt{3^2 + 4^2 + 2^2}}$	M1	1.1b
	$= \sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \ldots$ and $\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \ldots$	M1	2.1
	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = 0$ and $\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = 0$ $\therefore \ -\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = -3 + 12 + 2$	M1	1.1b
	$\sqrt{(-1)^2 + (-3)^2 + 1^2}\sqrt{(3)^2 + (-4)^2 + 2^2}\cos\theta = 11$ $\Rightarrow \cos\theta = \dfrac{11}{\sqrt{(-1)^2 + (-3)^2 + 1^2}\sqrt{(3)^2 + (-4)^2 + 2^2}}$	M1	2.1
	So angle between planes $\theta = 52°$ *	A1*	2.4
		(3)	
		(8 marks)	

Notes:

(a)

M1: Realises the need to and so attempts the scalar product between the normal and the position vector

M1: Correct method for the perpendicular distance

A1: Correct distance

(b)

M1: Recognises the need to calculate the scalar product between the given vector and both direction vectors

A1: Obtains zero both times and makes a conclusion

(c)

M1: Calculates the scalar product between the two normal vectors

M1: Applies the scalar product formula with their 11 to find a value for $\cos\theta$

A1*: Identifies the correct angle by linking the angle between the normal and the angle between the planes

Question	Scheme	Marks	AOs
3(i)(a)	$\lvert \mathbf{M} \rvert = 2(1+2) - a(-1-1) + 4(2-1) = 0 \Rightarrow a = \ldots$	M1	2.3
	The matrix $\mathbf{M}$ has an inverse when $a \neq -5$	A1	1.1b
		(2)	
(b)	Minors : $\begin{pmatrix} 3 & -2 & 1 \\ -a-8 & 2 & a+4 \\ 4-a & -6 & -2-a \end{pmatrix}$ or Cofactors : $\begin{pmatrix} 3 & 2 & 1 \\ a+8 & 2 & -a-4 \\ 4-a & 6 & -2-a \end{pmatrix}$	B1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{\lvert \mathbf{M} \rvert} \operatorname{adj}(\mathbf{M})$	M1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{2a+10} \begin{pmatrix} 3 & a+8 & 4-a \\ 2 & 2 & 6 \\ 1 & -a-4 & -2-a \end{pmatrix}$ — 2 correct rows or columns. Follow through their det$\mathbf{M}$	A1ft	1.1b
	All correct. Follow through their det$\mathbf{M}$	A1ft	1.1b
		(4)	
(ii)	When $n = 1$, lhs $= \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$, rhs $= \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ So the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	M1	2.4
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 3 \times 3^k & 0 \\ 3 \times 3(3^k-1)+6 & 1 \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
		(12 marks)	

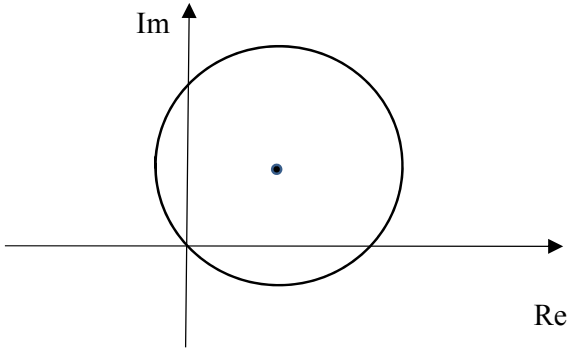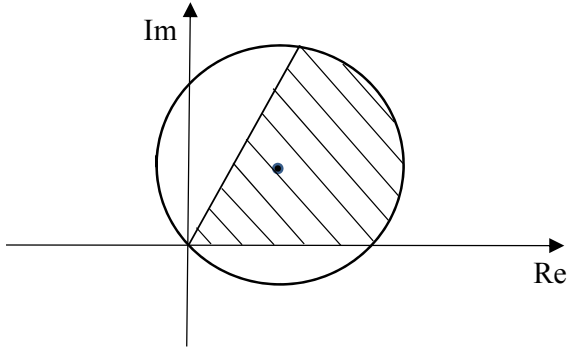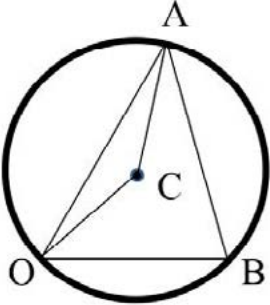| **Question 3 notes:**	
(i)(a)	
M1:	Attempts determinant, equates to zero and attempts to solve for a in order to establish the restriction for a
A1:	Provides the correct condition for a if **M** has an inverse
(i)(b)	
B1:	A correct matrix of minors or cofactors
M1:	For a complete method for the inverse
A1ft:	Two correct rows following through their determinant
A1ft:	Fully correct inverse following through their determinant
(ii)	
B1:	Shows the statement is true for $n = 1$
M1:	Assumes the statement is true for $n = k$
M1:	Attempts to multiply the correct matrices
A1:	Correct matrix in terms of k
A1:	Correct matrix in terms of $k + 1$
A1:	Correct complete conclusion

Question	Scheme	Marks	AOs
4(a)	$z^n + z^{-n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$	M1	2.1
	$= 2\cos n\theta\,*$	A1*	1.1b
		(2)	
(b)	$\left(z + z^{-1}\right)^4 = 16\cos^4\theta$	B1	2.1
	$\left(z + z^{-1}\right)^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	M1	2.1
	$\quad = z^4 + z^{-4} + 4\left(z^2 + z^{-2}\right) + 6$	A1	1.1b
	$\quad = 2\cos 4\theta + 4\left(2\cos 2\theta\right) + 6$	M1	2.1
	$\cos^4\theta = \dfrac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)\,*$	A1*	1.1b
		(5)	
		(7 marks)	

Notes:

(a)
M1: Identifies the correct form for z^n and z^{-n} and adds to progress to the printed answer
A1*: Achieves printed answer with no errors

(b)
B1: Begins the argument by using the correct index with the result from part (a)

M1: Realises the need to find the expansion of $\left(z + z^{-1}\right)^4$

A1: Terms correctly combined
M1: Links the expansion with the result in part (a)
A1*: Achieves printed answer with no errors

Question	Scheme	Marks	AOs
5(a)	$\dfrac{dy}{dx} = \sin x \cosh x + \cos x \sinh x$	M1	1.1a
	$\dfrac{d^2 y}{dx^2} = \cos x \cosh x + \sin x \sinh x + \cos x \cosh x - \sin x \sinh x$ $\left(= 2 \cos x \cosh x\right)$	M1	1.1b
	$\dfrac{d^3 y}{dx^3} = 2 \cos x \sinh x - 2 \sin x \cosh x$	M1	1.1b
	$\dfrac{d^4 y}{dx^4} = -4 \sinh x \sin x = -4y \,*$	A1*	2.1
		(4)	
(b)	$\left(\dfrac{d^2 y}{dx^2}\right)_0 = 2, \;\; \left(\dfrac{d^6 y}{dx^6}\right)_0 = -8, \;\; \left(\dfrac{d^{10} y}{dx^{10}}\right)_0 = 32$	B1	3.1a
	Uses $y = y_0 + x y_0' + \dfrac{x^2}{2!} y_0'' + \dfrac{x^3}{3!} y_0''' + \ldots$ with their values	M1	1.1b
	$= \dfrac{x^2}{2!}(2) + \dfrac{x^6}{6!}(-8) + \dfrac{x^{10}}{10!}(32)$	A1	1.1b
	$= x^2 - \dfrac{x^6}{90} + \dfrac{x^{10}}{113400}$	A1	1.1b
		(4)	
(c)	$2(-4)^{n-1} \dfrac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
		(10 marks)	

Notes:

(a)
M1: Realises the need to use the product rule and attempts first derivative
M1: Realises the need to use a second application of the product rule and attempts the second derivative
M1: Correct method for the third derivative
A1*: Obtains the correct 4th derivative and links this back to y

(b)
B1: Makes the connection with part **(a)** to establish the general pattern of derivatives and finds the correct non-zero values
M1: Correct attempt at Maclaurin series with their values
A1: Correct expression un-simplified
A1: Correct expression and simplified

(c)
M1: Generalising, dealing with signs, powers and factorials
A1: Correct expression

Question	Scheme	Marks	AOs
6(a)(i)		M1	1.1b
		A1	1.1b
(a)(ii)	$\|z-4-3i\|=5 \Rightarrow \|x+iy-4-3i\|=5 \Rightarrow (x-4)^2+(y-3)^2=...$	M1	2.1
	$(x-4)^2+(y-3)^2=25$ or any correct form	A1	1.1b
	$(r\cos\theta-4)^2+(r\sin\theta-3)^2=25$ $$\Rightarrow r^2\cos^2\theta-8r\cos\theta+16+r^2\sin^2\theta-6r\sin\theta+9=25$$ $$\Rightarrow r^2-8r\cos\theta-6r\sin\theta=0$$	M1	2.1
	$\therefore r=8\cos\theta+6\sin\theta\,*$	A1*	2.2a
		(6)	
(b)(i)		B1	1.1b
		B1ft	1.1b
(b)(ii)	$A=\dfrac{1}{2}\int r^2\,d\theta=\dfrac{1}{2}\int (8\cos\theta+6\sin\theta)^2\,d\theta$ $$=\dfrac{1}{2}\int (64\cos^2\theta+96\sin\theta\cos\theta+36\sin^2\theta)\,d\theta$$	M1	3.1a
	$=\dfrac{1}{2}\int (32(\cos 2\theta+1)+96\sin\theta\cos\theta+18(1-\cos 2\theta))\,d\theta$	M1	1.1b
	$=\dfrac{1}{2}\int (14\cos 2\theta+50+48\sin 2\theta)\,d\theta$	A1	1.1b
	$=\dfrac{1}{2}\left[7\sin 2\theta+50\theta-24\cos 2\theta\right]_0^{\frac{\pi}{3}}=\dfrac{1}{2}\left\{\left(\dfrac{7\sqrt{3}}{2}+\dfrac{50\pi}{3}+12\right)-(-24)\right\}$	M1	2.1
	$=\dfrac{7\sqrt{3}}{4}+\dfrac{25\pi}{3}+18$	A1	1.1b
		(7)	

Question	Scheme	Marks	AOs
	(b)(ii) Alternative: Candidates may take a geometric approach e.g. by finding sector + 2 triangles		
	Angle $ACB = \left(\dfrac{2\pi}{3}\right)$ so area sector $ACB = \dfrac{1}{2}(5)^2 \dfrac{2\pi}{3}$ Area of triangle $OCB = \dfrac{1}{2} \times 8 \times 3$	M1	3.1a
	Sector area ACB + triangle area $OCB = \dfrac{25\pi}{3} + 12$	A1	1.1b
	Area of triangle OAC: Angle $ACO = 2\pi - \dfrac{2\pi}{3} - \cos^{-1}\left(\dfrac{5^2 + 5^2 - 8^2}{2 \times 5 \times 5}\right)$ so area $OAC = \dfrac{1}{2}(5)^2 \sin\left(\dfrac{4\pi}{3} - \cos^{-1}\left(\dfrac{-7}{25}\right)\right)$	M1	1.1b
	$= \dfrac{25}{2}\left(\sin\dfrac{4\pi}{3}\cos\left(\cos^{-1}\left(\dfrac{-7}{25}\right)\right) - \cos\dfrac{4\pi}{3}\sin\left(\cos^{-1}\left(\dfrac{-7}{25}\right)\right)\right)$ $= \dfrac{25}{2}\left(\left(\dfrac{7\sqrt{3}}{50}\right) + \dfrac{1}{2}\sqrt{1 - \left(\dfrac{7}{25}\right)^2}\right) = \dfrac{7\sqrt{3}}{4} + 6$ Total area $= \dfrac{25\pi}{3} + \dfrac{1}{2} \times 8 \times 3 + 6 + \dfrac{7\sqrt{3}}{4}$	M1	2.1
	$= \dfrac{7\sqrt{3}}{4} + \dfrac{25\pi}{3} + 18$	A1	1.1b
		(13 marks)	

Question 6 notes:	
(a)(i)	
M1:	Draws a circle which passes through the origin
A1:	Fully correct diagram
(a)(ii)	
M1:	Uses $z = x + \mathrm{i}y$ in the given equation and uses modulus to find equation in x and y only
A1:	Correct equation in terms of x and y in any form – may be in terms of r and θ
M1:	Introduces polar form, expands and uses $\cos^2\theta + \sin^2\theta = 1$ leading to a polar equation
A1*:	Deduces the given equation (ignore any reference to $r = 0$ which gives a point on the curve)
(b)(i)	
B1:	Correct pair of rays added to their diagram
B1ft:	Area between their pair of rays and inside their circle from (a) shaded, as long as there is an intersection
(b)(ii)	
M1:	Selects an appropriate method by linking the diagram to the polar curve in (a), evidenced by use of the polar area formula
M1:	Uses double angle identities
A1:	Correct integral
M1:	Integrates and applies limits
A1:	Correct area
(b)(ii) Alternative:	
M1:	Selects an appropriate method by finding angle ACB and area of sector ACB and finds area of triangle OCB to make progress towards finding the required area
A1:	Correct combined area of sector ACB + triangle OCB
M1:	Starts the process of finding the area of triangle OAC by calculating angle ACO and attempts area of triangle OAC
M1:	Uses the addition formula to find the exact area of triangle OAC and employs a full correct method to find the area of the shaded region
A1:	Correct area

Question	Scheme	Marks	AOs
7(a)	$r = 10\dfrac{df}{dt} - 2f \Rightarrow \dfrac{dr}{dt} = 10\dfrac{d^2 f}{dt^2} - 2\dfrac{df}{dt}$	M1	2.1
	$10\dfrac{d^2 f}{dt^2} - 2\dfrac{df}{dt} = -0.2f + 0.4\left(10\dfrac{df}{dt} - 2f\right)$	M1	2.1
	$\dfrac{d^2 f}{dt^2} - 0.6\dfrac{df}{dt} + 0.1f = 0*$	A1*	1.1b
		(3)	
(b)	$m^2 - 0.6m + 0.1 = 0 \Rightarrow m = \dfrac{0.6 \pm \sqrt{0.6^2 - 4 \times 0.1}}{2}$	M1	3.4
	$m = 0.3 \pm 0.1\mathrm{i}$	A1	1.1b
	$f = \mathrm{e}^{\alpha t}\left(A\cos \beta t + B\sin \beta t\right)$	M1	3.4
	$f = \mathrm{e}^{0.3t}\left(A\cos 0.1t + B\sin 0.1t\right)$	A1	1.1b
		(4)	
(c)	$\dfrac{df}{dt} = 0.3\mathrm{e}^{0.3t}\left(A\cos 0.1t + B\sin 0.1t\right) + 0.1\mathrm{e}^{0.3t}\left(B\cos 0.1t - A\sin 0.1t\right)$	M1	3.4
	$r = 10\dfrac{df}{dt} - 2f$ $= \mathrm{e}^{0.3t}\left((3A+B)\cos 0.1t + (3B-A)\sin 0.1t\right) - 2\mathrm{e}^{0.3t}\left(A\cos 0.1t + B\sin 0.1t\right)$	M1	3.4
	$r = \mathrm{e}^{0.3t}\left((A+B)\cos 0.1t + (B-A)\sin 0.1t\right)$	A1	1.1b
		(3)	
(d)(i)	$t = 0,\ f = 6 \Rightarrow A = 6$	M1	3.1b
	$t = 0,\ r = 20 \Rightarrow B = 14$	M1	3.3
	$r = \mathrm{e}^{0.3t}\left(20\cos 0.1t + 8\sin 0.1t\right) = 0$	M1	3.1b
	$\tan 0.1t = -2.5$	A1	1.1b
	2019	A1	3.2a
(d)(ii)	3750 foxes	B1	3.4
(d)(iii)	e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible	B1	3.5a
		(7)	
		(17 marks)	

Question 7 notes:	
(a)	
M1:	Attempts to differentiate the first equation with respect to t
M1:	Proceeds to the printed answer by substituting into the second equation
A1*:	Achieves the printed answer with no errors
(b)	
M1:	Uses the model to form and solve the auxiliary equation
A1:	Correct values for m
M1:	Uses the model to form the CF
A1:	Correct CF
(c)	
M1:	Differentiates the expression for the number of foxes
M1:	Uses this result to find an expression for the number of rabbits
A1:	Correct equation
(d)(i)	
M1:	Realises the need to use the initial conditions in the model for the number of foxes
M1:	Realises the need to use the initial conditions in the model for the number of rabbits to find both unknown constants
M1:	Obtains an expression for r in terms of t and sets $= 0$
A1:	Rearranges and obtains a correct value for tan
A1:	Identifies the correct year
(d)(ii)	
B1:	Correct number of foxes
(d)(iii)	
B1:	Makes a suitable comment on the outcome of the model