4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

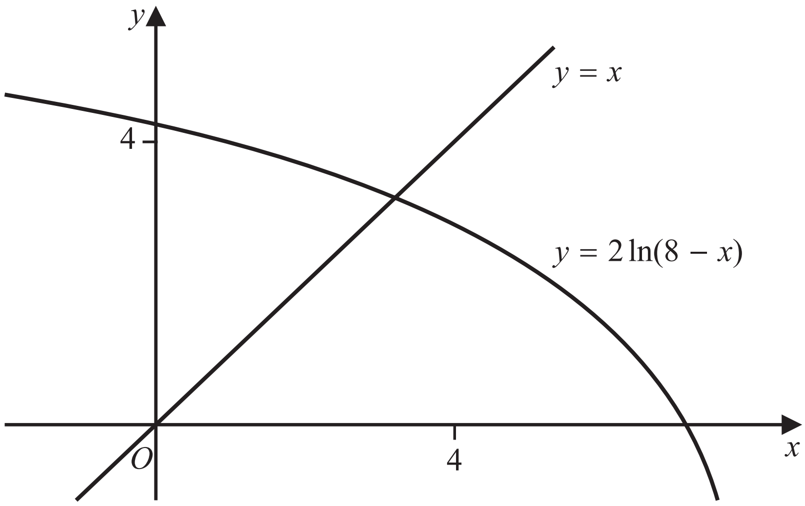

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

11.

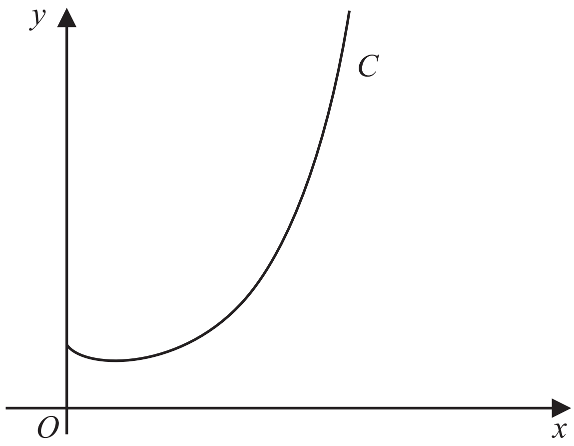

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{\,1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

7.

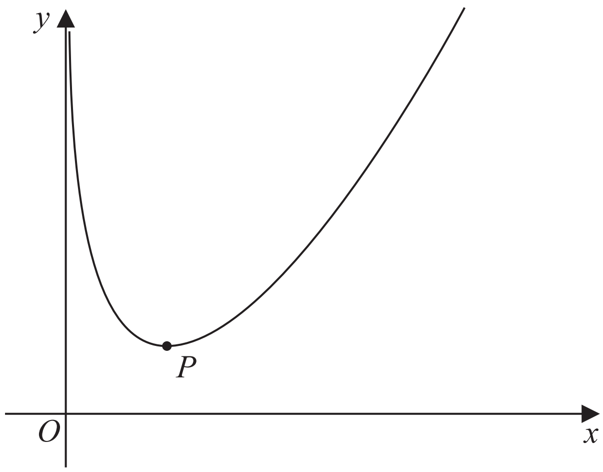

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

Leave blank

7.

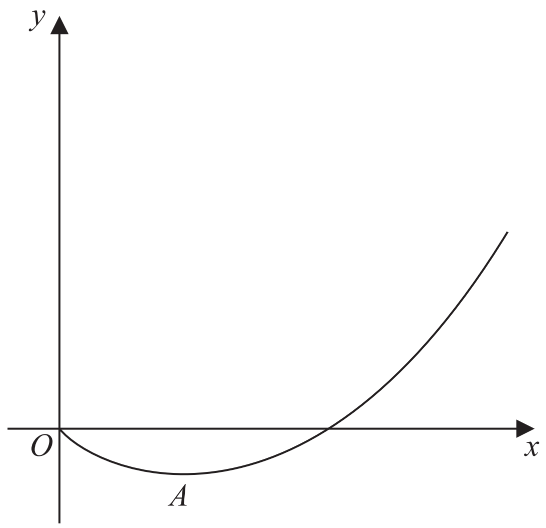

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x(1 + x)\ln x, \quad x > 0$$

The curve has a minimum turning point at A.

(a) Find $\mathrm{f}'(x)$

(3)

(b) Hence show that the x coordinate of A is the solution of the equation

$$x = \mathrm{e}^{-\frac{1+x}{1+2x}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \mathrm{e}^{-\frac{1+x_n}{1+2x_n}}, \qquad x_0 = 0.46$$

to find the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(d) Use your answer to part (c) to estimate the coordinates of A to 2 decimal places.

(2)

Leave blank

1. $$f(x) = 2x^3 + x - 10$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.5, 2]$

(2)

The only real root of $f(x) = 0$ is α

The iterative formula

$$x_{n+1} = \left(5 - \frac{1}{2}x_n\right)^{\frac{1}{3}}, \quad x_0 = 1.5$$

can be used to find an approximate value for α

(b) Calculate x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 1.6126$ correct to 4 decimal places.

(2)

Leave blank

10.

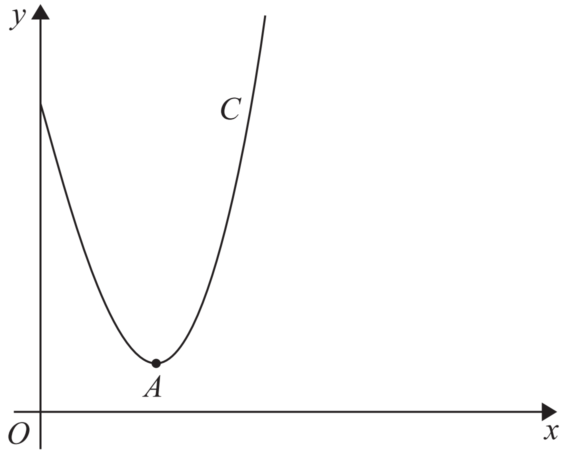

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

Point A is the minimum turning point on the curve.

(a) Show, by using calculus, that the x coordinate of point A is a solution of

$$x = \frac{6}{1 + \ln(x^2)}$$

(5)

(b) Starting with $x_0 = 2.27$, use the iteration

$$x_{n+1} = \frac{6}{1 + \ln(x_n^{\ 2})}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of point A to one decimal place.

(2)

Leave blank

5.

$$f(x) = -x^3 + 4x^2 - 6$$

(a) Show that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{6}{4-x}\right)}$$

(2)

(c) Starting with $x_1 = 1.5$ use the iteration $x_{n+1} = \sqrt{\left(\frac{6}{4-x_n}\right)}$ to calculate the values of x_2, x_3 and x_4 giving all your answers to 4 decimal places.

(3)

(d) Using a suitable interval, show that 1.572 is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

Leave blank

10. (a) Given that $-\dfrac{\pi}{2} < \mathrm{g}(x) < \dfrac{\pi}{2}$, sketch the graph of $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \arctan x, \qquad x \in \mathbb{R}$$

(2)

(b) Find the exact value of x for which

$$3\mathrm{g}(x + 1) - \pi = 0$$

(3)

The equation $\arctan x - 4 + \dfrac{1}{2}x = 0$ has a positive root at $x = \alpha$ radians.

(c) Show that $5 < \alpha < 6$

(2)

The iteration formula

$$x_{n+1} = 8 - 2 \arctan x_n$$

can be used to find an approximation for α

(d) Taking $x_0 = 5$, use this formula to find x_1 and x_2, giving each answer to 3 decimal places.

(2)

Leave blank

2.

$$f(x) = x^3 - 5x + 16$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = (ax + b)^{\frac{1}{3}}$$

giving the values of the constants a and b.

(2)

The equation $f(x) = 0$ has exactly one real root α, where $\alpha = -3$ to one significant figure.

(b) Starting with $x_1 = -3$, use the iteration

$$x_{n+1} = (ax_n + b)^{\frac{1}{3}}$$

with the values of a and b found in part (a), to calculate the values of x_2, x_3 and x_4, giving all your answers to 3 decimal places.

(3)

(c) Using a suitable interval, show that $\alpha = -3.17$ correct to 2 decimal places.

(2)

Leave blank

11. $$y = (2x^2 - 3)\tan\left(\frac{1}{2}x\right), \qquad 0 < x < \pi$$

(a) Find the exact value of x when $y = 0$

(1)

Given that $\frac{dy}{dx} = 0$ when $x = \alpha$,

(b) show that

$$2\alpha^2 - 3 + 4\alpha \sin \alpha = 0$$

(6)

The iterative formula

$$x_{n+1} = \frac{3}{(2x_n + 4\sin x_n)}$$

can be used to find an approximation for α.

(c) Taking $x_1 = 0.7$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 0.7283$ to 4 decimal places.

(2)

Leave blank

1. $$\mathrm{f}(x) = x^5 + x^3 - 12x^2 - 8, \qquad x \in \mathbb{R}$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{4(3x^2 + 2)}{x^2 + 1}}$$

(3)

(b) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\frac{4(3x_n^{\ 2} + 2)}{x_n^{\ 2} + 1}}$$

with $x_0 = 2$, to find x_1, x_2 and x_3 giving your answers to 3 decimal places.

(3)

The equation $\mathrm{f}(x) = 0$ has a single root, α.

(c) By choosing a suitable interval, prove that $\alpha = 2.247$ to 3 decimal places.

(2)