Question	Scheme	Marks	AOs
12(a)(i)	$y\times\frac{\mathrm{d}x}{\mathrm{d}t}=5\sin 2t\times 6\cos t$ or $5\times 2\sin t\cos t\times 6\cos t$	M1	1.2
	(Area =) $\int 5\sin 2t\times 6\cos t\,\mathrm{d}t=\int 5\times 2\sin t\cos t\times 6\cos t\,\mathrm{d}t$ or $\int 5\sin 2t\times 6\cos t\,\mathrm{d}t=\int 60\sin t\cos^2 t\,\mathrm{d}t$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,\mathrm{d}t$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 60\sin t\cos^2 t\,\mathrm{d}t=-20\cos^3 t$	M1 A1	1.1b 1.1b
	Area $=\left[-20\cos^3 t\right]_0^{\frac{\pi}{2}}=0-(-20)=20$ *	A1*	2.1
		(3)	
(b)	$5\sin 2t=4.2\Rightarrow\sin 2t=\frac{4.2}{5}$	M1	3.4
	$t=0.4986...,1.072...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t=0.4986...\Rightarrow x=2.869...$ $t=1.072\Rightarrow x=5.269...$	A1	1.1b
	Width of path = 2.40 metres	A1	3.2a
		(5)	
			(11 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dx}{dt}=\frac{1}{t+2}$, Area of R = $\int y\,dx=\int \frac{4}{t^2}\times\frac{1}{(t+2)}(dt)$	B1, M1
	Correct proof with limits and no errors Area = $\int_1^3 \frac{4}{t^2(t+2)}dt$	A1*
		(3)
(b)	$\frac{4}{t^2(t+2)}=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{(t+2)}$ or $\frac{4}{t^2(t+2)}=\frac{A}{t^2}+\frac{B}{(t+2)}$	B1
	$4=At(t+2)+B(t+2)+Ct^2$	
	Sub $t=0\Rightarrow B=2$	
	Sub $t=-2\Rightarrow C=1$	
	Compare t^2 $A+C=0\Rightarrow A=-1$	M1A1
	$\int_1^3 \frac{4}{t^2(t+2)}dt=\int_1^3 \frac{-1}{t}+\frac{2}{t^2}+\frac{1}{(t+2)}dt=\left[-\ln t-\frac{2}{t}+\ln(t+2)\right]_1^3$	M1A1
	$=\left(-\ln 3-\frac{2}{3}+\ln 5\right)-\left(-\ln 1-\frac{2}{1}+\ln 3\right)$	
	$=\ln\left(\frac{5}{9}\right)+\frac{4}{3}$	dM1A1
		(7)
(c)	Sub $t=e^x-2$ into $y=\frac{4}{t^2}\Rightarrow y=\frac{4}{\left(e^x-2\right)^2}$, $(x>\ln 2)$	M1A1
		(2)
		(12 marks)

(a)

B1 States or implies $\frac{dx}{dt}=\frac{1}{t+2}$. Accept $dx=\frac{1}{t+2}dt$

You may award this if embedded within an integral **before the final answer** is given

For example accept Area = $\int y\,dx=\int_1^3 \frac{4}{t^2}\times\frac{1}{t+2}dt$

Question Number	Scheme		Marks
12.(a)	$V = \int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = \int (2\sin 2t)^2 3\cos t \,\mathrm{d}t$ M1: Attempts $\int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t$ where $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm k\cos t$ May be implied by e.g. $\int (2\sin 2t)^2 3\cos t$ A1: $= \int (2\sin 2t)^2 3\cos t\,(\mathrm{d}t)$ (dt can be missing as long as the M is scored)		M1A1
	$= \int (4\sin t\cos t)^2 3\cos t \,\mathrm{d}t$	Uses sin2t = 2sin$t$cos$t$	M1
	$x = \frac{3}{2} \Rightarrow t = \frac{\pi}{6}$ or $k = 48$	Correct value for a (must be exact) or a correct value for k	B1
	$V = \int \pi y^2 \mathrm{d}x = 48\pi \int_0^{\frac{\pi}{6}} \sin^2 t\cos^3 t \,\mathrm{d}t$ *	Achieves printed answer including "dt" (even if lost earlier) with correct limits and 48π in place with no errors. Or achieves the printed answer with the letters a and k and states the correct values of a and k.	A1*
			(5)

(b)	$u=\sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t}=\cos t$	States $\frac{\mathrm{d}u}{\mathrm{d}t}=\cos t$ or equivalent. May be implied.	B1
	$V=k\int \sin^2 t\cos^3 t\,\mathrm{d}t=k\int u^2\cos^2 t\,\mathrm{d}u=k\int u^2(1-\sin^2 t)\,\mathrm{d}u=k\int u^2(1-u^2)\,\mathrm{d}u$ M1: Substitutes **fully** including for dt using $u=\sin t$ and $\cos^2 t=\pm1\pm\sin^2 t$ to produce an integral just in terms of u. A1ft: Fully correct integral in terms of u - follow through on incorrect k's and ignore inclusion or omission of π so look for e.g. $k\int u^2(1-u^2)\,\mathrm{d}u$ or equivalent and allow the letter k.		M1A1ft
	$=k\left[\frac{u^3}{3}-\frac{u^5}{5}\right]$	Multiplies out to form a polynomial in u and integrates with $u^n\to u^{n+1}$ for at least one of their powers of u.	M1
	Volume = $48\pi\left[\frac{u^3}{3}-\frac{u^5}{5}\right]_0^{\frac{1}{2}}=\frac{17\pi}{10}$	**d**M1: All methods must have been scored. It is for using the limits 0 and $\frac{1}{2}$ and subtracting or for using the limits 0 and $\frac{\pi}{6}$ if they return to sin t. However, in both cases the substitution of 0 does not need not be seen. A1: $V=\frac{17\pi}{10}$ oe such as $V=\frac{51\pi}{30}$	**d**M1A1
			(6)
	If $\frac{\mathrm{d}u}{\mathrm{d}t}=-\cos t$ is used, maximum B0M1A0M1M1A0 is possible		
			(11 marks)

Question Number	Scheme	Marks
14 (a)	$(1, 4.5)$	B1B1
		(2)
(b)	Attempts $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{12\sin\theta\cos\theta}{-24\cos^2\theta\sin\theta} = \left(-\frac{1}{2\cos\theta}\right)$	M1A1
	Subs $\theta = \frac{\pi}{3}$ into $\frac{dy}{dx} = (-1)$	M1
	Uses gradient of normal with $(1, 4.5) \Rightarrow (y - 4.5) = 1(x-1)$	ddM1
	$y = x + 3.5$	A1*
		(5)
(c)	Attempts $\int y \frac{dx}{d\theta} d\theta = \int 6\sin^2\theta \times -24\cos^2\theta\sin\theta \, d\theta$	M1A1
	Uses $\sin^2\theta = 1 - \cos^2\theta \Rightarrow \int y\frac{dx}{d\theta} d\theta = \int A\left(\cos^2\theta\sin\theta - \cos^4\theta\sin\theta\right) d\theta$	dM1
	Area of trapezium= $\frac{1}{2}(3.5+4.5) = 4$	B1
	Attempts trapezium + area under curve $= \frac{1}{2}(3.5+4.5) - 144\int_{\frac{\pi}{3}}^{0} \sin^3\theta\cos^2\theta \, d\theta$	ddM1
	$Area = 4 + 144\int_0^{\frac{\pi}{3}} \left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right) d\theta$	A1*
		(6)
(d)	Area of S $= 4 + 144\left[-\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{3}} = 4 + 144\left(\left(-\frac{1}{24} + \frac{1}{160}\right) - \left(-\frac{1}{3} + \frac{1}{5}\right)\right)$	M1A1
	$= \frac{181}{10}$	A1
		(3)
		(16 marks)

(a)

B1: Either of $(1, 4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…(May be seen on the diagram)

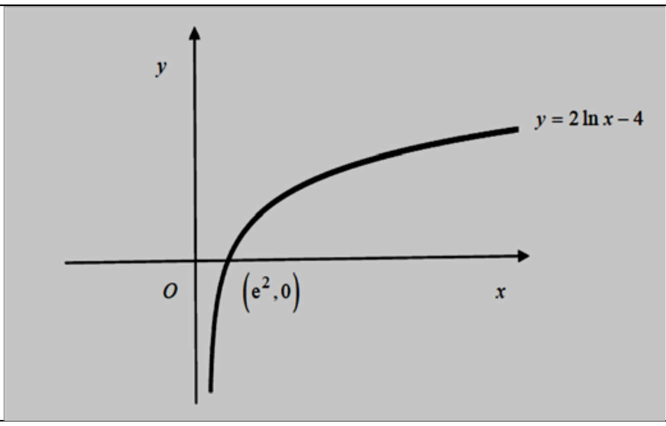
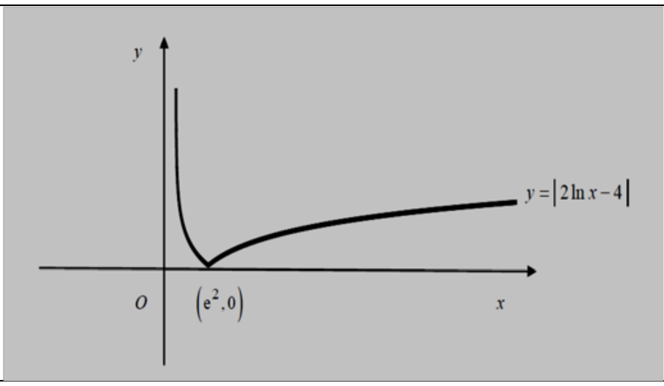
B1: Both $(1, 4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…

(b)

M1: Attempts $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ (Allow poor differentiation on y and/or x provided the functions are both "changed")

A1: $\frac{dy}{dx} = -\frac{12\sin\theta\cos\theta}{24\cos^2\theta\sin\theta} = \left(-\frac{1}{2\cos\theta}\right)$

M1: Subs $\theta = \frac{\pi}{3}$ into their $\frac{dy}{dx} = (-1)$

Question Number	Scheme		Marks
9(a)(i)		Shape	B1
		$(e^2, 0)$	B1
		Asymptote $x = 0$	B1
			(3)
(a)(ii)		Shape	B1ft
		Asymptote and coordinate	B1ft
			(2)
(b)	$2\ln x - 4 = 4 \Rightarrow \ln x = 4 \Rightarrow x = e^4$		M1A1
	$2\ln x - 4 = -4 \Rightarrow \ln x = 0 \Rightarrow x = 1$		M1A1
			(4)
(c)	$\text{gf}(x) = e^{2\ln x - 4 + 5} - 2, = e^1 \times e^{2\ln x} - 2 = ex^2 - 2$		M1,dM1A1
			(3)
(d)	$\text{gf}(x) > -2$		B1
			(1)
			(13 marks)

(a)(i)

B1: For a logarithmic shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1: Intersection with the x axis at $(e^2, 0)$.

Allow e^2 marked on the x axis. Condone $(0, e^2)$ being marked on the positive x axis.

Do not allow e^2 appearing as 7.39 for this mark unless e^2 is seen in the body of the script.
Allow if the coordinate is given in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then the ones on the curve take precedence.

B1: **Equation** of asymptote is $x = 0$ (do not allow " y-axis"). Note that the curve must appear to have an asymptote at $x = 0$

(a)(ii)

B1ft: For either the correct shape or a reflection of their "negative" curve in (a) in the x-axis. For this to be scored it must have appeared both above and below the x-axis. The curve to the lhs of the intercept must appear to have the correct curvature

B1ft: Score for the correct coordinates and asymptote. Alternatively follow through on the coordinates and asymptote given in part (a) as long as the curve appeared both above and below the x-axis and the curve approaches the same asymptote stated in (a)(i). Do not penalise "y-axis" given as the asymptote twice – i.e. penalise in (a)(i) only.

If the curves are sketched on the same axes – it must be clear which curve is which – if in doubt use review.

Question Number	Scheme	Notes	Marks
8.	$x = 3\theta\sin\theta,\ y = \sec^3\theta,\quad 0 \le \theta < \frac{\pi}{2}$		
(a)	$\{\text{When } y=8,\}\ 8 = \sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	Sets $y=8$ to find θ **and** attempts to substitute their θ into $x = 3\theta\sin\theta$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		**[2]**
(b)	$\frac{dx}{d\theta} = 3\sin\theta + 3\theta\cos\theta$	$3\theta\sin\theta \to 3\sin\theta + 3\theta\cos\theta$ Can be implied by later working	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\} = \int (\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{d\theta\}$	Applies $(\pm K\sec^3\theta)\left(\text{their } \frac{dx}{d\theta}\right)$ Ignore integral sign and $d\theta$; $K \ne 0$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\ d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. **Must have** integral sign and $d\theta$ in their final answer.	A1 *
	$x=0$ and $x=k \Rightarrow \underline{\alpha=0}$ and $\underline{\beta=\frac{\pi}{3}}$	$\alpha=0$ and $\beta=\frac{\pi}{3}$ or evidence of $0\to0$ and $k\to\frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		**[4]**
(c) **Way 1**	$\left\{\int \theta\sec^2\theta d\theta\right\} = \theta\tan\theta - \int\tan\theta\{d\theta\}$	$\theta\sec^2\theta \to A\theta g(\theta) - B\int g(\theta),\ A>0,\ B>0,$ where $g(\theta)$ is a trigonometric function in θ **and** $g(\theta) = \text{their} \int\sec^2\theta d\theta$. [Note: $g(\theta) \ne \sec^2\theta$]	M1
		dependent on the previous M mark **Either** $\lambda\theta\sec^2\theta \to A\theta\tan\theta - B\int\tan\theta,\ A>0,\ B>0$ **or** $\theta\sec^2\theta \to \theta\tan\theta - \int\tan\theta$	dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec\theta)$ or $\theta\tan\theta + \ln(\cos\theta)$ **or** $\lambda\theta\sec^2\theta \to \lambda\theta\tan\theta - \lambda\ln(\sec\theta)$ or $\lambda\theta\tan\theta + \lambda\ln(\cos\theta)$	A1
	Note: Condone $\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec x)$ or $\theta\tan\theta + \ln(\cos x)$ for A1		
	$\left\{\int\tan\theta\sec^2\theta d\theta\right\}$	$\tan\theta\sec^2\theta$ or $\lambda\tan\theta\sec^2\theta \to \pm C\tan^2\theta$ or $\pm C\sec^2\theta$ or $\pm Cu^{-2}$, where $u = \cos\theta$	M1
	$= \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u=\cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	$\tan\theta\sec^2\theta \to \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2\cos^2\theta}$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ or $0.5u^{-2}$, where $u=\cos\theta$ or $0.5u^2$, where $u=\tan\theta$ or $\lambda\tan\theta\sec^2\theta \to \frac{\lambda}{2}\tan^2\theta$ or $\frac{\lambda}{2}\sec^2\theta$ or $\frac{\lambda}{2\cos^2\theta}$ or $0.5\lambda u^{-2}$, where $u=\cos\theta$ or $0.5\lambda u^2$, where $u=\tan\theta$	A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$		
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$		
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$		A1 o.e.
			[6]
			12

Question Number	Scheme	Notes	Marks
8. (c) **Way 2**	**Way 2 for the first 5 marks:** Applying integration by parts on $\int(\theta + \tan\theta)\sec^2\theta\,\mathrm{d}\theta$		
	$\int(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta = \int(\theta + \tan\theta)\sec^2\theta\,\mathrm{d}\theta,$	$\left\{ u = \theta + \tan\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = 1 + \sec^2\theta \quad \frac{\mathrm{d}v}{\mathrm{d}\theta} = \sec^2\theta \Rightarrow v = \tan\theta = \mathrm{g}(\theta) \right\}$	
	$\mathrm{h}(\theta)$ and $\mathrm{g}(\theta)$ are trigonometric functions in θ **and** $\mathrm{g}(\theta) =$ their $\int\sec^2\theta\,\mathrm{d}\theta$. [**Note:** $\mathrm{g}(\theta) \neq \sec^2\theta$]		
	$= (\theta + \tan\theta)\tan\theta - \int(1 + \sec^2\theta)\tan\theta\{\mathrm{d}\theta\}$	$A(\theta + \tan\theta)\mathrm{g}(\theta) - B\int(1 + \mathrm{h}(\theta))\mathrm{g}(\theta),\ A > 0,\ B > 0$	M1
		dependent on the previous M mark **Either** $\pm\left[(\theta + \tan\theta)\sec^2\theta\right] \rightarrow$ $A(\theta + \tan\theta)\tan\theta - B\int(1 + \mathrm{h}(\theta))\tan\theta,\ A \neq 0,\ B > 0$ **or** $(\theta + \tan\theta)\tan\theta - \int(1 + \mathrm{h}(\theta))\tan\theta$	dM1
	$= (\theta + \tan\theta)\tan\theta - \int(\tan\theta + \tan\theta\sec^2\theta)\{\mathrm{d}\theta\}$		
	$= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \int\tan\theta\sec^2\theta\{\mathrm{d}\theta\}$	$(\theta + \tan\theta)\tan\theta - \ln(\sec\theta)$ o.e. or $\pm\left[(\theta + \tan\theta)\tan\theta - \ln(\sec\theta)\right]$ o.e.	A1
	$= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \frac{1}{2}\tan^2\theta$	$\tan\theta\sec^2\theta \rightarrow \pm C\tan^2\theta$ or $\pm C\sec^2\theta$	M1
	or $= (\theta + \tan\theta)\tan\theta - \ln(\sec\theta) - \frac{1}{2}\sec^2\theta$ etc.	$(\theta + \tan\theta)\tan\theta - \frac{1}{2}\tan^2\theta$ or $(\theta + \tan\theta)\tan\theta - \frac{1}{2}\sec^2\theta$	A1
	Note	Allow the first two marks in part (c) for $\theta\tan\theta - \int\tan\theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta\tan\theta - \ln(\sec\theta)$ embedded in their working	
	Note	Allow 3rd M1 2nd A1 marks for either $\tan^2\theta - \frac{1}{2}\tan^2\theta$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ embedded in their working	
	Question 8 Notes		
8. (a)	**Note**	Allow M1 for an answer of $k =$ awrt 2.72 without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	Allow M1 for an answer of $k = 3\left(\arccos(\tfrac{1}{2})\right)\sin\left(\arccos(\tfrac{1}{2})\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	
	Note	E.g. allow M1 for $\theta = 60°$, leading to $k = 3(60)\sin(60)$ or $k = 90\sqrt{3}$	

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \; \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{array}{ll} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

Question Number	Scheme		Marks
5. (b)	**Alternative Method 1 of Equating Coefficients**		
	$y = \dfrac{x^2 + ax + b}{x - 3} \Rightarrow y(x-3) = x^2 + ax + b$ $y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$		
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $24 + 4a = 32 \Rightarrow a = 2$ constant: $9 + 3a + b = 10 \Rightarrow b = -5$	Equates their coefficients in t and finds both $a = \ldots$ and $b = \ldots$	dM1
		$a = 2$ and $b = -5$	A1
			[3]
5. (b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\} y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3} \Rightarrow y = x + 5 + \dfrac{10}{(x-3)}$ $\underline{\underline{y(x-3)}} = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = \underline{\underline{(x+5)(x-3) + 10}}$		dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x - 3}$ **or** equating coefficients to give $a = 2$ and $b = -5$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1 cso
			[3]

Question Number	Scheme		Marks
7.	$x = 3\tan\theta,\quad y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta,\quad 0 \leqslant \theta < \frac{\pi}{2}.$		
(a)	$\frac{dx}{d\theta} = 3\sec^2\theta,\quad \frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}\quad \left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$	M1
		Correct $\frac{dy}{dx}$	A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\quad \left\{= -\frac{2}{3}\right\}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$	M1
	So, $m(\mathbf{N}) = \frac{3}{2}$	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	Either **N:** $y - 2 = \text{"}\frac{3}{2}\text{"}(x - 3)$ **or** $2 = \left(\text{"}\frac{3}{2}\text{"}\right)(3) + c$	**see notes**	M1
	$\{$At Q, $y = 0$, so, $-2 = \frac{3}{2}(x - 3)\}$ **giving** $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso**
			[6]
(b)	$\left\{\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta\right\} = \left\{\int\right\}(4\cos^2\theta)^2 3\sec^2\theta\ \{d\theta\}$	**see notes**	M1
	So, $\pi\int y^2 dx = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\{d\theta\}$	**see notes**	A1
	$\int y^2 dx = \int 48\cos^2\theta\, d\theta$	$\int 48\cos^2\theta\{d\theta\}$	A1
	$= \{48\}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta\quad \left\{= \int(24 + 24\cos 2\theta)d\theta\right\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$	M1
	$= \{48\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\quad \{= 24\theta + 12\sin 2\theta\}$	**Dependent on the first method mark.** For $\pm\alpha\theta \pm \beta\sin 2\theta$	dM1
		$\cos^2\theta \to \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{= 48\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}\right\} = \{48\}\left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0 + 0)\right)\quad \{= 6\pi + 12\}$	**Dependent on the third method mark.**	dM1
	$\{$So $V = \pi\int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi\}$		
	$V_{\text{cone}} = \frac{1}{3}\pi(2)^2\left(3 - \frac{5}{3}\right)\quad \left\{= \frac{16\pi}{9}\right\}$	$V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their }(a))$	M1
	$\left\{\text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9}\right\} \Rightarrow \text{Vol}(S) = \frac{92}{9}\pi + 6\pi^2$	$\frac{92}{9}\pi + 6\pi^2$	A1
		$\left\{p = \frac{92}{9},\ q = 6\right\}$	**[9]**
			15

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Question Number	Scheme		Marks
5.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \rightarrow \frac{2^t}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \rightarrow \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \rightarrow \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**

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5. (a)	**M1:** Applies $x = 0$ and obtains a value of t.
	A1: For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$
	Alternative Solution 1:
	M1: For substituting $t = 2$ into either x or y.
	A1: $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$
	Alternative Solution 2:
	M1: Applies $y = 3$ and obtains a value of t.
	A1: For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.
	Alternative Solution 3:
	M1: Applies $y = 3$ or $x = 0$ and obtains a value of t.
	A1: Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b).
	A1: For finding $x = 1$.
	Note: Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working.
	M1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** their $\frac{dy}{dt}$ must be a function of t.
	M1: Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$.
	M1: $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent.
	A1: $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here.
	Working in decimals is ok for the three method marks. B1, A1 require exact values.
(d)	**M1:** Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1).\left(\text{their } \frac{dx}{dt}\right)$
	B1: Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1.
	M1*: Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$
	... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$.
	A1: Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$.
	dM1*: Depends upon the previous method mark. Substitutes their limits in t and subtracts either way round.
	A1: Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.