Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

In A Level Further Mathematics (9FM0)

Paper 01 Core Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

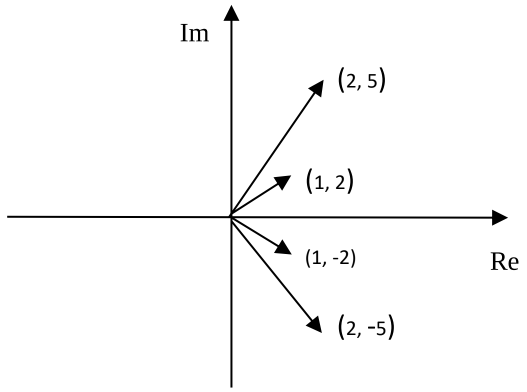
- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$z = 2 - 5\text{i}$	B1	1.2
	$(z-(2+5\text{i}))(z-(2-5\text{i})) = ...$ or Sum of roots = 4, Product of roots = 29 $\rightarrow z^2 + ...$	M1	3.1a
	$= z^2 - 4z + 29$	A1	1.1b
	$z^4 - 6z^3 + az^2 + bz + 145 = (z^2 - 4z + 29)(z^2 + cz + 5)$	M1	3.1a
	$z^2 - 2z + 5 = 0$	A1	2.2a
	$z^2 - 2z + 5 = 0 \Rightarrow z = ...$	M1	1.1b
	$z = 1 \pm 2\text{i}$	A1	1.1b
		(7)	
(b)		B1ft	1.1b
		B1	1.1b
		(2)	
		(9 marks)	

Notes

(a)
Note: if there are multiple attempts or attempts which start in main scheme before switching to an Alt, then mark the most complete attempt.
B1: Identifies the correct complex conjugate as another root.
M1: Formulates a correct strategy – main scheme. Uses the conjugate pair and a correct method to find a quadratic factor.
A1: Correct quadratic.
M1: Uses the given quartic and their quadratic to establish the other quadratic factor – by inspection or long division etc. resulting in a 3TQ
A1: Deduces the correct second quadratic.
M1: Solves their second quadratic.
A1: Correct second conjugate pair (isw if they try to write in a factorised equation).
(b)
B1ft: A pair of complex roots plotted correctly, either both $2 \pm 5\text{i}$ or follow through their second pair of *complex* roots. Just the points are needed. Allow if there is no labelling for this mark, as long as there is a pair of roots symmetric in the real axis.
B1: Fully correct and labelled sketch with both sets of roots shown in approximately the correct locations. Just the points are needed. Labelling may be by coordinates or labels on axes, and accept tick marks as unit increments for labels. The $(1, \pm 2)$ roots must be within the sector spanned by $(2, \pm 5)$ (as in the diagram).

(a) **Alt 1**	$z = 2 - 5\text{i}$	B1	1.2
	$\alpha + \beta = 4, \alpha\beta = 2^2 + 5^2 = ...$ and $\alpha + \beta + \gamma + \delta = \pm 6, \alpha\beta\gamma\delta = \pm 145$	M1	3.1a
	$\alpha + \beta = 4, \alpha\beta = 29$ and $\alpha + \beta + \gamma + \delta = 6, \alpha\beta\gamma\delta = 145$	A1	1.1b
	$\Rightarrow \gamma + \delta = 2,\ \gamma\delta = \frac{"145"}{"29"} \Rightarrow (2 - \delta)\delta = \frac{145}{29}$	M1	3.1a
	$\delta^2 - 2\delta + 5 = 0$	A1	2.2a
	$\delta^2 - 2\delta + 5 = 0 \Rightarrow \delta = ...$	M1	1.1b
	$= 1 \pm 2\text{i}$	A1	1.1b
		(7)	

Notes

(a)
B1: Identifies the correct complex conjugate as another root
M1: Formulates a correct strategy – sum and product approach. Attempts formulae for sum and product of the two know roots and all four roots – allow sign errors, $\alpha + \beta = 4, \alpha\beta = 2^2 + 5^2 = ...$ $\alpha + \beta + \gamma + \delta = \pm 6, \alpha\beta\gamma\delta = \pm 145$
A1: Correct equations $\alpha + \beta = 4, \alpha\beta = 29$ $\alpha + \beta + \gamma + \delta = 6, \alpha\beta\gamma\delta = 145$ (seen or implied).
M1: Uses the sum and product of known roots to reduce to equations in just the unknown roots and attempts to solve simultaneously. Note allow if they assume complex roots and use $\lambda \pm \mu\text{i}$ for the two unknown roots.
A1: Deduces the correct quadratic for remaining roots.
M1: Solves their quadratic A1: Correct second conjugate pair

(a) **Alt 2**	$z = 2 - 5\text{i}$	B1	1.2
	$\text{f}(2 \pm 5\text{i}) = 0 \Rightarrow -21a + 2b + 1038 \pm (-20a - 5b + 450)\text{i} = 0$ $\Rightarrow 21a - 2b - 1038 = 0, 20a + 5b - 450 = 0$	M1 A1	3.1a 1.1b
	$\Rightarrow a = ...,\ b = ...$	M1	3.1a
	$a = 42$ **and** $b = -78$	A1	2.2a
	$z^4 - 6z^3 + "42"z^2 - "78"z + 145 = 0 \Rightarrow z = ...$	M1	1.1b
	$z = 1 \pm 2\text{i}, (2 \pm 5\text{i})$	A1	1.1b
		(7)	

Notes

(a)
B1: Identifies the correct complex conjugate as another root- seen anywhere
M1: Formulates a correct strategy – factor theorem approach. Applies the factor theorem with either complex root and equates real and imaginary terms to form simultaneous equations in *a* and *b*
A1: Correct equations need not be simplified.
M1: Solves the equations to find values for *a* and *b*. Do not be concerned with the algebra.
A1: Correct values
M1: Solves the resulting quartic – may be by calculator. Note that if roots are stated by calculator they must correspond to their found *a* and *b*, but allow for rounding to nearest integer.
A1: Correct second conjugate pair from fully correct work.

Question	Scheme	Marks	AOs
2(a)	$\alpha+\beta+\gamma=\frac{3}{2},\ \alpha\beta+\alpha\gamma+\beta\gamma=6, \alpha\beta\gamma=-\frac{7}{2}$	B1	1.1b
		(1)	
(b)(i)	$\frac{2}{\alpha}+\frac{2}{\beta}+\frac{2}{\gamma}=\frac{2(\alpha\beta+\alpha\gamma+\beta\gamma)}{\alpha\beta\gamma}=2\times\frac{"6"}{"-7/2"}$	M1	1.1b
	$=-\frac{24}{7}$ oee	A1ft	1.1b
(ii)	$(\alpha-1)(\beta-1)(\gamma-1)=(\alpha\beta-(\alpha+\beta)+1)(\gamma-1)=...$	M1	1.1b
	$=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+\alpha+\beta+\gamma-1$	A1	1.1b
	$=-9$	A1	1.1b
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=\left("\frac{3}{2}"\right)^2-2"6"$	M1	3.1a
	$=\frac{9}{4}-2(6)=-\frac{39}{4}$ oee	A1ft	1.1b
		(7)	
		(8 marks)	

Notes

(a)
B1: Correct values stated.
(b) Note question requires use of (a) so other methods will score no marks.
(i)
M1: Use a **correct** identity with an attempt to substitute their values into the correct places (allowing a slip) to find the value required. If identity is not shown it may be implied by the working.
A1ft: Correct value (follow through their part (a)) Note that this means the error.
$\Sigma\alpha_i=3, \Sigma\alpha_i\beta_j=12, \Pi\alpha_i=-7$ will score M1A1ft for the correct answer albeit from incorrect values.
(ii)
M1: Attempts to expand the product fully (allow sign slips and at most one incorrect or missing term).
A1: Correct expansion in terms of product, pair sum and sum - must be seen grouped or implied by substitution of values seen.
A1: Correct value.
(iii)
M1: Use a **correct** identity with an attempt to substitute their values into the correct places (allowing a slip) to find the value required. If identity is not shown it may be implied by the working.
A1ft: Correct value from correct working (follow through their part (a)). Note this means the error $\Sigma\alpha_i=-\frac{3}{2}$ will still give the A1ft here if correct identity is used.

Question	Scheme	Marks	AOs
3(a)	Max $r = 4 + a = 5.5 \Rightarrow a = ...$	M1	3.4
	$a = 1.5$	A1	2.2a
		(2)	
(b)	$\text{Pool area} = \frac{1}{2}\int_0^{2\pi} \left(4 - \text{"1.5"}\sin 3\theta\right)^2 \, d\theta$	M1	3.1a
	$\left(4 - \text{"1.5"}\sin 3\theta\right)^2 = 16 - \text{"12"}\sin 3\theta + \text{"2.25"}\sin^2 3\theta$ $= 16 - \text{"12"}\sin 3\theta + \text{"2.25"}\left(\frac{1 - \cos 6\theta}{2}\right)\left[= 16 - 8a\sin 3\theta + a^2\left(\frac{1 - \cos 6\theta}{2}\right)\right]$	M1	2.1
	$\int \left(4 - \text{"1.5"}\sin 3\theta\right)^2 \, d\theta = 16\theta + \text{"4"}\cos 3\theta + \text{"}\frac{9}{8}\text{"}\left(\theta - \frac{\sin 6\theta}{6}\right)$ $\left[= 16\theta + \frac{8a}{3}\cos 3\theta + \frac{a^2}{2}\left(\theta - \frac{\sin 6\theta}{6}\right)\right]$	A1ft	1.1b
	$\frac{1}{2}\left[\text{"}\frac{137}{8}\text{"}\theta + \text{"4"}\cos 3\theta - \text{"}\frac{3}{16}\text{"}\sin 6\theta\right]_0^{2\pi} = ...\left(\frac{137}{8}\pi\right)$	dM1	3.1a
	Area of $T = \pi \times 36 - \frac{137}{8}\pi$	DM1	1.1b
	$= \frac{151}{8}\pi \ \left(\text{m}^2\right)$ oe	A1	1.1b
		(6)	
			(8 marks)

Notes

(a)

M1: Uses all the information given for the model and realises the maximum value of r is $(4 + a)$ and uses the radius of the circle to find a value for a.

A1: Deduces the correct value of a. Note $a = -1.5$ can potentially gain M1A0.

(b)

Note accept with their a, a made up a or even a itself for the first 5 marks. Note use of $a = -1.5$ can score full marks.

M1: Adopts a correct strategy for the area of the pool. This requires the correct use of the polar area formula including the ½ .

Note the ½ may be implied by choice of limits (e.g. any span of π radians without the half implies an attempt at doubling so the ½ may not appear (even if the symmetry is incorrect).)

M1: Squares the bracket, achieving three terms, and applies $\sin^2 3\theta = \frac{\pm 1 \pm \cos 6\theta}{2}$ in order to reach an integrable form. Condone numerical slips when expanding.

A1ft: Correct integration in any form (½ not needed here) (follow through their a). ie as shown in scheme or if gathered it is $\left(32 + a^2\right)\frac{\theta}{2} + \frac{8a}{3}\cos 3\theta - a^2\frac{\sin 6\theta}{12}$

dM1: Depends on previous M. Uses appropriate limits for their integrated function. Allow limits other than 0 and 2π by using symmetry provided the correct multiple is used, e.g. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ followed by doubling (which may be cancelled with the $\frac{1}{2}$) or $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ and multiplying by 3

DM1: Depends on first M. Fully correct strategy for obtaining the area of *T*. Must have a correct attempt at the circle area (maybe be via integration) and area inside the curve. Symmetry may have been used.

A1: Correct area from fully correct work (all previous marks scored). Units not required. The decimal answer 18.875π is acceptable, but answer must be exact, not a rounded decimal.

Note: $\sin^2 3\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$ use will score a maximum of M1M0A0dM0DM1A0

Note: if $\frac{1}{2}\int_0^{2\pi} 6^2 - \left(4 - a\sin 3\theta\right)^2 \,\mathrm{d}\theta$ is used the scheme will follow as above with A1ft for $10\theta - \frac{8a}{6}\cos 3\theta - \frac{a^2}{4}\left(\theta - \frac{\sin 6\theta}{6}\right)$ and dM1DM1 gained together.

Note: Integrating over 0 to 2π means the trig terms will disappear so watch out for incorrect trig terms in the integration, which will lead to the correct answer but will lose both A marks.

Question	Scheme	Marks	AOs
4(a)	$z^n + \frac{1}{z^n} \equiv e^{in\theta} + \frac{1}{e^{in\theta}} \equiv e^{in\theta} + e^{-in\theta}$	M1	1.1b
	$\equiv \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta \equiv 2\cos n\theta$*	A1*	2.1
		(2)	
(b)	$\left(z + z^{-1}\right)^5 = 32\cos^5\theta$	B1	2.2a
	$\left(z + z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1 A1	1.1b 1.1b
	$32\cos^5\theta = \left(z^5 + z^{-5}\right) + 5\left(z^3 + z^{-3}\right) + 10\left(z + z^{-1}\right)$ $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1	2.1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$*	A1*	1.1b
		(5)	
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta \Rightarrow 16\cos^5\theta = -2\cos\theta$	B1	3.1a
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0 \Rightarrow \theta = \ldots$	M1	1.1b
	$8\cos^4\theta + 1 = 0$ has no solution so $\cos\theta = 0$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1	2.2a
		(3)	
		(10 marks)	

Notes

(a)

M1: Substitutes *z* into the LHS and simplifies the powers as shown. Allow if they go direct to trigonometric expressions without exponentials. The mark is for sorting out the negative index.

A1*: Converts the exponential form to trigonometric form correctly and correctly completes the proof with no errors seen. The trigonometric expansion must be clearly seen. Condone missing brackets in e.g $\cos - n\theta$ terms if intent is clear. Note the LHS of the identity may be implied.

(b)

B1: Deduces that $\left(z + z^{-1}\right)^5 = 32\cos^5\theta$ Do not accept 2^5 for 32. May be implied.

M1: Attempts to expand $\left(z + z^{-1}\right)^5$. Correct binomial coefficients must be used, terms need not be simplified. Condone at most one slip in powers.

A1: Correct expansion, terms need not be gathered but powers must have been simplified.

M1: Sets their expressions equal and applies the result from (a) – grouping must be shown.

A1*: Reaches the printed answer with no errors and relevant steps all shown.

(c)

B1: Uses the result from (b) to deduce the correct equation.

M1: Must have attempted to use part (b) to obtain $\alpha\cos^5\theta = \beta\cos\theta$ or equivalent. Collects to one side and attempts to factorise and solve. Note dividing through by cos θ is M0.

A1: Rejects the inappropriate solution and selects $\cos\theta = 0$ and obtains the correct values only. The equation must have been correct. There must have been some consideration of the $8\cos^4\theta + 1$ e.g. stating $8\cos^4\theta > 0$ so no solutions, or attempting to find complex roots and deducing no answers. May be minimal, but some consideration that no roots arise from this part must have been given.

Note: The correct answer will appear from incorrect attempts – the M must be gained in order to award the A. E.g. assuming the equation reduces to $\cos^5\theta = 0$ will score B0M0A0.

Likewise, answers only scores B0M0A0 (questions says hence so use of (b) must be seen).

Alt (a)	$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta + \frac{1}{\cos n\theta + \mathrm{i}\sin n\theta} = \frac{\cos^2 n\theta + 2\mathrm{i}\cos n\theta \sin n\theta - \sin^2 n\theta + 1}{\cos n\theta + \mathrm{i}\sin \theta}$	M1	1.1b
	$= \frac{2\cos^2 n\theta + 2\mathrm{i}\cos n\theta \sin n\theta}{\cos n\theta + \mathrm{i}\sin n\theta} = \frac{2\cos n\theta\left(\cos n\theta + \mathrm{i}\sin n\theta\right)}{\cos n\theta + \mathrm{i}\sin n\theta} = 2\cos n\theta$ *	A1*	2.1
		(2)	
Notes			
Alt: by De Moivre M1: Applies De Moivre on both terms and puts over a common denominator. A1*: Complete correctly, using $1 - \sin^2 n\theta = \cos^2 n\theta$ and cancelling $\cos n\theta + \mathrm{i}\sin n\theta$. No errors seen.			
Alt (b)	$\cos 5\theta = \mathrm{Re}\left(\cos 5\theta + \mathrm{i}\sin 5\theta\right) = \mathrm{Re}\left(\cos\theta + \mathrm{i}\sin\theta\right)^5$	B1	2.2a
	$\left(\cos\theta + \mathrm{i}\sin\theta\right)^5 = c^5 + 5\mathrm{i}c^4 s + 10\mathrm{i}^2 c^3 s^2 + 10\mathrm{i}^3 c^2 s^3 + 5\mathrm{i}^4 c s^4 + \mathrm{i}^5 s^5$	M1	1.1b
	$\mathrm{Re}\left(\cos\theta + \mathrm{i}\sin\theta\right)^5 = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1	1.1b
	$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\left(1 - \cos^2\theta\right) + 5\cos\theta\left(1 - \cos^2\theta\right)^2$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ $= 16\cos^5\theta - \frac{20}{4}\left(\cos 3\theta + 3\cos\theta\right) + 5\cos\theta$	M1	2.1
	$\Rightarrow \cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$*	A1*	1.1b
		(5)	
Notes			
Alt: by De Moivre B1: Correctly stated or clearly implied De Moivre statement for $\cos 5\theta$ M1: Attempts to expand $\left(\cos\theta + \mathrm{i}\sin\theta\right)^5$ Correct coefficients but allow one slip per main scheme. The powers of i need not be simplified for the attempt at expansion, accept if only the real terms are shown. Allow *c* and *s* notation. A1: Correct real terms extracted with the i's removed. M1: Applies $\sin^2\theta = 1 - \cos^2\theta$ to reduce to an equation in $\cos\theta$ **and** applies $\cos^3\theta = \frac{1}{4}\left(\cos 3\theta + 3\cos\theta\right)$ (quoted or derived – allow a slip if derived) to get to an equation without powers of cos terms. A1*: Reaches the printed answer with no errors and relevant steps all shown.			

Question	Scheme	Marks	AOs
5(a)	$(t+2)\frac{\mathrm{d}v}{\mathrm{d}t}+3v=k(t+2)-3 \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t}+\frac{3v}{t+2}=k-\frac{3}{t+2}$ $I=\mathrm{e}^{\int \frac{3}{t+2}\mathrm{d}t}=(t+2)^3$	M1	3.1b
	$\Rightarrow v(t+2)^3=\int\left(k(t+2)^3-3(t+2)^2\right)\mathrm{d}t=...$	M1	1.1b
	$v(t+2)^3=\frac{k}{4}(t+2)^4-(t+2)^3(+c)$ $\left[\text{Alt } (t+2)^3v=\frac{kt^4}{4}+2kt^3-t^3+6kt^2-6t^2+8kt-12t(+c)\right]$	A1	1.1b
	$t=0,\ v=0 \Rightarrow c=8-4k$	M1	3.4
	$v(t+2)^3=\frac{k}{4}(t+2)^4-(t+2)^3+8-4k$ $\Rightarrow v=\frac{k}{4}(t+2)-1+\frac{4(2-k)}{(t+2)^3}$ *	A1*	2.1
		(5)	
(b)	$v=4,\ t=2 \Rightarrow 4=k-1+\frac{4(2-k)}{64} \Rightarrow k=...(5.2)$	M1	3.1b
	$\Rightarrow v=\frac{"5.2"}{4}(5+2)-1+\frac{4(2-"5.2")}{(5+2)^3}=...$	dM1	3.4
	Velocity is $8.06\,\mathrm{ms}^{-1}$ (awrt)	A1	3.2a
		(3)	
(c)	E.g. • The model suggests the velocity increases indefinitely which is unlikely • The raindrop will reach the ground/doesn't fall forever so not valid • The raindrop will reach a terminal velocity after a finite time so not good model	B1	3.5a
		(1)	
			(9 marks)

Notes
(a) M1: Begins to solve the problem with a correct process to find the integrating factor for the model. Look for an attempt at $\mathrm{e}^{\int \frac{A}{t+2}\,\mathrm{d}t} = ...$ where *A* is a constant (positive or negative). For attempts at IF by recognition they must achieve form $A(t+2)^3$ M1: Attempts the solution of the differential equation using a correct method with their integrating factor. The d*t* may be implied. They must attempt the integration as part of the method, though it need not be correct – any changed function will do. So score for $v \times \text{their IF} = \text{their attempt at integrating their IF} \times \left(k - \frac{3}{t+2}\right)$ allowing for errors expanding this bracket. A1: Correct solution. The + *c* may be missing for this mark. M1: Interprets the initial conditions to find the constant of integration – must have a constant of integration for this mark and the correct initial conditions must be clear. A1*: Obtains the printed answer with no errors and at least one intermediate line between $v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 + c$ and the stated answer. Allow terms in different order, but the $8-4k$ must be factorised. (b) M1: Uses the given conditions to establish the value of *k* in the model dM1: Uses their value of *k* and the equation of the model with *t* = 5 to find a value for *v*. If a value of *k* has been found accept "*t* = 5 *v* =…" with any value for an attempt. A1: Correct value **including units** (allow awrt 8.06 ms^{-1}). Accept exact answer 5531/686 (c) B1: Evaluates the model by making a suitable comment – see scheme for examples. Accept answers relating to the size of velocity increasing beyond what is reasonable or answers that allude to the situation not being able to continue for ever. Accept if symbols are used, e.g. "as $t \to \infty, v \to \infty$ which cannot happen" but must include a comment on validity, not just a statement about what happens (e.g. $t \to \infty, v \to \infty$ with no comment). Do not accept e.g. "*v* does not tend to a value" as this is just a statement about what happens. If contrary information is given then award B0. Some examples of borderline answers seen. "Not valid as rain is limited to how fast its velocity is therefore can't be big." Scores B1 "Due to gravitational force and not huge height between cloud and ground, the model would be inaccurate for large period of time, because the velocity of raindrops would be a big number, which is unlikely." B1 "Not valid for very large value of *t* as raindrop will not move that fast." B0 "Raindrop has very little mass so effect due to gravity is lower. Unlikely to have a $1000\mathrm{ms}^{-1}$ raindrop. Not very valid model for large values of *t*" B1 "The water droplet wouldn't always follow the model in cases where it hits a surface" B0 "No valid as raindrop will hit ground soon. Speed can be infinitely big which is not plausible." B1 "Would not be valid as the raindrop likely has already hit the ground." B1 You may need to use your judgement on whether their comments are acceptable.

Question	Scheme	Marks	AOs
6	If $n=1$ $\sum_{r=1}^{n}(2r-1)^2=(2-1)^2=1$ and $\frac{1}{3}n(4n^2-1)=\frac{1}{3}(1)(4(1)^2-1)=1$ **(LHS=RHS)** so true for $n = 1$	B1	2.4
	(Assume true for $n = k$ so $\sum_{r=1}^{k}(2r-1)^2=\frac{1}{3}k(4k^2-1)$ then) $\sum_{r=1}^{k+1}(2r-1)^2=\frac{1}{3}k(4k^2-1)+\left(2(k+1)-1\right)^2$	M1	2.1
	E.g $=\frac{1}{3}(2k+1)(2k^2+5k+3)$ or $=\frac{4}{3}k^3-\frac{1}{3}k+4k^2+4k+1$	dM1	1.1b
	$\frac{1}{3}(k+1)(2k+3)(2k+1)$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{4k^3}{3}+4k^2+\frac{11k}{3}+1$	A1	1.1b
	$=\frac{1}{3}(k+1)(4(k+1)^2-1)$ Or see notes	A1	2.2a
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n.	A1	2.4
		(6)	

(6 marks)

Notes

B1: Demonstrates the statement is true for $n = 1$. Accept as minimum $(1)^2=1$ and $\frac{1}{3}(3)=1$ for the check (both sides must clearly be evaluated as 1), and for the conclusion "true for $n=1$" or broken "if/when $n=1$ <check> (hence) true/shown/tick" – must be part of the initial check (not just in the conclusion).
M1: Assume the result for $n = k$ and attempts to add the $(k + 1)^{th}$ term to the result for the sum to k terms. An assumption may be clearly made, but accept tacit assumptions. Allow if there are minor slips in the expressions if the intent is clear.
dM1: Makes progress towards proving the inductive step by either factorising the common factor $(2k+1)$ or expanding fully. There may be variations so score for equivalent progress.
A1: Either for a correct factorised form with $(k + 1)$ as a factor or a fully correct expansion with gathered terms. Must have come from correct work from the inductive step, not backwards worked from the $n = k + 1$ expression.
A1: Completes the inductive steps by obtaining the correct expression in terms of $(k + 1)$ with suitable intermediate step e.g. clear factorisation of $(k+1)$ first, OR by expanding the required expression for $n = k + 1$ to achieve equal expressions. Must have been completely correct work, no errors.
A1: Depends on the MMAA marks having been scored with at least an attempt to check one side of the $n = 1$ case having been made. Correct complete conclusion. Must include the notions of "true for $n = 1$", "true for $n = k$ implies true for $n = k+1$" and "hence true for all n" though the exact wording will vary. The conclusion should be given at the end with the exception that the "true for $n = 1$" may be stated with the initial check.
Note: stating "true for $n = k$ and $n = k + 1$" will be A0.
Note: Accept use of n instead of k for all except the final A mark.
Note: Attempts at using summation formulae without induction score no marks.

Question	Scheme	Marks	AOs
7(a)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(2\mathbf{i}+\mathbf{j}-4\mathbf{k})=6-10+4=0$ $(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=18-10-8=0$	M1	1.1b
	So $3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ is perpendicular to Π	A1	2.1
		(2)	
(b)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=3+20-3=20$	M1	1.1b
	$3x-10y-z=20$	A1	2.5
		(2)	
(c)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(5\mathbf{i}+p\mathbf{j}-7\mathbf{k})=\text{"}20\text{"}$ $\Rightarrow 15-10p+7=\text{"}20\text{"}\Rightarrow p=...$	M1	3.1a
	$p=0.2$ (oe)	A1	1.1b
		(2)	
(d)	E.g. $1+2\lambda=5+6\mu,\ 3-4\lambda=-7+8\mu\Rightarrow\lambda=...$ or $\mu=...$ $\mu=0.1\ (\text{or }\lambda=2.3)\Rightarrow A(5.6,\ 0.3,-6.2)$	M1	1.1b
	$12\mathbf{i}-11\mathbf{j}+6\mathbf{k}-(5.6\mathbf{i}+0.3\mathbf{j}-6.2\mathbf{k})=6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k}$ $(6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k})\cdot(3\mathbf{i}-10\mathbf{j}-\mathbf{k})=19.2+113-12.2=120$	M1	3.1a
	$120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\cos\alpha\Rightarrow\alpha=...$ or e.g. $120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\sin\alpha\Rightarrow\alpha=...$	M1	1.1b
	Angle between AB and plane $\theta=40°$ (awrt)	A1	1.1b
		(4)	
		(10 marks)	

Notes

(a)
M1: Attempts the scalar product between the given vector and the 2 direction vectors (calculation should be shown for at least one).
A1: Obtains zero for both with sufficient working shown and concludes perpendicular (each one or states both are). Accept normal to as equivalent to perpendicular to.
(b)
M1: Attempts the scalar product between the normal vector and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$. Allow if the vector product is used again from scratch. Allow equivalent approaches such as using the normal to form the equation $3x-10y-z=d$ and substituting (1, −2, 3) to find d. Maybe be implied by a correct equation if no incorrect working is seen.
A1: Correct equation in Cartesian form (allow equivalent Cartesian equations).
(c)
M1: Attempts the scalar product between the normal vector and $5\mathbf{i}+p\mathbf{j}-7\mathbf{k}$ (or any point on the line), sets = their 20 and solves a linear equation in p. Alternatively substitutes $x=5$, $y=p$ and $z=-7$ into their Cartesian equation and solves for p, or any other complete method to find p.
A1: For $p=0.2$ (oe)
(d)
M1: A complete method to find the coordinates of A. Allow if a slip in transcribing a coordinate occurs – it is a mark for the method. If the correct vectors are clearly intended, give credit.
M1: Attempts the vector AB and attempts the scalar product with this and the normal vector. Note that if the correct formulae are seen then allow the method for any value appearing afterwards.
M1: Complete method to find the required angle or its complement.
A1: For awrt 40°

Note: Accept alternative vector notations throughout. Where method is not shown for finding coordinates, it may be implied by two correct coordinates.			
Alt (a)	$(2\mathbf{i}+\mathbf{j}-4\mathbf{k})\times(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=(8+4)\mathbf{i}-(16+24)\mathbf{j}+(2-6)\mathbf{k}$	M1	1.1b
	$=12\mathbf{i}-40\mathbf{j}-4\mathbf{k}=4(3\mathbf{i}-10\mathbf{j}-\mathbf{k})$ So $3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ is perpendicular to Π	A1	2.1
		(2)	
Notes			
M1: Attempts the vector product between the direction vectors of the lines, allowing at most one slip in the brackets. A1: Obtains the correct vectors and shows it is a multiple of the required answer and concludes perpendicular.			
Alt II (a)	Normal to plane is $\mathbf{n}=\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k}\Rightarrow(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})\bullet(2\mathbf{i}+\mathbf{j}-4\mathbf{k})=0$ and $(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})\bullet(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=0$ So $2\alpha+\beta-4\gamma=0,\ 6\alpha+\beta+8\gamma=0$ $\alpha=3\Rightarrow\beta=4\gamma-6,\beta=-8\gamma-18\Rightarrow\beta=..,\gamma=..$	M1	1.1b
	$\gamma=-1,\beta=-10\Rightarrow 3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ perpendicular to Π	A1	2.1
		(2)	
Notes			
M1: Sets up a normal vector to the plane in terms of variables and attempts the vector product between the direction vectors of the lines and the normal to form simultaneous equations and attempts to solve by choosing one of the values and solving for the other two. A1: Correct values, and obtains the correct vector, or a multiple of it and shows it is a multiple of the required answer and concludes perpendicular.			

Alt (d)	E.g. $1+2\lambda=5+6\mu,\ 3-4\lambda=-7+8\mu\Rightarrow\lambda=...$ or $\mu=...$ $\mu=0.1$ (or $\lambda=2.3$) $\Rightarrow A(5.6,\ 0.3,-6.2)$	M1	1.1b
	$\overrightarrow{BX}=m\mathbf{n}\Rightarrow\overrightarrow{OX}=\overrightarrow{OB}+m\mathbf{n}=(12-3m)\mathbf{i}-(11+10m)\mathbf{j}+(6-m)\mathbf{k}$ $\Rightarrow 3(12-3m)-10(-11-10m)-(6-m)=20\Rightarrow m=...\left(-\frac{12}{11}\right)$ $\left[\Rightarrow X=\left(\frac{96}{11},-\frac{1}{11},\frac{78}{11}\right)\right]$	M1	3.1a
	$\sin\alpha=\frac{BX}{AB}=\frac{\frac{12}{11}\times\sqrt{3^2+10^2+1^2}}{\sqrt{6.4^2+11.3^2+12.2^2}}\Rightarrow\alpha=...$	M1	1.1b
	Angle between AB and plane $\theta=40°$ (awrt)	A1	1.1b
		(4)	
Notes			
M1: A complete method to find the coordinates of A. M1: Forms equation for $\overrightarrow{OX}=\overrightarrow{OB}+m\mathbf{n}$ and proceeds to find where normal to plane through B intersects the plane, or the distance BX ($=m\lvert\mathbf{n}\rvert$) M1: Complete method to find the required angle or its complement. May find X first to find BX etc, or may use different sides of the triangle. A1: For awrt 40°			

Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}+3y-2x$	M1	1.1b
	$=\frac{\mathrm{d}x}{\mathrm{d}t}+3\left(\frac{\mathrm{d}x}{\mathrm{d}t}-x\right)-2x$	M1	2.1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-4\frac{\mathrm{d}x}{\mathrm{d}t}+5x=0$*	A1*	1.1b
		(3)	
(b)	$m^2-4m+5=0\Rightarrow m=...$	M1	3.4
	$m=2\pm\mathrm{i}$	A1	1.1b
	$x=\mathrm{e}^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)$	M1	3.4
	$x=\mathrm{e}^{2t}\left(A\cos t+B\sin t\right)$	A1	1.1b
		(4)	
(c)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}-x=\mathrm{e}^{2t}\left(B\cos t-A\sin t+2A\cos t+2B\sin t\right)-\mathrm{e}^{2t}\left(A\cos t+B\sin t\right)$	M1	3.4
	$y=\mathrm{e}^{2t}\left((A+B)\cos t+(B-A)\sin t\right)$	A1	1.1b
		(2)	
(d)	$A=100,\ 275=A+B\Rightarrow B=175$	M1	3.3
	$x=y\Rightarrow 100\cos t+175\sin t=275\cos t+75\sin t\Rightarrow\tan t=...$	dM1	3.1a
	$\tan t=1.75$	A1	1.1b
	$T=24\tan^{-1}(1.75)=...$	M1	3.2a
	= 25.24	A1	1.1b
		(5)	
(e)	E.g. • Both populations become negative for some times which is not possible	B1	3.5b
		(1)	
			(15 marks)

Notes

Note: Accept the dot notation $\dot{x}$ etc, as equivalents for the derivatives wrt *t* throughout.

(a)

M1: Differentiates the first equation and substitutes for $\frac{\mathrm{d}y}{\mathrm{d}t}$ from the second equation.

M1: Proceeds to an equation in *x* and $\frac{\mathrm{d}x}{\mathrm{d}t}$ only by substituting for *y*.

A1*: Achieves the printed answer with no errors.

(b)

M1: Uses the model to form and solve the Auxiliary Equation.

A1: Correct roots of the AE.

M1: Uses the model to form a Complementary Function appropriate for their roots. Accept complex index form.

A1: Correct General Solution. Accept complex index form. Must include the *x*

(c)
M1: Uses the model and their answer to part (b) to give y in terms of t. Must involve an attempt at the product rule. Alternatively, they may repeat the whole process of parts (a) and (b) again on y – score for a full process leading to the solution for y in terms of t only.
A1: Correct simplified equation though may be with expanded brackets but like terms should be gathered (ie the $A\cos t$ and $B\sin t$ terms). If starting over, allow if names of constants are the same as for (b) (ie treat independently) but they will lose later marks. Accept complex index form.
(d)
M1: Realises the need to use the initial conditions to establish the values of their constants. But must have consistent constants (ie four different constants if they did part (c) from scratch). Solutions with complex indices are unlikely to achieve this mark. If unsure, use the Review system to consult your team leader.
dM1: Sets $x = y$ and collects and reaches $\tan t = \ldots$. Alternatively may convert into sine or cosine and reach an equivalent equation.
A1: Correct equation – accept equivalent if other trig approaches are used. Must come from correct work. Note $\sin t = \frac{7}{\sqrt{65}}$ and $\cos t = \frac{4}{\sqrt{65}}$ are the other ratios.
M1: Solves their $\tan t = \ldots$ and multiplies by 24. Must be working in radians. If degrees mode is used score M0.
A1: Correct value. Ignore units.
(e)
B1: **Must have at least one correct general equation for x or y for this mark to be awarded.**
Rote answers about generic models without evidence to support any claims should not be given credit.
Suggests a suitable correct limitation of the model. This **must** focus on the fact the equations produce negative values. Award if a correct answer is given and ignore extra comments.
E.g. "gives negative values" would be a minimal acceptable answer.

Note: Answers about unlimited growth that imply the populations increase indefinitely or about scarcity of resource **without** an accompanying correct statement are B0.

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