

2. Prove by induction that for all positive integers n ,

$$f(n) = 2^{3n+1} + 3(5^{2n+1})$$

is divisible by 17

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

6. Given that

$$y = e^{2x} \sinh x$$

prove by induction that for $n \in \mathbb{N}$

$$\frac{d^n y}{dx^n} = e^{2x} \left(\frac{3^n + 1}{2} \sinh x + \frac{3^n - 1}{2} \cosh x \right)$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



3. Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



8. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



8. (a) Prove by induction that, for all positive integers n ,

$$\sum_{r=1}^n r(r+1)(2r+1) = \frac{1}{2}n(n+1)^2(n+2) \quad (6)$$

- (b) Hence, show that, for all positive integers n ,

$$\sum_{r=n}^{2n} r(r+1)(2r+1) = \frac{1}{2}n(n+1)(an+b)(cn+d)$$

where a , b , c and d are integers to be determined.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



7. Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}^n = \begin{pmatrix} 1 - 6n & 9n \\ -4n & 1 + 6n \end{pmatrix}$$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

