1. The transformation P is an enlargement, centre the origin, with scale factor k, where $k > 0$

The transformation Q is a rotation through angle θ degrees anticlockwise about the origin.

The transformation P followed by the transformation Q is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$$

(a) Determine

(i) the value of k,

(ii) the smallest value of θ

(4)

A square S has vertices at the points with coordinates $(0, 0)$, $(a, -a)$, $(2a, 0)$ and (a, a) where a is a constant.

The square S is transformed to the square S' by the transformation represented by $\mathbf{M}$.

(b) Determine, in terms of a, the area of S'

(2)

2. $\mathbf{A} = \begin{pmatrix} 4 & -2 \\ 5 & 3 \end{pmatrix}$

The matrix **A** represents the linear transformation *M*.

Prove that, for the linear transformation *M*, there are no invariant lines.

(5)

3. $\mathbf{M} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ where a is a constant

(a) Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\mathbf{M}^n = \begin{pmatrix} 3^n & \frac{a}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}$$

(6)

Triangle T has vertices A, B and C.

Triangle T is transformed to triangle T' by the transformation represented by $\mathbf{M}^n$ where $n \in \mathbb{N}$

Given that

- triangle T has an area of $5\,\text{cm}^2$
- triangle T' has an area of $1215\,\text{cm}^2$
- vertex $A(2, -2)$ is transformed to vertex $A'(123, -2)$

(b) determine

(i) the value of n

(ii) the value of a

(5)

3. $$\mathbf{M} = \begin{pmatrix} -2 & 5 \\ 6 & k \end{pmatrix}$$

where k is a constant.

Given that

$$\mathbf{M}^2 + 11\mathbf{M} = a\mathbf{I}$$

where a is a constant and $\mathbf{I}$ is the 2×2 identity matrix,

(a) (i) determine the value of a

(ii) show that $k = -9$

(3)

(b) Determine the equations of the invariant lines of the transformation represented by $\mathbf{M}$.

(6)

(c) State which, if any, of the lines identified in (b) consist of fixed points, giving a reason for your answer.

(1)

5.

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **A**.

(3)

The transformation V, represented by the 2×2 matrix **B**, is a reflection in the line $y = -x$

(b) Write down the matrix **B**.

(1)

Given that U followed by V is the transformation T, which is represented by the matrix **C**,

(c) find the matrix **C**.

(2)

(d) Show that there is a real number k for which the point $(1, k)$ is invariant under T.

(4)

1.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix **M** is non-singular.

(2)

The transformation T of the plane is represented by the matrix **M**.

The triangle R is transformed to the triangle S by the transformation T.

Given that the area of S is 63 square units,

(b) find the area of R.

(2)

(c) Show that the line $y = 2x$ is invariant under the transformation T.

(2)

6. (i)

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$$

where a and b are non-zero constants.

Given that the matrix **A** is self-inverse,

(a) determine the value of b and the possible values for a.

(5)

The matrix **A** represents a linear transformation M.

Using the smaller value of a from part (a),

(b) show that the invariant points of the linear transformation M form a line, stating the equation of this line.

(3)

(ii)

$$\mathbf{P} = \begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix}$$

where p is a positive constant.

The matrix **P** represents a linear transformation U.
The triangle T has vertices at the points with coordinates (1, 2), (3, 2) and (2, 5).
The area of the image of T under the linear transformation U is 15

(a) Determine the value of p.

(4)

The transformation V consists of a stretch scale factor 3 parallel to the x-axis with the y-axis invariant followed by a stretch scale factor –2 parallel to the y-axis with the x-axis invariant. The transformation V is represented by the matrix **Q**.

(b) Write down the matrix **Q**.

(2)

Given that U followed by V is the transformation W, which is represented by the matrix **R**,

(c) find the matrix **R**.

(2)

1. $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(a) (i) Describe fully the single geometrical transformation P represented by the matrix **P**.

(ii) Describe fully the single geometrical transformation Q represented by the matrix **Q**.

(4)

The transformation P followed by the transformation Q is the transformation R, which is represented by the matrix **R**.

(b) Determine **R**.

(1)

(c) (i) Evaluate the determinant of **R**.

(ii) Explain how the value obtained in (c)(i) relates to the transformation R.

(2)

3. *With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the y-axis is represented by the matrix*

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

The point P has coordinates (8, 3, 2)

The point Q is the image of P under the transformation reflection in the plane $y = 0$

(a) Write down the coordinates of Q

(1)

The point R is the image of P under the transformation rotation through 120° anticlockwise about the y-axis, with respect to the **right-hand rule**.

(b) Determine the exact coordinates of R

(2)

(c) Hence find $|\overrightarrow{PR}|$ giving your answer as a simplified surd.

(2)

(d) Show that $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular.

(1)

(e) Hence determine the exact area of triangle PQR, giving your answer as a surd in simplest form.

(2)

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3.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometric transformation A represented by the matrix **A**.

(2)

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

The transformation B is represented by the matrix **B**.
The transformation A followed by the transformation B is the transformation C, which is represented by the matrix **C**.

To determine matrix **C**, a student attempts the following matrix multiplication.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

(b) State the error made by the student.

(1)

(c) Determine the correct matrix **C**.

(1)

9. (i) $\mathbf{P} = \begin{pmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{pmatrix}$ where k is a constant

Show that **P** is non-singular for all real values of k.

(4)

(ii) $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$

The matrix **Q** represents a linear transformation T

Under T, the point $A(a, 2)$ and the point $B(4, -a)$, where a is a constant, are transformed to the points A' and B' respectively.

Given that the distance $A'B'$ is $\sqrt{58}$, determine the possible values of a.

(5)