



**...day June 20XX – Morning/Afternoon**

**AS Level Mathematics A**

**H230/02 Pure Mathematics and Mechanics**

**SAMPLE MARK SCHEME**

**Duration:** 1 hour 30 minutes

**MAXIMUM MARK    75**



**This document consists of 12 pages**

## Text Instructions

## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

## 2. Subject-specific Marking Instructions for AS Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

### **M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### **A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### **B**

Mark for a correct result or statement independent of Method marks.

### **E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for  $g$ . E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(a)	(i)	Coordinates $(-3, 0)$ , $(-1, 0)$ and $(-2, 2)$ seen	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1</b> <b>1.1</b>	A horizontal translation only is seen	
1	(a)	(ii)	Reflection in the $x$ -axis	<b>B1</b> <b>[1]</b>	<b>1.2</b>	Must be a complete statement	
1	(b)	(i)	$(2, 12)$	<b>B1</b> <b>[1]</b>	<b>1.1</b>		
1	(b)	(ii)	$(\frac{1}{2}, 3)$	<b>B1</b> <b>[1]</b>	<b>1.1</b>		
2			<b>DR</b> $2(1 - \sin^2 x) = 2 - \sin x$ $2\sin^2 x - \sin x = 0$ $\sin x(2\sin x - 1) = 0$  $\sin x = \frac{1}{2}$ so $x = 30$ or $x = 150$ $\sin x = 0$ so $x = 0$ or $x = 180$	<b>M1</b> <b>A1</b> <b>M1</b>  <b>A1</b> <b>A1</b> <b>[5]</b>	<b>3.1a</b> <b>1.1</b> <b>1.1a</b>  <b>1.1</b> <b>1.1</b>	Use $\cos^2 x = 1 - \sin^2 x$ and simplify  Obtain $2\sin^2 x - 1\sin x = 0$ Attempt to solve a 2 term quadratic in $\sin x$ and use correct order of operations to obtain $x$ Both values are required Both values are required	One step of simplification must be seen  Use any valid method Must be seen

Question		Answer	Marks	AOs	Guidance
3	(a)	The model is exponential so the rate of change of $m$ is proportional to $m$ In this case, the rate of change of $m$ is $2m$	<b>M1</b> <b>E1</b> <b>[2]</b>	<b>1.1</b> <b>2.2a</b>	Gradient of $e^{kx} = ke^{kx}$ In context
3	(b)	The initial membership	<b>B1</b> <b>[1]</b>	<b>1.1</b>	
3	(c)	$60000 = 150e^{2t}$ $\ln 400 = 2t$ $2.995 = t$ and hence 3	<b>M1</b> <b>A1</b> <b>A1</b> <b>[3]</b>	<b>3.4</b> <b>1.1</b> <b>1.1</b>	Correct equation and use correct order of operations Obtain correct intermediate step Or $\ln 60000 = \ln 150 + 2t$ Obtain correct answer
3	(d)	E.g. When the graph reaches 60 000 the graph becomes constant.	<b>B1</b> <b>[1]</b>	<b>3.5c</b>	Correct suggestion
4	(a)	$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \mathbf{d} - \mathbf{a} = \overrightarrow{AD}$ $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	<b>B1</b> <b>M1</b> <b>A1</b> <b>[3]</b>	<b>1.1</b> <b>3.1a</b> <b>1.1</b>	soi
4	(b)	$\overrightarrow{OM} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ $ \overrightarrow{AM}  = \sqrt{6^2 + 3^2} = 3\sqrt{5}$	<b>B1</b> <b>M1</b> <b>A1</b> <b>[3]</b>	<b>1.1</b> <b>1.1</b> <b>2.2a</b>	soi Accept 6.71

Question		Answer	Marks	AOs	Guidance
5	(a)	$\log_{10} y = \log_{10} p + x \log_{10} q$ $m = \log_{10} q, c = \log_{10} p$	<b>B1</b> <b>B1</b> [2]	<b>2.1</b> <b>2.4</b>	
5	(b)	E.g. $\log_{10} q = \frac{2.4-1.6}{1-5} = -0.2$ $q = 10^{-0.2} = 0.63$ $\log_{10} p = 2.5$ so $p = 320$	<b>M1</b> <b>A1</b> <b>B1</b> [3]	<b>3.3</b> <b>1.1</b> <b>1.1</b>	Measure gradient from graph and identify it as $\log q$  Accept $q$ in $[0.6, 0.7]$ Accept $p$ in $[300, 400]$
5	(c)	$\log_{10} 20 = 1.3$ so week 7 E.g. Extrapolation is unjustified because it assumes that the assumptions made in the model will hold true in the long term	<b>B1</b> <b>E1</b>  [2]	<b>3.4</b> <b>3.5b</b>	One valid explanation
6	(a)	31 gives $3^2 + 1^2 = 10$  10 is even and hence the suggestion is false	<b>M1</b> <b>E1</b> [2]	<b>2.1</b> <b>2.1</b>	<b>OR</b> <b>M1</b> 37 gives $3^2 + 7^2 = 58$ <b>E1</b> 58 is even and hence the suggestion is false
6	(b)	$n^2 + (n+1)^2 + (n+2)^2$  $3n^2 + 6n + 5$ $3(n^2 + 2n + 1) + 2$ which always leaves a remainder of 2 and so cannot be divided by 3	<b>M1</b> <b>A1FT</b> <b>E1</b>  [3]	<b>2.1</b> <b>1.1</b> <b>2.1</b>	Any valid expressions for three consecutive integers FT <i>their</i> expressions Correct conclusion.

Question		Answer	Marks	AOs	Guidance
7		$f(x+h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$ $\frac{f(x+h) - f(x)}{h} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$ $= 4x^3 + 6x^2h + 4xh^2 + h^3$ <p>As <math>h \rightarrow 0</math> all the terms in <math>h</math> tend to zero.</p> <p>Therefore <math>f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 4x^3</math></p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>E1</b></p> <p><b>[5]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>2.4</b></p> <p><b>2.1</b></p>	<p>Attempt at expansion with product of powers of <math>x</math> and <math>h</math> summing to 4 and some attempt at coefficients, not necessarily correct</p> <p>Attempt <math>\frac{f(x+h) - f(x)}{h}</math></p> <p>Allow at most two errors</p> <p>All terms correct</p> <p>Accept some indication that as <math>h</math> tends to 0, the terms involving <math>h</math> vanish and leave <math>4x^3</math></p> <p>Award for good use of language, and of limit and function notation</p> <p>Only requires the two M1 marks to be awarded.</p>



Question	Answer	Marks	AOs	Guidance
8	$2x + 3y = 0$ $\Rightarrow y = -\frac{2}{3}x$ and gradient $-\frac{2}{3}$ Hence, gradient of the tangent is $\frac{3}{2}$  $\frac{dy}{dx} = \frac{3}{2}kx^{\frac{1}{2}}$ At $x = 4$ , $\frac{3}{2}k(4)^{\frac{1}{2}} = 3k$  Hence $3k = \frac{3}{2}$ , so $k = \frac{1}{2}$  At $P$ , $y = \frac{1}{2}(4)^{\frac{3}{2}} = 4$ so $P = (4, 4)$ so equation of normal through $P$ is $(y - 4) = -\frac{2}{3}(x - 4)$ When $y = 0$ , $x = 10$ so $Q = (10, 0)$  Using $P(4, 4)$ and $Q(10, 0)$ $PQ^2 = (10 - 4)^2 + (0 - 4)^2$ Circle equation is $(x - 4)^2 + (y - 4)^2 = 52$	<b>M1</b>  <b>A1FT</b>  <b>M1</b>  <b>A1</b>  <b>M1</b>  <b>E1</b>  <b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1FT</b>  <b>[10]</b>	<b>3.1a</b>  <b>1.1</b>  <b>1.1a</b>  <b>1.1</b>  <b>1.1</b>  <b>1.1</b>  <b>3.1a</b>  <b>1.1</b>  <b>1.1</b>  <b>1.1</b>	Identify gradient of line $(= -\frac{2}{3})$ anywhere Use $m_1m_2 = -1$ anywhere $(= \frac{3}{2})$ FT their gradient Attempt differentiation Obtain $\frac{3}{2}kx^{\frac{1}{2}}$ Substitute $x = 4$ and equate to the normal gradient AG Identify coordinates, gradient of normal and form equation with their coordinates Substitute $y = 0$ and obtain $x = 10$ Use Pythagoras to obtain length $PQ^2$ Accept equivalent forms FT their coordinates for $P$ and $Q$  Allow sign slip  The power must be seen to decrease  Tangent gradients may also be used i.e. $-\frac{1}{3k} = -\frac{2}{3}$  Accept $y = 4$

Question		Answer	Marks	AOs	Guidance	
9	(a)	$-\left(\frac{4-0}{8-5}\right)$	<b>M1</b>	<b>1.1</b>	Attempt at acceleration calculation with at most one error	Or use of $v = u + at$ with $v = 0, u = 4$ and $t = 3$ with at least 2 values correct
		Acceleration = $-\frac{4}{3}$	<b>A1</b>	<b>1.1</b>	Or equivalent	
9	(b)	Distance travelled $= \frac{1}{2}(5+8)(4) + \frac{1}{2}(12)(3) = 44$	<b>M1</b>	<b>1.1</b>	Attempt at both areas; the trapezium and triangle	
		Displacement $= \frac{1}{2}(5+8)(4) - \frac{1}{2}(12)(3) = 8$	<b>A1</b>	<b>1.1</b>	Either distance travelled or displacement correct	
		44 m > 40 m so distance travelled is more than five times the displacement	<b>E1</b>	<b>2.2a</b>	Must see relevant comparison	
			<b>[3]</b>			
10	(a)	$s = 9(6)^2 - 1.5(6)^3 = 0$	<b>M1</b>	<b>1.1</b>		
		E.g. The boomerang is at $O$ E.g. She catches the boomerang	<b>E1</b>	<b>3.4</b>		
			<b>[2]</b>			
10	(b)	$v = 18t - \frac{9}{2}t^2$	<b>M1</b>	<b>1.1</b>		
		When $v = 0$ , $t = 0$ or $t = 4$	<b>M1</b>	<b>1.1</b>	Imply deduction that greatest distance is when velocity = 0 and solve	
		At $t = 0$ , $s = 0$ , so maximum displacement must be when $t = 4$ giving $s = 48$ m	<b>A1</b>	<b>1.1</b>		
			<b>E1</b>	<b>3.4</b>		
		<b>[4]</b>				
10	(c)	$t = 5$	<b>B1</b>	<b>3.1b</b>		
		$v = -22.5$	<b>B1</b>	<b>3.4</b>		
			<b>[2]</b>			
10	(d)	$a = 18 - 9t$	<b>M1</b>	<b>1.1</b>		
		Acceleration = $-27(\text{ms}^{-2})$	<b>A1</b>	<b>1.1</b>		
			<b>[2]</b>			

Question		Answer	Marks	AOs	Guidance				
11	(a)	Resultant force from the tug boats is positive so it is moving east	E1	2.2a	(600i)				
		There is zero resultant force in the <b>j</b> direction, so it is not moving north or south	E1	2.2a					
			[2]						
11	(b)	$350 + 250 - 200 = 100000a$	M1	3.3	Use $F = ma$ . Allow sign errors and one missing force				
		Obtain $0.004 \text{ m s}^{-2}$	A1	1.1					
			[2]						
11	(c)	$400 = 1.5t + \frac{1}{2}(0.004)t^2$	M1	3.1b	Use $s = ut + \frac{1}{2}at^2$				
		$0.002t^2 + 1.5t - 400 = 0$	A1	1.1		Obtain correct quadratic. Any equivalent form			
		Obtain 209 (seconds)	M1	3.4			Use any method to solve their quadratic		
		$v^2 = 1.5^2 + 2(0.004)(400)$	A1	1.1				If negative root given (-958.63088) this must be clearly discarded	
		Obtain $2.33 \text{ (m s}^{-1}\text{)}$	M1	3.4					Use $v^2 = u^2 + 2as$ with their $a$ or $v = u + at$ with their $a$ and $t$
			A1	1.1					
	[6]								

## Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3 (PS)	AO3 (M)	Total
<b>Pure</b>					
<b>1ai</b>	2				<b>2</b>
<b>1aii</b>	1				<b>1</b>
<b>1bi</b>	1				<b>1</b>
<b>1bii</b>	1				<b>1</b>
<b>2</b>	4		1		<b>5</b>
<b>3a</b>	1	1			<b>2</b>
<b>3b</b>	1				<b>1</b>
<b>3c</b>	2			1	<b>3</b>
<b>3d</b>				1	<b>1</b>
<b>4a</b>	2		1		<b>3</b>
<b>4b</b>	2	1			<b>3</b>
<b>5a</b>		2			<b>2</b>
<b>5b</b>	2			1	<b>3</b>
<b>5c</b>			2		<b>2</b>
<b>6a</b>		2			<b>2</b>
<b>6b</b>	1	2			<b>3</b>
<b>7</b>	3	2			<b>5</b>
<b>8</b>	8		2		<b>10</b>
<b>Statistics</b>					
<b>9a</b>	2				<b>2</b>
<b>9b</b>	2	1			<b>3</b>
<b>10a</b>	1			1	<b>2</b>
<b>10b</b>	3			1	<b>4</b>
<b>10c</b>			1	1	<b>2</b>
<b>10d</b>	2				<b>2</b>
<b>11a</b>		2			<b>2</b>
<b>11b</b>	1			1	<b>2</b>
<b>11c</b>	3		1	2	<b>6</b>
<b>Totals</b>	<b>45</b>	<b>13</b>	<b>6</b>	<b>11</b>	<b>75</b>

PS = Problem Solving

M = Modelling

Summary of Updates

Date	Version	Change
October 2018	2	We've reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our "Exploring our question papers" brochures on our website.
November 2019	2.1	Amendment to Instructions rubric on front cover.