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1. In a survey it is found that barn owls occur randomly at a rate of 9 per 1000 km².

 - Find the probability that in a randomly selected area of 1000 km² there are at least 10 barn owls. (2)
 - Find the probability that in a randomly selected area of 200 km² there are exactly 2 barn owls. (3)
 - Using a suitable approximation, find the probability that in a randomly selected area of 50000 km² there are at least 470 barn owls. (6)



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Question 1 continued

Q1

(Total 11 marks)



2. The proportion of houses in Radville which are unable to receive digital radio is 25%. In a survey of a random sample of 30 houses taken from Radville, the number, X , of houses which are unable to receive digital radio is recorded.

(a) Find $P(5 \leq X < 11)$

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

(b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)



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Question 2 continued

Q2

(Total 8 marks)



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3. A random variable X has probability density function given by

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ k\left(1 - \frac{x}{6}\right) & 2 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- (a) Show that $k = \frac{1}{4}$ (4)

(b) Write down the mode of X . (1)

(c) Specify fully the cumulative distribution function $F(x)$. (5)

(d) Find the upper quartile of X . (4)



Question 3 continued

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Question 3 continued



Question 3 continued

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Q3

(Total 14 marks)



4. The continuous random variable L represents the error, in metres, made when a machine cuts poles to a target length. The distribution of L is a continuous uniform distribution over the interval $[0, 0.5]$

(a) Find $P(L < 0.4)$. (1)

(b) Write down $E(L)$. (1)

(c) Calculate $\text{Var}(L)$. (2)

A random sample of 30 poles cut by this machine is taken.

(d) Find the probability that fewer than 4 poles have an error of more than 0.4 metres from the target length. (3)

When a new machine cuts poles to a target length, the error, X metres, is modelled by the cumulative distribution function $F(x)$ where

$$F(x) = \begin{cases} 0 & x < 0 \\ 4x - 4x^2 & 0 \leq x \leq 0.5 \\ 1 & \text{otherwise} \end{cases}$$

(e) Using this model, find $P(X > 0.4)$ (2)

A random sample of 100 poles cut by this new machine is taken.

(f) Using a suitable approximation, find the probability that at least 8 of these poles have an error of more than 0.4 metres. (3)



Question 4 continued

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Question 4 continued



Question 4 continued

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5. *Liftsforall* claims that the lift they maintain in a block of flats breaks down at random at a mean rate of 4 times per month. To test this, the number of times the lift breaks down in a month is recorded.

- (a) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis that ‘the mean rate at which the lift breaks down is 4 times per month’. The probability of rejection in each of the tails should be as close to 2.5% as possible. (3)

Over a randomly selected 1 month period the lift broke down 3 times.

- (b) Test, at the 5% level of significance, whether *Liftsforall*'s claim is correct. State your hypotheses clearly. (2)

(c) State the actual significance level of this test. (1)

The residents in the block of flats have a maintenance contract with *Liftsforall*. The residents pay *Liftsforall* £500 for every quarter (3 months) in which there are at most 3 breakdowns. If there are 4 or more breakdowns in a quarter then the residents do not pay for that quarter.

Liftsforall installs a new lift in the block of flats.

Given that the new lift breaks down at a mean rate of 2 times per month,

- (d) find the probability that the residents do not pay more than £500 to *Liftsforall* in the next year. (6)



Question 5 continued

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Question 5 continued



Question 5 continued

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6. A continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} kx^n & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where k and n are positive integers.

- (a) Find k in terms of n .

(3)

- (b) Find $E(X)$ in terms of n .

(3)

- (c) Find $E(X^2)$ in terms of n .

(2)

Given that $n = 2$

- (d) find $\text{Var}(3X)$.

(3)



Question 6 continued

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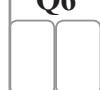
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Question 6 continued



Question 6 continued

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7. A bag contains a large number of 10p, 20p and 50p coins in the ratio 1:2:2

A random sample of 3 coins is taken from the bag.

Find the sampling distribution of the median of these samples.

(7)



Question 7 continued

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Question 7 continued

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TOTAL FOR PAPER: 75 MARKS

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