16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

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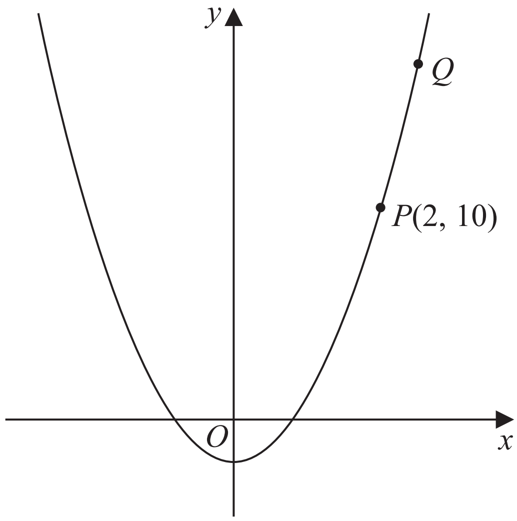

Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.
(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.
(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).
(1)

8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

10. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

14. A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at (2, 9)

(a) find $g(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for Question 2 is 4 marks)

6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

(Total for Question 6 is 4 marks)

15.

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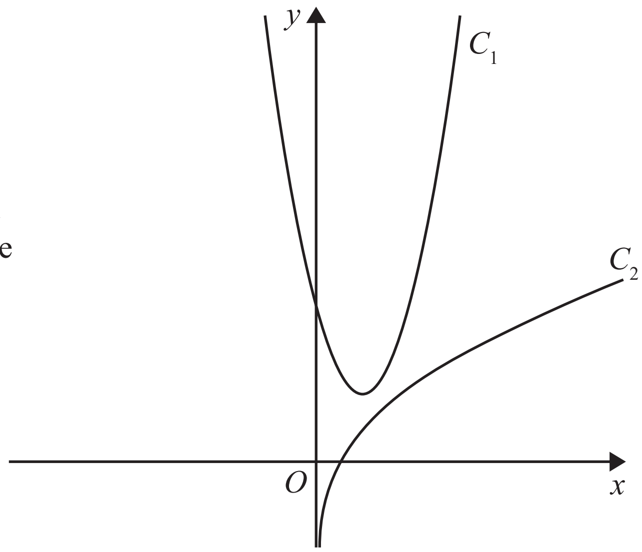

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2},\ 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

16.

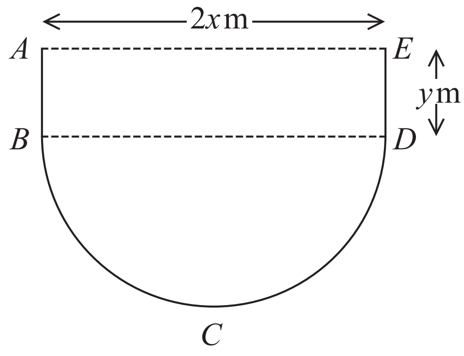

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

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9.

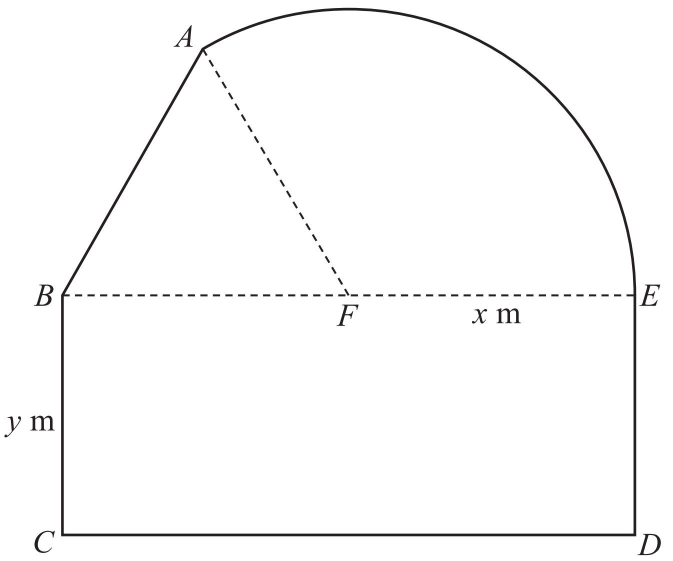

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\ m^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)

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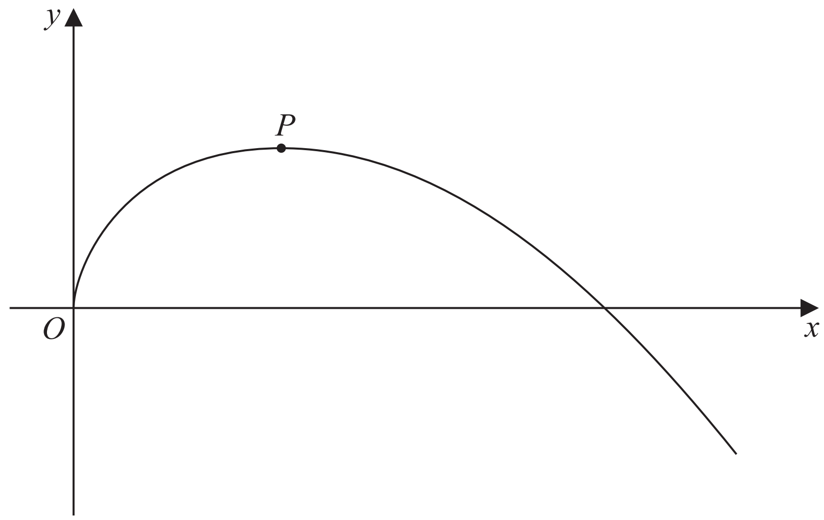

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 27\sqrt{x} - 2x^2, \qquad x \in \mathbb{R}, x > 0$$

(a) Find $\dfrac{dy}{dx}$

(3)

The curve has a maximum turning point P, as shown in Figure 2.

(b) Use the answer to part (a) to find the exact coordinates of P.

(5)

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16. $f(x) = ax^3 + bx^2 + 2x - 5$, where a and b are constants

The point $P(1, 4)$ lies on the curve with equation $y = f(x)$.

The tangent to $y = f(x)$ at the point P has equation $y = 12x - 8$

Calculate the value of a and the value of b.

(5)

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4. The curve C has equation $y = 4x\sqrt{x} + \dfrac{48}{\sqrt{x}} - \sqrt{8}, \quad x > 0$

(a) Find, simplifying each term,

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Use part (a) to find the exact coordinates of the stationary point of C.

(5)

(c) Determine whether the stationary point of C is a maximum or minimum, giving a reason for your answer.

(2)

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1. Given $y = \frac{x^3}{3} - 2x^2 + 3x + 5$

(a) find $\frac{dy}{dx}$, simplifying each term.

(3)

(b) Hence find the set of values of x for which $\frac{dy}{dx} > 0$

(4)

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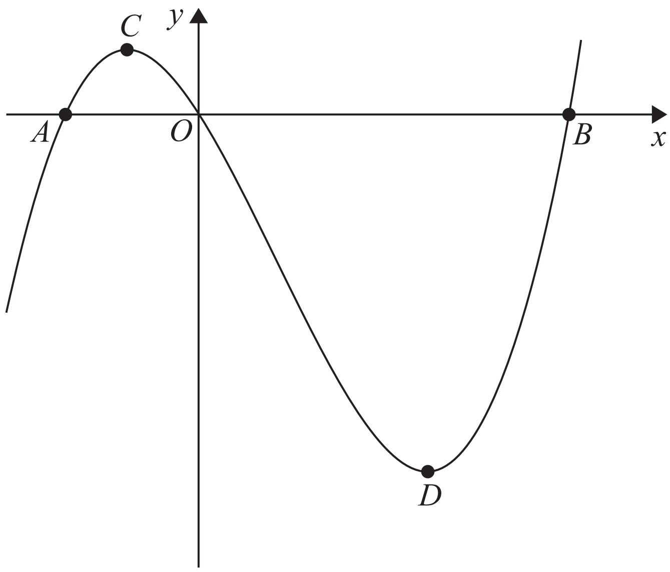

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Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{x^3 - 9x^2 - 81x}{27}$$

The curve crosses the x-axis at the point A, the point B and the origin O.
The curve has a maximum turning point at C and a minimum turning point at D.

(a) Use algebra to find exact values for the x coordinates of the points A and B.

(4)

(b) Use calculus to find the coordinates of the points C and D.

(6)

The graph of $y = \mathrm{f}(x + a)$, where a is a constant, has its minimum turning point on the y-axis.

(c) Write down the value of a.

(1)

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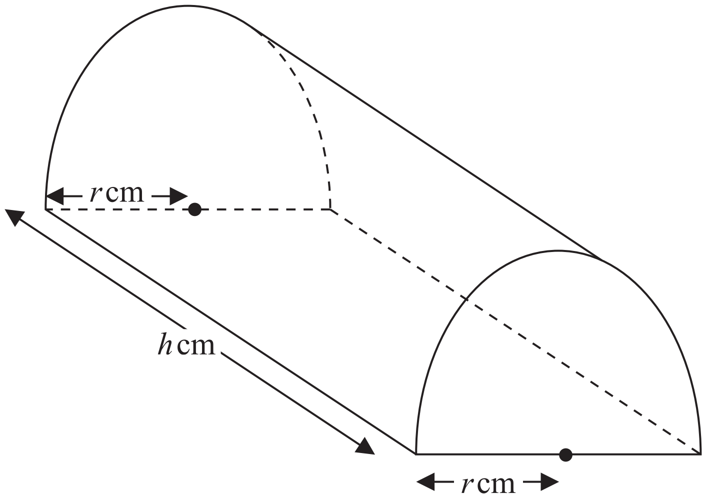

Figure 4

Figure 4 shows a solid wooden block. The block is a right prism with length h cm. The cross-section of the block is a semi-circle with radius r cm.

The total surface area of the block, including the curved surface, the two semi-circular ends and the rectangular base, is 200 cm^2

(a) Show that the volume V cm^3 of the block is given by

$$V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$$

(5)

(b) Use calculus to find the maximum value of V. Give your answer to the nearest cm^3.
(6)

(c) Justify, by further differentiation, that the value of V that you have found is a maximum.

(2)

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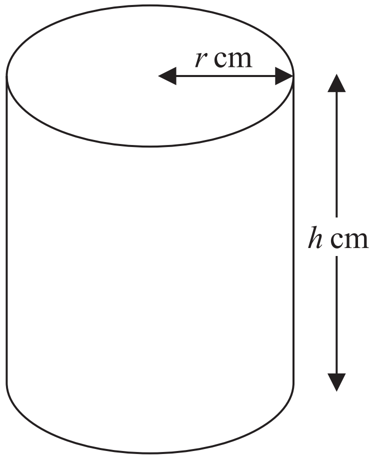

Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height h cm and radius r cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm^3 of water when full.

(a) Show that the total external surface area, S cm^2, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(c) Justify that the value of S you found in part (b) is a minimum.

(2)