Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Mechanics M2 (6678/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

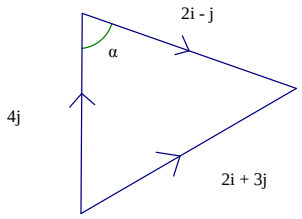
  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Q.	Scheme	Marks	Notes
1a	$\mathbf{I} = 0.5(\mathbf{2i} + 3\mathbf{j}) - 0.5(4\mathbf{j})$	M1	Impulse-momentum equation. Dimensionally correct. Condone subtraction in wrong order.
	$(= 0.5(2\mathbf{i} - \mathbf{j}))$	A1	Correct unsimplified
	$\lvert\mathbf{I}\rvert = \frac{1}{2}\sqrt{2^2 + 1^2}$	M1	Correct method for modulus. Follow their **I**
	$= \frac{1}{2}\sqrt{5}\,(= 1.12)\,\text{Ns}$	A1	1.1 or better (from correct solution only)
		(4)	
1b	$\tan^{-1}(\pm 2)$ or $\tan^{-1}\left(\pm\frac{1}{2}\right)$ or $\tan\theta = \pm 2$ or $\tan\theta = \pm\frac{1}{2}$ or equivalent	M1	Correct method for a relevant angle. Follow their **I**
	Required angle = 117° $(116.6°$ or better$)$	A1	Accept 243° (2.03 rads)
		(2)	
1balt			
	$\cos\alpha = \frac{16 + 5 - 13}{2\sqrt{5}\sqrt{16}} = \frac{1}{\sqrt{5}}$	M1	
	Required angle = 117° $(116.6°)$	A1	Accept 243°
		(2)	
		[6]	

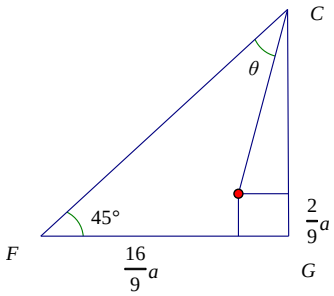
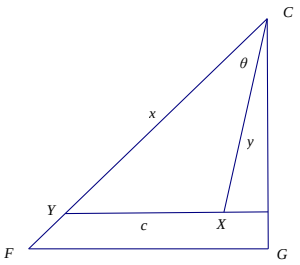
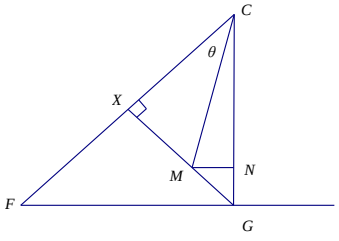
Q.	Scheme	Marks	Notes
2a	Constant speed $\Rightarrow$ no acceleration. Driving force $= 200 + 50 + 900g\sin\theta + 150g\sin\theta$	M1	Equation of motion of the truck. All terms required & dimensionally correct. Condone sin/cos confusion and sign error(s)
	0r $D - T - 200 - 900g\sin\theta = 0$ **and** $T - 50 - 150g\sin\theta = 0$		
		A1	At most one error Allow for 2 separate equations including T
		A1	Correct unsimplified expression for the driving force (no T)
	$= 250 + 1050g \times \frac{1}{9} \left(= 1393.3333...\right)$		$\left(\frac{4180}{3}\right)$
	$P = \left(250 + 1050g \times \frac{1}{9}\right) \times 15$	M1	Use of $P = Fv$ with their F Independent M1. Could appear in first equation as $F = \frac{P}{v}$.
	$= 20900$ W $(20.9$ kW$)$	A1	Accept 21000 W, 21kW. Maximum 3 s.f.
		(5)	
2b	$\left(their 1393\frac{1}{3}\right) - 200 - 900g \times \frac{1}{9} = 900a$	M1	Equation of motion for the truck at instant after the towbar breaks. All terms required & dimensionally correct. Allow for an equation to find acceleration down the slope
		A1ft	Correct for their driving force $\left(1393\frac{1}{3}\right)$.
	$a = 0.237$ m s^{-2}	A1	Accept 0.24 , not $\frac{32}{135}$ must be +ve
		(3)	
2c	$\frac{1}{2} \times 150 \times 15^2 = 50d + 150g\sin\theta d$	M1	**Must be using work-energy (for trailer only)** All terms required & dimensionally correct. Condone sin/cos confusion and sign error(s)
		A1	Unsimplified equation with at most one error
	$\left(16875 = 50d + \frac{150}{9}gd\right)$	A1	Correct unsimplified equation for d
	$d = 79$ m (79.1)	A1	Maximum 3 s.f.
		(4)	
		[12]	

<table>
<tr><th>Q.</th><th>Scheme</th><th>Marks</th><th>Notes</th></tr>
<tr><td>3a</td><td>
<table>
<tr><td></td><td>FGDE</td><td>2 of CFG</td><td>CBAG</td><td>L</td></tr>
<tr><td>Mass ratio</td><td>4</td><td>4</td><td>4</td><td>12</td></tr>
<tr><td>C of M from EF</td><td>a</td><td>$\frac{4}{3}a$</td><td>$3a$</td><td>d</td></tr>
</table>
</td><td>B1

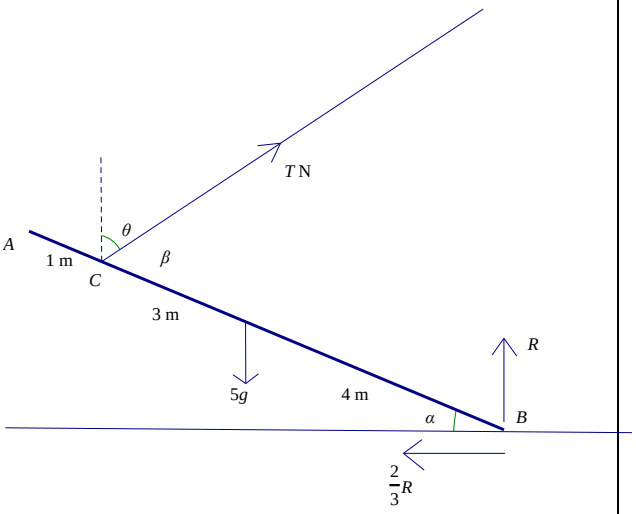
B1</td><td>Mass ratios

Distances from EF or an alternative vertical axis</td></tr>
<tr><td></td><td>$12d = 4a + 4\times\frac{4}{3}a + 4\times 3a$</td><td>M1</td><td>Moments about EF or equivalent
Need all terms and dimensionally correct</td></tr>
<tr><td></td><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>$12d = 16a\times\frac{4}{3},\ \ d = \frac{16}{9}a$</td><td>A1</td><td>Sufficient working to justify *given answer*</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>3a alt</td><td>Splitting the rectangle into a pair of trapeziums gives mass ratios 1: 1: 2</td><td>B1</td><td></td></tr>
<tr><td></td><td>Distance of c of m of ABCF is
$\frac{8}{9}a$ from AF and $\frac{22}{9}a$ from EF</td><td>B1</td><td></td></tr>
<tr><td></td><td>$2a - a\times\frac{8}{9} + a\times\frac{22}{9}\left(=\frac{32}{9}a\right) = 2d$</td><td>M1A1</td><td></td></tr>
<tr><td></td><td>$d = \frac{16}{9}a$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>3a alt</td><td>Square -square -triangle+triangle
<table>
<tr><td></td><td>L sq</td><td>S sq</td><td>-tri</td><td>+tri</td><td>L</td></tr>
<tr><td>mass</td><td>4</td><td>1</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>3</td></tr>
<tr><td>From EF</td><td>2a</td><td>3a</td><td>$\frac{2a}{3}$</td><td>$\frac{4a}{3}$</td><td>d</td></tr>
</table>
</td><td>B1

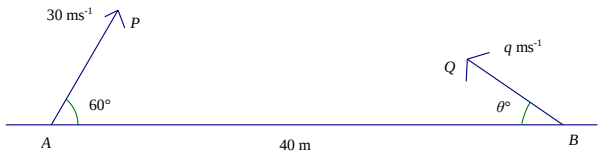
B1</td><td></td></tr>
<tr><td></td><td>$4\times 2a - 3a - \frac{1}{2}\times\frac{2a}{3} + \frac{1}{2}\times\frac{4a}{3}\left(=\frac{16a}{3}\right) = 3d$</td><td>M1A1</td><td></td></tr>
<tr><td></td><td>$d = \frac{16}{9}a$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
</table>

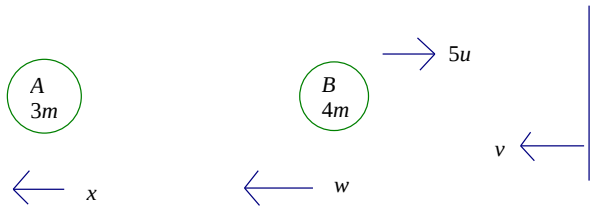
Q.	Scheme	Marks	Notes
3b			
	Symmetry $\Rightarrow$ c of m $\frac{16}{9}a$ from C	B1	For vertical distance – allow for $\frac{20}{9}a$ or equivalent
	$\tan^{-1}\frac{1}{8}$ $\left(\tan^{-1}8\right)$	M1	Correct trig to find relevant angle (using $\frac{2}{9}a$ horizontally and their vertical $\neq 2a$)
	$7.125°$	A1	$\left(7.1°, 82.9°, 0.124\text{rads}, 1.45\text{rads}\right)$
	$\theta = 37.9°$	A1 (4)	$38°$ or better $\left(37.874....°,\ 0.66\text{ rads}\right)$
3b alt			Using cosine rule With $x = \frac{16}{9}\sqrt{2}a$, $c = \frac{14}{9}a$ $y = \sqrt{65}\times\frac{2a}{9}$
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	
	$\cos\theta = \frac{x^2+y^2-c^2}{2xy} = \frac{9}{\sqrt{130}} = 0.789...$	M1A1	
	$\theta = 37.9°$	A1 (4)	
3balt			
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	i.e. on bisector of angle G
	$\tan\theta = \frac{MX}{CX} = \frac{\sqrt{2}a - \frac{2\sqrt{2}}{9}a}{\sqrt{2}a} = \frac{7}{9}$	M1A1	Using Isosceles triangles
	$\theta = 37.9°$	A1 (4)	
		[9]	

Q.	Scheme	Marks	Notes
4a	$v=0 \Rightarrow 3t^2-16t+21=0$	M1	Set $v=0$ and attempt to solve
	$\left((3t-7)(t-3)=0\right) \quad t_1=\frac{7}{3}, \quad t_2=3$	A1	
		(2)	
4b	$a=\frac{\mathrm{d}}{\mathrm{d}t}\left(3t^2-16t+21\right)$	M1	Differentiate *v* to obtain *a*
	$=6t-16$	A1	
	$t=t_1, \quad a=6\times\frac{7}{3}-16=-2$ (m s^{-2}) Magnitude 2 (m s^{-2})	A1	No errors seen. Must be positive - the Q asks for magnitude.
		(3)	
4c	$s=\int\left(3t^2-16t+21\right)\mathrm{d}t$	M1	Integrate *v* to find *s*
	$=t^3-8t^2+21t\,(+C)$	A1	
	$\pm\left(\left(3^3-8\times9+21\times3\right)-\left(\left(\frac{7}{3}\right)^3-8\times\frac{49}{9}+21\times\frac{7}{3}\right)\right)$	M1	Correct use of their limits
	$s=0.148$ (m) $\quad\left(\frac{4}{27}\right)$	A1	Final answer must be positive. 0.15 or better
		(4)	
4d	Return to $O \Rightarrow s=0=t\left(t^2-8t+21\right)$	B1	seen or implied
	Discriminant of quadratic $=64-4\times21\left(=-20\right)<0$	M1	Or equivalent. ***given answer so must show some evidence of method***
	No real roots $\Rightarrow$ does not return to *O*	A1	Sufficient correct working to justify ***given answer***
		(3)	
4dalt	Travels away until $t_1=\frac{7}{3}$, turns back at $t_2=3$ then turns away again	M1	Complete story
	$s_3=18$	B1	Seen or implied
	Complete argument	A1	
		(3)	
4dalt	Distance time graph	B1	
	Locate min turning point	M1	
	Complete argument	A1	
		(3)	
		[12]	

Q.	Scheme	Marks	Notes
5a			
	$F = \frac{2}{3}R$ seen or implied	B1	Use of $F = \mu R$. Could be on diagram. Allow in (b) if not seen before
	$\text{M}(C): 5g \times 3\cos\alpha + F \times 7\sin\alpha = 7\cos\alpha \times R$	M1	Moments about C or alternative complete method to find equation in F and R or R only. Dimensionally correct and all terms needed. Condone sin/cos confusion and sign error(s).
		A1	At most one error
		A1	Correct unsimplified equation
	$15g\cos\alpha = R\left(7\cos\alpha - \frac{14}{3}\sin\alpha\right)$		
	$15g \times \frac{4}{5} = R\left(7 \times \frac{4}{5} - \frac{14}{3} \times \frac{3}{5}\right) = \frac{14}{5}R$	dM1	Substitute for F and trig and solve for R Dependent on previous M1
	$R = \frac{30}{7}g = 42\,(\text{N})$	A1	
		(6)	
	e.g. of alternative for M1A1A1:		
	$\text{M}(A): T\sin\beta + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$ **and** $\text{M}(B)$: $7T\sin\beta = 20g\cos\alpha$	(M1)	
		(A1)	At most 1 error
	$\frac{20g}{7}\cos\alpha + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$	(A1)	Correct unsimplified equation in F and R or R only

Q.	Scheme	Marks	Notes
5b	Resolve $\updownarrow$: $T\cos\theta + R = 5g$ $R + T\sin(\beta - \alpha) = 5g$	M1	Need all terms. Condone sin/cos confusion and sign error(s).
		A1	Correct in R or *their R*
	Resolve $\leftrightarrow$: $T\sin\theta = F\ (=28)$ $F\left(=\frac{2}{3}R\right) = T\cos(\beta - \alpha)$	M1	Need both terms. Condone sin/cos confusion
		A1	Correct in R or *their R*
	Solve simultaneous equations for $\beta - \alpha$		
	$\tan(\beta - \alpha) = 4$, $\beta = 50.9°$ $(51°)$	A1	cso . Max 3 s.f.
		(5)	
Alt 5b	M(B): $7 \times T\sin\beta = 5g\cos\alpha \times 4$	M1	Moments equation. Dimensionally correct. Condone sin/cos confusion and sign error(s).
	$\left(T\sin\beta = \frac{16}{7}g\right)$	A1	
	OR: resolve perpendicular to the rod: $T\sin\beta + R\cos\alpha = 5g\cos\alpha + \frac{2}{3}R\sin\alpha$	(M1) (A1)	
	Resolve parallel to rod: $T\cos\beta + 5g\sin\alpha = F\cos\alpha + R\sin\alpha$ $\left(=\frac{2}{3}R\cos\alpha + R\sin\alpha\right)$	M1	All terms needed. Condone sin/cos confusion and sign error(s).
	$\left(T\cos\beta = \frac{13}{7}g\right)$	A1	
	Solve simultaneous equations for β		
	$\tan\beta = \frac{16}{13}$, $\beta = 50.9°$ $(51°)$	A1	cso. Max 3 s.f.
		(5)	
		[11]	

Q.	Scheme	Marks	Notes
6a			
	$30\cos 60\times 2+q\cos\theta\times 2=40$	M1	Equation for horizontal distance Need to be using the 40 m
		A1	Correct unsimplified
	$30\sin 60\times 2-4.9\times 4=q\sin\theta\times 2-4.9\times 4$ $30\sin 60=q\sin\theta$	M1	Equal vertical distance or initial vertical components of velocity
		A1	Correct unsimplified (no error seen)
	$q\cos\theta=\pm 5$ $q\sin\theta=15\sqrt{3}$		
	$\tan\theta=3\sqrt{3}$ $(\tan\theta=6\sin 60)$	DM1	Solve for q or θ Dependent on both preceding M marks
	$\theta=79.1$ (79)		(1.38 radians) or better
	$q=26.45...=26.5$	A1	(26 or better) $(10\sqrt{7})$ Both correct and no error seen
		(6)	
6b	Vertical component of speed =	M1	Must be working towards speed of P (or v^2) (condone if working on Q - they equal vertical components of velocity)
	$30\sin 60-2g\,(=6.38...)$	A1	Correct unsimplified. Accept ±
	speed $=\sqrt{(30\cos 60)^2+6.38^2}$	DM1	Use Pythagoras. Dependent on previous M Follow their vertical component.
		A1ft	Correct unsimplified equation in v or v^2.
	$=\sqrt{15^2+6.38^2}=16.3\,(\text{m s}^{-1})$	A1	or 16 2 or 3 sf only
		(5)	
6b alt	Vertical distance =	M1	Must be working towards speed of P
	$30\sin 60\times 2-4.9\times 4=32.36$	A1	Correct unsimplified
	Conservation of energy:	DM1	Dependent on previous M. Follow their vertical distance.
	$\frac{1}{2}mv^2+mg\times 32.36=\frac{1}{2}m\times 900$	A1ft	Correct unsimplified equation in v or v^2.
	$v=16.3$ (m s^{-1}) (16)	A1	
		(5)	
		[11]	

Q.	Scheme	Marks	Notes
7a			
	Impact with wall: $v=\frac{3}{5}\times 5u=3u$	B1	or $-3u$
	Impulse $\pm 4m\left(3u-\left(-5u\right)\right)$	M1	M0 if clearly using $mv+mu$, otherwise bod
	Magnitude $32mu$ (Ns)	A1	
		(3)	
7b	CLM: $3mx+4mw=4m\times 3u$	M1	Need all 4 terms. Condone sign errors. Use of $5u$ is M0
		A1ft	follow their $3u$
	Impact: $x-w=e\times 3u$	M1	Used the right way round. Use of $5u$ is M0
		A1ft	follow their $3u$ signs consistent with CLM equation
	$3m\left(w+3eu\right)+4mw=7mw+9emu=12mu$		
	$7w=u\left(12-9e\right)$	DM1	Solve for w or kw. Dependent on two preceding M marks
	Use of $e\leq 1$ in their w: $7w\geq 3u$	M1	Condone use of $<$
	Hence $w>0$ and A and B are moving in the same direction	A1 (7)	Complete argument leading to ***given answer***
7c	KE of B before collision $=\frac{1}{2}\times 4m\times\left(3u\right)^2\left(=18mu^2\right)$	B1	follow their $3u$. seen or implied
	$\Rightarrow\frac{1}{2}\times 4m\left(\frac{u}{7}\left(12-9e\right)\right)^2=\frac{1}{4}\left(\frac{1}{2}\times 4m\times 9u^2\right)$	M1	Follow their w. $\frac{1}{4}$ on the right side.
	$4\left(12-9e\right)^2=49\times 9,\ \left(4-3e\right)^2=\frac{49}{4}$	A1	Correct equation in m, u and e
	$e=\frac{1}{6}$	A1 (4)	
7c alt	KE of B before collision $=\frac{1}{2}\times 4m\times\left(3u\right)^2\left(=18mu^2\right)$	B1	follow their $3u$
	$\Rightarrow\frac{1}{2}4mw^2=\frac{1}{4}\times\frac{1}{2}\times 4m\left(3u\right)^2\qquad\left(w=\frac{1}{2}\times 3u\right)$	M1	$\frac{1}{4}$ on the right side.
	$\frac{3}{7}\left(4-3e\right)=\frac{1}{2}\times 3$	A1	Correct equation in m, u and e from correct work only
	$e=\frac{1}{6}$	A1 (4)	0.17 or better from correct work only
		[14]	