

10. A student is investigating the following statement about natural numbers.

“ $n^3 - n$ is a multiple of 4”

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

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2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

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15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

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16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

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