Answer ALL questions. Write your answers in the spaces provided.

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k.

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

1. The curve C has equation

$$y = 31\sinh x - 2\sinh 2x \qquad x \in \mathbb{R}$$

Determine, in terms of natural logarithms, the exact x coordinates of the stationary points of C.

(7)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0$$

Give your answers in the form $q\ln 2$ where q is rational and in simplest form.

(4)