Question	Scheme	Marks	AOs
13(a)	$y=\text{cosec}^3\theta \Rightarrow \frac{dy}{d\theta}=-3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$	M1	1.1b
	$\frac{dy}{dx}=\frac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y=8\Rightarrow \text{cosec}^3\theta=8\Rightarrow \sin^3\theta=\frac{1}{8}\Rightarrow\sin\theta=\frac{1}{2}$	M1	3.1a
	$\theta=\frac{\pi}{6}\Rightarrow\frac{dy}{dx}=\frac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)}=\ldots$ **or** $\sin\theta=\frac{1}{2}\Rightarrow\frac{dy}{dx}=\frac{\frac{-3}{\sin^3\theta}\times\frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)}=\frac{-3\times8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$=-24\sqrt{3}$	A1	2.2a
		(3)	
		(6 marks)	

Notes

(a)

B1: Correct expression for $\frac{dy}{d\theta}$ seen or implied in any form e.g. $\frac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\frac{dx}{d\theta}=k\cos 2\theta$ or $\alpha\cos^2\theta+\beta\sin^2\theta$ (from product rule on sinθcosθ)

and attempts $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$

A1: Correct expression in any form.

May see e.g. $\frac{-3\cos\theta}{2\sin^4\theta\cos 2\theta}$, $-\frac{3}{4\sin^4\theta\cos\theta-2\sin^3\theta\tan\theta}$

(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when $y = 8$ and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta=\frac{1}{2}$ or e.g. $\theta=\frac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\frac{dy}{dx}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\frac{dy}{dx}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\frac{dy}{dx}$.

A1: Deduces the correct gradient

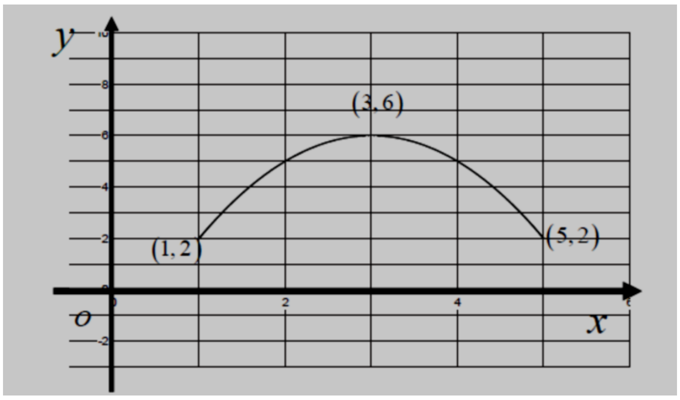
Question	Scheme	Marks	AOs
13	$(x-3)^2+y^2=\left(\frac{t^2+5}{t^2+1}-3\right)^2+\left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\frac{(2-2t^2)^2+16t^2}{(t^2+1)^2}=\frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2}=\frac{4(t^2+1)^2}{(t^2+1)^2}=4$*	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.
dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.
A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x=\frac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\frac{5-x}{x-1}$ $y=\frac{4t}{t^2+1}\Rightarrow y^2=\frac{16t^2}{(t^2+1)^2}=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}=16\left(\frac{5-x}{x-1}\right)\times\left(\frac{(x-1)}{5-x+x-1}\right)^2\Rightarrow y^2=(5-x)(x-1)$	dM1	1.1b
	$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step $\Rightarrow(x-3)^2+y^2=4$*	A1*	2.1
		(3)	
		(3 marks)	

Notes

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.
Other methods exist which also lead to an appropriate equation. E.g using $t=\frac{y}{x-1}$
dM1: Uses correct processing to eliminate the fractions and start to simplify
A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \frac{y-4}{2} = 1-2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y-4 = 2-4\times\frac{(x-3)^2}{4} \Rightarrow y = 6-(x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3+2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ $\Rightarrow k-x = 6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b
	Uses $b^2-4ac=0 \Rightarrow \quad 49-4\times1\times(k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

Question	Scheme		Marks	AOs
4	$C_1:\ x=10\cos t\ ,\ y=4\sqrt{2}\sin t,\ \ 0\le t<2\pi;\ \ C_2: x^2+y^2=66$			
Way 1	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100(1-\sin^2 t)+32\sin^2 t=66$	$100\cos^2 t+32(1-\cos^2 t)=66$	M1 A1	2.1 1.1b
	$100-68\sin^2 t=66\ \Rightarrow \sin^2 t=\frac{1}{2}$ $\Rightarrow\ \sin t=...$	$68\cos^2 t+32=66\ \Rightarrow \cos^2 t=\frac{1}{2}$ $\Rightarrow\ \cos t=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, -4)		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t+\sin^2 t=1\Rightarrow\}\ \left(\frac{x}{10}\right)^2+\left(\frac{y}{4\sqrt{2}}\right)^2=1\ \{\Rightarrow 32x^2+100y^2=3200\}$		M1	3.1a
	$\frac{x^2}{100}+\frac{66-x^2}{32}=1$	$\frac{66-y^2}{100}+\frac{y^2}{32}=1$	M1 A1	2.1 1.1b
	$32x^2+6600-100x^2=3200$ $x^2=50\ \Rightarrow\ x=...$	$2112-32y^2+100y^2=3200$ $y^2=16\ \Rightarrow\ y=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, -4)		A1	3.2a
			(6)	
Way 3	$\{C_2: x^2+y^2=66\Rightarrow\}\ \ x=\sqrt{66}\cos\alpha,\ y=\sqrt{66}\sin\alpha$ $\{C_1=C_2\ \Rightarrow\}\ \ 10\cos t=\sqrt{66}\cos\alpha,\ \ 4\sqrt{2}\sin t=\sqrt{66}\sin\alpha$ $\{\cos^2\alpha+\sin^2\alpha=1\Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2+\left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2=1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right)+32\left(\frac{1-\cos 2t}{2}\right)=66$		M1 A1	2.1 1.1b
	$50+50\cos 2t+16-16\cos 2t=66\ \Rightarrow\ 34\cos 2t+66=66$ $\Rightarrow\ \cos 2t=...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=$(awrt 7.07, -4)		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x=5\sqrt{2},\ y=-4$ followed by $S=(-4,5\sqrt{2})$			
				(6 marks)

Notes for Question 4

Question Number	Scheme	Marks
11(a)	$6\sin t = 3 \Rightarrow \sin t = 0.5 \Rightarrow t = \dfrac{\pi}{6}$	B1
		(1)
(b)	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{6\cos t}{-20\sin 2t}$	M1A1
	Sub $t = \dfrac{\pi}{6}$ into $\dfrac{dy}{dx} = \dfrac{6\cos t}{-20\sin 2t} = \dfrac{6\cos\left(\frac{\pi}{6}\right)}{-20\sin\left(\frac{\pi}{3}\right)} = -\dfrac{3}{10}$	M1A1
	Uses normal gradient with (5, 3) $\Rightarrow \dfrac{y-3}{x-5} = \dfrac{10}{3}$	M1
	$\Rightarrow 3y = 10x - 41$	A1*
		(6)
©	Sub $x = 10\cos 2t, \quad y = 6\sin t,$ into $3y = 10x - 41$	
	$\Rightarrow 18\sin t = 100\cos 2t - 41$	M1
	$\Rightarrow 18\sin t = 100(1 - 2\sin^2 t) - 41$	
	$\Rightarrow 200\sin^2 t + 18\sin t - 59 = 0$	M1, A1
	$\Rightarrow (2\sin t - 1)(100\sin t + 59) = 0$	
	$\Rightarrow \sin t = -\dfrac{59}{100}\ (\Rightarrow t = -0.63106...)$	M1A1
	Using either their t or $\sin t$ to find either coord of B	M1
	Hence B has co-ordinates (3.038, - 3.54). These are exact values	A1,A1
	The equivalent fractional answers are $\left(\dfrac{1519}{500}, -\dfrac{177}{50}\right)$	**(8)**
		(15 marks)

Question Number	Scheme	Marks
9(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$	M1
	When $t = 3, \quad x = 15$	A1
	$A = (3, 0)$	B1
		(3)
Or	**Special case** - uses answer - $t^2 + 2t = 15 \Rightarrow t = 3 \quad$ (-5)	M1
	When $t = 3, \quad y = 0$	A1
	$A = (3, 0)$	B1
		(3)
(b)	$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{3t^2 - 9}{2t + 2}$	M1A1
	Substitutes $t = 3$ into $\dfrac{dy}{dx} = \dfrac{3t^2 - 9}{2t + 2} \Rightarrow$ gradient $= \left(\dfrac{9}{4}\right)$	M1
	Uses their $\left(\dfrac{9}{4}\right)$ and $(15, 0)$ to produce tangent equation $\quad 9x - 4y - 135 = 0$ *	M1 A1*
		(5)
(c)	Substitutes $x = t^2 + 2t,\ y = t^3 - 9t$, into $9x - 4y - 135 = 0$	
	$\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$	M1
	$\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$	
	$\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0 \quad$ or $\Rightarrow (t - 3)(t - 3)(4t + 15) = 0$	dM1
	$t = -\dfrac{15}{4}$	A1
	Coordinates of X are $\left(\dfrac{105}{16}, -\dfrac{1215}{64}\right)$ or $\left(6\dfrac{9}{16}, -18\dfrac{63}{64}\right)$	ddM1A1cso
	Accept awrt $(6.56, -18.98)$	
		(5)
		(13 marks)

Question Number	Scheme	Marks
13 (a)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-12\sin 2t}$	M1
	$= \frac{2\cos t}{-24\sin t \cos t}$	dM1
	$= \frac{2\cancel{\cos t}}{-24\sin t\cancel{\cos t}} = -\frac{1}{12}\ \text{cosec}\ t$	M1 A1 [4]
(b)	When $t = \frac{\pi}{3}$, $\frac{dy}{dx} = -\frac{1}{12\times\sqrt{3}/2} = \left(-\frac{\sqrt{3}}{18}\right)$	M1 A1
	So Normal has gradient $-\frac{1}{m} = 6\sqrt{3}$	M1
	When $t = \frac{\pi}{3}$, $x = -3$ and $y = \sqrt{3}$	B1
	Equation of normal is $y - \sqrt{3} = 6\sqrt{3}(x+3)$ so $y = 6\sqrt{3}x + 19\sqrt{3}$	M1 A1 [6]
(c)	$x = 6(1-2\sin^2 t) \Rightarrow x = f(y)$	M1
	So $x = 6 - 3y^2$ or $f(y) = 6 - 3y^2$	dM1 A1 [3]
(d)	$-2 < y < 2$ or $k = 2$	B1 [1] **(14 marks)**
Alt (a)	Via cartesian must start with $x = A \pm By^2$ or $y = \sqrt{C \pm Dx}$	
	$\frac{dx}{dy} = ky$ or $\frac{dy}{dx} = b\left(2 - \frac{x}{3}\right)^{-\frac{1}{2}}$ then as before	M1
	followed by correct (double angle) substitution	dM1
Alt (b)	Must start with $x = A \pm By^2$ or $y = \sqrt{C \pm Dx}$	
	$\frac{dx}{dy} = -6y$ or $\frac{dy}{dx} = -\frac{1}{6}\left(2 - \frac{x}{3}\right)^{-\frac{1}{2}}$	
	For substituting their $y = \sqrt{3}$ into a $\frac{dx}{dy}$ of the form Py	
	Or alternatively substituting their $x = -3$ into a $\frac{dy}{dx}$ of the form $P(Q \pm Rx)^{-\frac{1}{2}}$	1st M1
	For using the 'correct numerical' grad of the normal either $-\frac{dx}{dy}$ or $-\frac{1}{dy/dx}$	2nd M1

Qu	Scheme	Marks
13 (a)	Puts $x = 0$ and obtains $\theta = -\frac{\pi}{6}$	B1
	Substitutes their θ to obtain $y = \frac{10\sqrt{3}}{3}$ or $\left(0, \frac{10\sqrt{3}}{3}\right)$	M1 A1 **(3)**
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{5\sec\theta\tan\theta}{\sqrt{3}\sec^2\theta}$	M1 A1
	$= \frac{5\times \sin\theta/\cos\theta}{\sqrt{3}\times 1/\cos\theta}$	B1
	$= \frac{5}{\sqrt{3}}\sin\theta$ or $\lambda = \frac{5}{\sqrt{3}}$ oe	A1 **(4)**
(c)	Puts $\frac{dy}{dx} = 0$ and obtains θ and calculates x and y **or** deduces correct answer	M1
	Obtains (1, 5)	A1 **(2)**
(d)	$\tan\theta = \frac{x-1}{\sqrt{3}}$ and $\sec\theta = \frac{y}{5}$	M1
	Uses $1+\tan^2\theta = \sec^2\theta$ to give $1 + \text{"}\left(\frac{x-1}{\sqrt{3}}\right)^2\text{"} = \text{"}\left(\frac{y}{5}\right)^2\text{"}$	M1
	$\frac{3+x^2-2x+1}{3} = \left(\frac{y}{5}\right)^2$ **so** $y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	A1* **(3)** **(12 marks)**
Alt 1(d)	$y = 5\sqrt{1+\tan^2\theta}, = 5\sqrt{1+\left(\frac{x-1}{\sqrt{3}}\right)^2}$	M1, M1
	$y = \frac{5}{3}\sqrt{3}\sqrt{(x^2-2x+4)}$ *	A1* (3)
Alt 2 (d)	Assume $y = k\sqrt{(x^2-2x+4)}$ and sub both $x = 1+\sqrt{3}\tan\theta$ and $y = 5\sec\theta$	
	$5\sec\theta = k\times\sqrt{3+3\tan^2\theta}$	M1
	$5\sec\theta = k\times\sec\theta\sqrt{3}$	M1
	$k = \frac{5}{3}\sqrt{3}$ AND conclusion "hence true"	A1* (3)

(a)

B1: For $\theta = -\frac{\pi}{6}$ or $-30°$ or awrt -0.52 but may be awarded for $\cos\theta = \frac{\sqrt{3}}{2}$ or $\sec\theta = \frac{2}{\sqrt{3}}$ if θ is not explicitly found

M1: Substitutes their θ (or their $\cos\theta$ or $\sec\theta$) found from an attempt at $x = 0$ to give y

A1: cao. Accept $y = \frac{10}{\sqrt{3}}$ Correct answer with no incorrect working scores all 3 marks.

Note that $\theta = \frac{\pi}{6}$ also gives $y = \frac{10\sqrt{3}}{3}$ but scores B0 M1 A0

Question Number	Scheme	Marks
10 (a)	$t(t-4)=0 \Rightarrow t=4$ Hence $x=\dfrac{20\times 4}{2\times 4+1}=\dfrac{80}{9}$	M1A1
		(2)
(b)	$x=\dfrac{20t}{2t+1} \Rightarrow \dfrac{dx}{dt}=\dfrac{20(2t+1)-20t\times 2}{(2t+1)^2}=\left(\dfrac{20}{(2t+1)^2}\right)$	M1A1
	$\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{(2t-4)}{20/(2t+1)^2}, =\dfrac{(t-2)(2t+1)^2}{10}$	M1A1,A1
		(5)
	Mark c(i) and (ii) together:	
(c)(i)	$x=\dfrac{20t}{2t+1} \Rightarrow 2tx+x=20t \Rightarrow t(20-2x)=x \Rightarrow t=\dfrac{x}{20-2x}$ or $\dfrac{-x}{2x-20}$	M1A1
		(2)
(ii)	Sub $t=\dfrac{x}{20-2x}$ into $y=t(t-4) \Rightarrow y=\dfrac{x}{20-2x}\left(\dfrac{x}{20-2x}-4\right)$	M1
	$\Rightarrow y=\dfrac{x}{20-2x}\times\left(\dfrac{x}{20-2x}-\dfrac{4(20-2x)}{20-2x}\right)$	**d**M1
	$\Rightarrow y=\dfrac{x}{20-2x}\times\left(\dfrac{9x-80}{20-2x}\right)$	
	$\Rightarrow y=\dfrac{x(9x-80)}{(20-2x)^2}$, oe $\quad 0<x<10$ or $k=10$	A1, B1
		(4)
		(13 marks)

(a)
M1: Attempts to find x when $t=4$
A1: $\frac{80}{9}$ (Not 8.88… but isw if $\frac{80}{9}$ is seen)
(Ignore any attempts to find x when $t = 0$)
(b)
M1: Attempts to apply the quotient rule on $\frac{20t}{2t+1}$ with $u=20t, v=2t+1$

Alternatively applies the product rule on $20t(2t+1)^{-1}$ OR writes $\frac{20t}{2t+1}$ as $A-\frac{B}{2t+1}$ and uses the chain rule

A1: $\dfrac{dx}{dt}=\dfrac{20(2t+1)-20t\times 2}{(2t+1)^2}$ or $\dfrac{dx}{dt}=20(2t+1)^{-1}+20t\times -2(2t+1)^{-2}$

Question Number	Scheme	Notes	Marks
1.	$x = 3t - 4,\ \ y = 5 - \dfrac{6}{t},\ \ t > 0$		
(a)	$\dfrac{dx}{dt} = 3,\ \ \dfrac{dy}{dt} = 6t^{-2}$		
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ to give $\dfrac{dy}{dx}$ in terms of t **or** their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$ to give $\dfrac{dy}{dx}$ in terms of t	M1
		$\dfrac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\dfrac{dx}{dt}$ **and** $\dfrac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \dfrac{18}{x+4} \Rightarrow \dfrac{dy}{dx} = \dfrac{18}{(x+4)^2} = \dfrac{18}{(3t)^2}$	Writes $\dfrac{dy}{dx}$ in the form $\dfrac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\dfrac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	$x = -\dfrac{5}{2},\ y = -7$ **or** $P\left(-\dfrac{5}{2}, -7\right)$ seen or implied.	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - \text{"}-7\text{"} = \text{"}8\text{"}\left(x - \text{"}-\frac{5}{2}\text{"}\right)$ • $\text{"}-7\text{"} = (\text{"}8\text{"})(\text{"}-\frac{5}{2}\text{"}) + c$ So, $y = (\text{their } m_T)x + \text{"}c\text{"}$	**Some** attempt to substitute $t = 0.5$ into their $\dfrac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 **cso**
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**
(c) **Way 1**	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\} y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	Þ $y = 5 - \dfrac{18}{x+4}$ Þ $y = \dfrac{5(x+4) - 18}{x+4}$		
	So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 **cso**
			[3]
(c) **Way 2**	$\left\{t = \dfrac{6}{5-y} \Rightarrow\right\} x = \dfrac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	Þ $(x+4)(5-y) = 18$ Þ $5x - xy + 20 - 4y = 18$		
	$\{$Þ $5x + 2 = y(x+4)\}$ So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 **cso**
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		**8**

Question Number	Scheme	Notes	Marks
8. (c) **Way 2**	**Way 2 for the first 5 marks:** Applying integration by parts on $\int(\theta+\tan\theta)\sec^2\theta\,\mathrm{d}\theta$		
	$\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\,\mathrm{d}\theta = \int(\theta+\tan\theta)\sec^2\theta\,\mathrm{d}\theta,$	$\left\{ u=\theta+\tan\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta}=1+\sec^2\theta \quad \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec^2\theta \Rightarrow v=\tan\theta=\mathrm{g}(\theta) \right\}$	
	$\mathrm{h}(\theta)$ and $\mathrm{g}(\theta)$ are trigonometric functions in θ **and** $\mathrm{g}(\theta)=$ their $\int\sec^2\theta\,\mathrm{d}\theta$. [**Note:** $\mathrm{g}(\theta)\neq\sec^2\theta$]		
	$=(\theta+\tan\theta)\tan\theta-\int(1+\sec^2\theta)\tan\theta\{\mathrm{d}\theta\}$	$A(\theta+\tan\theta)\mathrm{g}(\theta)-B\int(1+\mathrm{h}(\theta))\mathrm{g}(\theta),\ A>0,\ B>0$	M1
		dependent on the previous M mark **Either** $/\left[(\theta+\tan\theta)\sec^2\theta\right]\to$ $A(\theta+\tan\theta)\tan\theta-B\int(1+\mathrm{h}(\theta))\tan\theta,\ A\neq0,\ B>0$ **or** $(\theta+\tan\theta)\tan\theta-\int(1+\mathrm{h}(\theta))\tan\theta$	dM1
	$=(\theta+\tan\theta)\tan\theta-\int(\tan\theta+\tan\theta\sec^2\theta)\{\mathrm{d}\theta\}$		
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\int\tan\theta\sec^2\theta\{\mathrm{d}\theta\}$	$(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)$ o.e. or $/\left[(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)\right]$ o.e.	A1
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\tan^2\theta$ or $=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\sec^2\theta$ etc.	$\tan\theta\sec^2\theta\to\pm C\tan^2\theta$ or $\pm C\sec^2\theta$	M1
		$(\theta+\tan\theta)\tan\theta-\frac{1}{2}\tan^2\theta$ or $(\theta+\tan\theta)\tan\theta-\frac{1}{2}\sec^2\theta$	A1
	Note	Allow the first two marks in part (c) for $\theta\tan\theta-\int\tan\theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta\tan\theta-\ln(\sec\theta)$ embedded in their working	
	Note	Allow 3rd M1 2nd A1 marks for either $\tan^2\theta-\frac{1}{2}\tan^2\theta$ or $\tan^2\theta-\frac{1}{2}\sec^2\theta$ embedded in their working	
	Question 8 Notes		
8. (a)	**Note**	Allow M1 for an answer of $k=$ awrt 2.72 without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	Allow M1 for an answer of $k=3\left(\arccos(\tfrac{1}{2})\right)\sin\left(\arccos(\tfrac{1}{2})\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	E.g. allow M1 for $\theta=60°$, leading to $k=3(60)\sin(60)$ or $k=90\sqrt{3}$	

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y =$ awrt 8.7	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	$\left(4, 5\sqrt{3}\right)$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

		Question 5 Notes
5. (a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin\left(2(45)\right)$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $\left(5\sqrt{3}, 4\right)$ is A0.

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

Question Number	Scheme		Marks
5.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 **o.e.**
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b",\ g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow\ y = \dfrac{x^2 + 2x - 5}{x-3},\ \ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3] **6**

Question Number	Scheme		Marks
5. (b)	**Alternative Method 1 of Equating Coefficients**		
	$y = \dfrac{x^2 + ax + b}{x - 3} \Rightarrow y(x-3) = x^2 + ax + b$ $y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$		
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $24 + 4a = 32 \Rightarrow a = 2$ constant: $9 + 3a + b = 10 \Rightarrow b = -5$	Equates their coefficients in t and finds both $a = ...$ and $b = ...$	dM1
		$a = 2$ and $b = -5$	A1
			[3]
5. (b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\} y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3} \Rightarrow y = x + 5 + \dfrac{10}{(x-3)}$ $\underline{\underline{y(x-3)}} = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = \underline{\underline{(x+5)(x-3) + 10}}$		dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x - 3}$ **or** equating coefficients to give $a = 2$ and $b = -5$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3]

Question Number	Scheme		Marks
8.	$x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3} \quad A(k, 1)$ lies on the curve, $k > 0$		
(a)	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k \text{ (or } x) = \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)$ **or** $x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \frac{\pi}{2}$ or $\frac{8-\pi}{2}$	x or $k = 4 - \frac{\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 4\cos t, \quad \frac{dy}{dt} = 2\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$ At $t = -\frac{\pi}{2}$, $\frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{2}\right)}$; $= -2$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$.	M1;
		Correct value for $\frac{dy}{dx}$ of -2	A1 **cao cso**
			[4]
(c)	$\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	gives $4\sin t - 4\cos t = -1$	**See notes**	A1
	So $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ $;= -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$ $;= -1$	**See notes**	M1; A1
	$t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}$ or $t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$	**See notes**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] **12**

	Question 8 Notes	
(c)	**NOTE**	**<u>VERY IMPORTANT NOTE FOR PART (c)</u>** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

Question 8: Alternative Methods for Part (c)

8. (c) **Alternative Method 1:**

Working	Notes	Marks
$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
	Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
	Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$ or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.	M1
• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
		[6]

8. (c) **Alternative Method 2:**

Working	Notes	Marks
$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$ leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
	$16 - 16\sin 2t = 1$ or equivalent.	A1
$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\}\ t = \frac{\sin^{-1}(....)}{2}$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
		[6]

Question Number	Scheme		Marks
5.	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**Main Scheme**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**Main Scheme**		
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	**[2]**
(b)	**Alternative Method 1**		
	$(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x+y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x+y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

Question Number	Scheme		Marks
4.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
	At $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2] **9**

Notes for Question 4

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.
This mark may be implied by their final answer.
Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$ which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Question Number	Scheme	Marks
6.	(a) $\frac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\frac{dy}{dt} = -8\cos t \sin t$	M1 A1
	$\frac{dy}{dx} = \frac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{dy}{dx} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right)$	A1 **(5)**
	(b) When $t = \frac{\pi}{3}$ $\quad x = \frac{3}{2},\ y = 1$ can be implied	B1
	$m = -\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (= 2)$	M1
	$y - 1 = 2\left(x - \frac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**
	(c) $x = \sqrt{3}\sin 2t = \sqrt{3}\times 2\sin t\cos t$	M1
	$x^2 = 12\sin^2 t\cos^2 t = 12\left(1-\cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1-\frac{y}{4}\right)\frac{y}{4}$ or equivalent	M1 A1 **(3)**
		[12]
	Alternative to (c)	
	$y = 2\cos 2t + 2$	M1
	$\sin^2 2t + \cos^2 2t = 1$	
	$\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$	M1 A1 **(3)**

Question Number	Scheme	Marks
5.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1 $\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}, \quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] **8**
(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. *<u>Alternative differentiation in part (a)</u>* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t \sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t \sin t - 6\sin t \cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t \sin t)$	