Question	Scheme	Marks	AOs
14 (a)	$f(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + ...$	M1	1.1b
	$= -3(x-2)^2 + ...$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) dx$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
		(10 marks)	
Alt(c)	$\int 3x^2 - 12x + 12 \, dx = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12 \, dx$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + ...$
Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + ...$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

Question	Scheme	Marks	AOs
9	$\int_k^9 \frac{6}{\sqrt{x}}\,dx = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36-12\sqrt{k}=20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36-12\sqrt{k}=20 \Rightarrow k=$	dM1	3.1a
	$\Rightarrow k=\frac{16}{9}$ oe	A1	1.1b
		(4)	
		(4 marks)	

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36-12\sqrt{k}=20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}}<0$

A1: $k=\frac{16}{9}$

Question	Scheme	Marks	AOs
3	$\int \frac{3x^4 - 4}{2x^3}\,\mathrm{d}x = \int \frac{3}{2}x - 2x^{-3}\,\mathrm{d}x$	M1 A1	1.1b 1.1b
	$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{-2} \quad (+\,c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e	A1	1.1b
		(4)	
			(4 marks)

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,\mathrm{d}x$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,\mathrm{d}x$ o.e such as $\frac{1}{2}\int \left(3x - 4x^{-3}\right)\,\mathrm{d}x$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

AS Mathematics

Paper 8MA0 01 June 2018 Mark Scheme

Question	Scheme	Marks	AOs
1	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x = \frac{2}{3}\times\frac{x^4}{4} + ... + x$	A1	1.1b
	$= ... - 6\frac{x^{\frac{3}{2}}}{3/2} + ...$	A1	1.1b
	$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n \to x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power**' terms (may be un-simplified at this stage)

A1: Correct '**fractional power**' term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line. Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6} - 4x\sqrt{x} + 1x^1 + c$

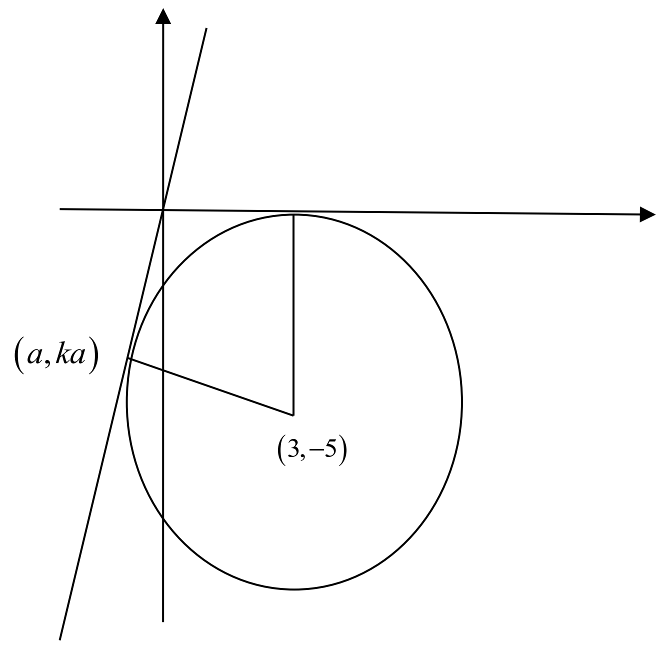
Accept $\frac{x^4 - 24x^{\frac{3}{2}} + 6x}{6} + c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$

A1: Deduces $k<0, k>\frac{15}{8}$. This must be in terms of *k*. Allow exact equivalents such as $k<0 \cup k>1.875$ but not allow $0>k>\frac{15}{8}$ or the above with AND, & or $\cap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$



Alt (b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Distance from (a,ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2)=9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k\times\frac{ka+5}{a-3}=-1 \Rightarrow a(1+k^2)=3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k=\frac{15}{8}$	A1	1.1b
	Deduces $k<0, k>\frac{15}{8}$	A1	2.2a
		(6)	

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the *x*-axis. Key points that must be seen are • Attempt at differentiation	M1	3.1a

	• Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis		
	$y=\frac{32}{x^2}+3x-8\Rightarrow\frac{dy}{dx}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6=\text{"}-\frac{1}{2}\text{"}(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6=\text{"}-\frac{1}{2}\text{"}(x-4)\Rightarrow x=...$ or an attempt using just gradients $\text{"}-\frac{1}{2}\text{"}=\frac{6}{a-4}\Rightarrow a=...$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{\text{"}16\text{"}}\text{"}\left(-\frac{1}{2}x+8\right)\text{"}dx$	M1	3.1a
	$\int\frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area $=10+36=46$ *	A1*	2.1
		(10)	
			(10 marks)

(a)

The first 5 marks are for finding the normal to the curve cuts the x - axis

M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme

M1: Differentiates with at least one index reduced by one

A1: $\frac{dy}{dx}=-\frac{64}{x^3}+3$

dM1: Method of finding

either the equation of the normal at (4, 6) .

or where the equation of the normal at (4, 6) cuts the x - axis

See scheme. It is dependent upon having gained the M mark for differentiation.

Question	Scheme	Marks	AOs
3(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{4}{x^3} + kx\right) dx = -\frac{2}{x^2} + \frac{1}{2}kx^2 + c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2} + \frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2} + \frac{1}{2}k \times 4\right) - \left(-\frac{2}{(0.5)^2} + \frac{1}{2}k \times (0.5)^2\right) = 8$	M1	1.1b
	$7.5 + \frac{15}{8}k = 8 \Rightarrow k = ...$	dM1	1.1b
	$k = \frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone " unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).

Accept $4 \times \frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.

Ignore spurious notation e.g. answer appearing with an $\int$ sign or with dx on the end.

Accept $-\frac{2}{x^2} + \frac{1}{2}kx^2 + c$ or exact simplified equivalent such as $-2x^{-2} + k\frac{x^2}{2} + c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2} + \frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k = \frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n} = \frac{4}{15}$
Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

	Scheme	Marks	AOs
13.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{dy}{dx} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = ...$	M1	1.1b
	Chooses $x = 4$ ~~$x = \frac{5}{3}$~~	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	
			(7 marks)

Notes

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.
If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x - \frac{5}{3}\right)$

A1: Chooses $x = 4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**
The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0.
So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.
Expect to see embedded or calculated values.
Don't accept $\int_0^4 2x^3 - 17x^2 + 40x \, dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

Question	Scheme	Marks	AOs
7 (a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9}, \cancel{9}$	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with + c and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

Question	Scheme	Marks	AOs
10 (a)	$g(5) = 2\times5^3 + 5^2 - 41\times5 - 70 = ...$	M1	1.1a
	$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3 + x^2 - 41x - 70 = (x-5)(2x^2 ... x \pm 14)$	M1	1.1b
	$= (x-5)(2x^2 + 11x + 14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x)) = (x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3} - \frac{190}{3}$	M1	2.2a
	Area $= 571\frac{2}{3}$	A1	2.1
		(4)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0

Look for evidence of embedded values or two correct terms of $g(5) = 250 + 25 - 205 - 70 = ...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$

$g(5) = 0 \Rightarrow (x-5)$ is a factor ✓

Do not allow if candidate states

$f(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**

$g(x) = 0 \Rightarrow (x-5)$ is a factor **(It is not g(x) and there is no conclusion)**

This may be seen in a preamble before finding $g(5) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2 + 11x + 14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2 + 11x + 14)$

Question	Scheme	Marks	AOs
5	$f(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)dx = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times 2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes:

B1: Correct function with numerical powers

M1: Allow for raising power by one. $x^n \to x^{n+1}$

A1: Correct three terms

M1: Substitutes limits and rationalises denominator

A1*: Completely correct, no errors seen

Question Number	Scheme	Marks
10. (a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
	Either: Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$ Or: Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$	A1
	Either: So turning point (all correct work so far) Or: Deduce $x = 1$ from correct work	A1cso (3)
(b) Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle $ABP = \frac{1}{2}\times 1\times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)\,dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x\ \{+c\}$ *or* $x^4 + 3x^3 - 15x^2 - 8x\ \{+c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{1} = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ $= (-19) - \frac{261}{256}$ or $-19 - 1.02$	dM1
	So Area $=$ "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + " 20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	$= 32.52$ (NOT -32.52)	A1 (7) **[10]**
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2 For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the -25	B1
	For second mark: Integrating to find triangle area $\int_1^2 (25x - 50)\,dx = \left[\frac{25}{2}x^2 - 50x\right]_1^2 = -50 + 37.5 = -12.5$ so area is 12.5 Then mark as before if they use Method in original scheme	B1
(b) Way 2	**Way 2:** Those who use area for **original curve between -1/4 and 2** and subtract area **between line and curve between 1 and 2** have a correct (long) method . The first B1 (if y=−25 is not seen) is for equation of straight line $y = 25x - 50$	B1
	The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5	B1
	$\int (4x^3 + 9x^2 - 55x + 42)\,dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\ \{+c\}$ (or integration as in Way 1)	M1A1
	The dM1 is for correct use of the different correct limits for each of the two areas: i.e. $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2} = (16 + 24 - 60 - 16) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ **And** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2 = 16 + 24 - 110 + 84 - (1 + 3 - 27.5 + 42)$	dM1
	So Area = their $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2}$ **minus their** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2$ i.e. "*their* 37.0195" − "*their* 4.5" (with both sets of limits correct for the integral)	ddM1
	Reaching $= 32.52$ (NOT -32.52) See over for special case with wrong limits	A1

Question Number	Scheme		Marks
7. (a)	$\left\{\int\left(3x - x^{\frac{3}{2}}\right)dx\right\} = \frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\{+c\}$	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$	M1
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
			[3]
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$	Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	M1
	$\left\{\text{Area}(S) = \left[\frac{3x^2}{2} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$		
	$= \left(\frac{3(9)^2}{2} - \left(\frac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$	Applies the limit 9 on an integrated function with **no wrong lower limit** .	ddM1
	$\left\{= \left(\frac{243}{2} - \frac{486}{5}\right) - \{0\}\right\} = \frac{243}{10}$ or 24.3	$\frac{243}{10}$ or 24.3	A1 oe
			[3]
			6

Question 7 Notes

(a)	**M1**	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$
	1st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.
	2nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the n+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+ c$". Ignore integral signs in their answer.
(b)	**1st M1**	Sets $y = 0$, and reaches the **correct** $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$) Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$. Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit **9** substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.
	A1	$\frac{243}{10}$ or 24.3
	Common Error	**Common Error** $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ *so* $x = \sqrt{3}$ **Then** uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3

Question Number	Scheme	Marks
3.(a)	$\frac{x^3+4}{2x^2}=\frac{x^3}{2x^2}+\frac{4}{2x^2}=\frac{1}{2}x+2x^{-2}$	M1A1A1 **[3]**
(b)	$\int\frac{x^3+4}{2x^2}\,dx=\int\frac{1}{2}x+2x^{-2}\,dx=\frac{1}{4}x^2-2x^{-1}+c$	M1A1A1 **[3]** **(6 marks)**

(a)

M1 For an attempt to divide by $2x^2$. It may be implied if either index or either coefficient is correct.

A1 One correct term. Either $\frac{1}{2}x$ or $+2x^{-2}$. Allow $\frac{1}{2}x^1=0.5x$ or, for this mark only, $+2x^{-2}=+\frac{2}{x^2}$

A1 $\frac{1}{2}x+2x^{-2}$ or $0.5x+2x^{-2}$ Accept $x^1=x$ A final answer of $\frac{1}{2}x+\frac{2}{x^2}$ is M1 A1 A0

(b)

M1 Raises any of the indices by one for their Ax^p+Bx^q

A1 One term both correct and simplified. Accept either $\frac{1}{4}x^2\,/\,0.25x^2$ or $-2x^{-1}\,/-\frac{2}{x}\,/-\frac{2}{x^1}$

A1 $\frac{1}{4}x^2-2x^{-1}+c$ including the +c. Accept equivalents such as $0.25x^2-\frac{2}{x^1}+c$ or $\frac{x^3-8}{4x}+c$

Do not accept expressions like $\frac{1}{4}x^2+-2x^{-1}+c$

Question Number	Scheme	Marks
15. (a)(i)	18	B1
(ii)	3	B1
		[2]
(b)	$\dfrac{(x-3)^2(x+4)}{2} = '18'$	M1
	$(x^2-6x+9)(x+4)=36$	
	$\Rightarrow x^3-2x^2-15x+36=36$	dM1
	$\Rightarrow x^3-2x^2-15x=0 \Rightarrow x^2-2x-15=0$	A1*
		[3]
(c)	$x=5$	B1
	$y=\dfrac{(5-3)^2(5+4)}{2} \Rightarrow (5,18)$	M1A1
		[3]
(d)	**Method 1** $\int\left(\frac{1}{2}x^3-x^2-\frac{15}{2}x+18\right)dx=\frac{1}{8}x^4-\frac{1}{3}x^3-\frac{15}{4}x^2+18x$ — **Method 2** OR $\int\left(-\frac{1}{2}x^3+x^2+\frac{15}{2}x\right)dx=-\frac{1}{8}x^4+\frac{1}{3}x^3+\frac{15}{4}x^2$	M1A1
	Method 1: Uses their 5 as the upper limit (and subtracts 0) to obtain an area — Method 2: Uses their 5 as the upper limit (and subtracts 0) to obtain area	M1
	Method 1: Area of rectangle = 90 — Method 2: Implied by correct answer $57\frac{7}{24}$	B1
	Method 1: Use = Area of rectangle – Area beneath curve — Method 2: Implied by subtraction in the integration	dM1
	Method 1: $=90-32\frac{17}{24}=57\frac{7}{24}$ $\left(\frac{1375}{24}\right)$ — Method 2: $=57\frac{7}{24}$ $\left(\frac{1375}{24}\right)$	A1cso
		[6]
		(14 marks)

Question Number	Scheme	Marks
8 (a)	$\int (3x^2+4x-15)\,dx = x^3+2x^2-15x+c$	M1A1A1
		(3)
(b)	$\int_b^4 (3x^2+4x-15)\,dx = \left[x^3+2x^2-15x+(c)\right]_b^4 = 36$	
	$(64+32-60)-(b^3+2b^2-15b)=36$	M1
	$b^3+2b^2-15b=0$	A1* **(2)**
(c)	$b=0$	B1
	$b^2+2b-15=0 \Rightarrow (b+5)(b-3)=0$	M1
	$b=-5, 3$	A1
		(3)
		(8 marks)

(a)
M1 Raises the index of any term in x by one
A1 Two of the three algebraic terms correct (unsimplified). For example accept $2x^2 = \frac{4}{2}x^{1+1}$
A1 cao including the +c

(b)
M1 Substitutes 4 and b into their integrated expression, subtracts either way around and sets equal to 36
A1* Simplifies to the given solution. This is a given answer and therefore the intermediate line(s) must be correct. Minimum expectation for an intermediate line is $36-(b^3+2b^2-15b)=36$ or equivalent with the bracket removed.

(c)
B1 $b=0$
M1 Factorises/ attempts to solve the quadratic
A1 $b=-5, 3$

Question Number	Scheme	Marks
12(a)	$y = x^3 - 9x^2 + 26x - 18 \Rightarrow \frac{dy}{dx} = 3x^2 - 18x + 26$	M1A1
	At $x = 4 \Rightarrow \frac{dy}{dx} = 3\times4^2 - 18\times4 + 26 (= 2)$	M1
	Equation of normal is $y - 6 = -\frac{1}{2}(x-4) \Rightarrow 2y + x = 16$	dM1A1*
		(5)
(b)	Sub $x = 1$ in $(y) = 1 - 9 + 26 - 18 = 0$	B1*
		(1)
(c)	$\int x^3 - 9x^2 + 26x - 18 dx = \left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x\right]$	M1A1
	Normal meets x axis at $x = 16$	B1
	Area of triangle $= \frac{1}{2}\times(16-4)\times6 = (36)$	M1
	Correct method for area = $\left[\frac{1}{4}x^4 - 3x^3 + 13x^2 - 18x\right]_1^4 + \frac{1}{2}\times(16-4)\times6$	dM1
	$= 51.75$	A1
		(6)
		(12 marks)

(a)

M1 Two of the three terms correct (may be unsimplified).

A1 $\left(\frac{dy}{dx} =\right) 3x^2 - 18x + 26$, need not be simplified. You may not see the $\frac{dy}{dx}$

M1 Substitutes $x = 4$ into their $\frac{dy}{dx}$

dM1 The candidate must have scored both M's. It is for the correct method of finding the equation of a normal. Look for $y - 6 = -\left.\frac{dx}{dy}\right|_{x=4} \times (x-4)$

If the form $y = mx + c$ is used it is for proceding as far as $c = ..$

A1* cso $2y + x = 16$ Note that this is a given answer. $x + 2y = 16$ is ok

(b)

B1* Either substitute $x = 1$ in $(y) = 1^3 - 9\times1^2 + 26\times1 - 18 = 0$ or $(y) = 1 - 9 + 26 - 18 = 0$

Or substitute $y = 0$ reach $(x-1)(x^2 - 8x + 18)$ by inspection or division and state $x = 1$

Question Number	Scheme		Marks
7 (i)	$\frac{2+4x^3}{x^2}=\frac{2}{x^2}+4x=2x^{-2}+4x$	Attempts to split the fraction. This can be awarded for $\frac{2}{x^2}$ or $\frac{4x^3}{x^2}$ or **may** be implied by the sight of one correct index e.g px^{-2} or qx **providing one of these terms is obtained correctly. So for example** $\frac{2+4x^3}{x^2}=2+4x^3+x^{-2}$ would be M0 as the x^{-2} **has been obtained incorrectly.**	M1
	$\int 2x^{-2}+4x\,dx=2\times\frac{x^{-1}}{-1}+4\times\frac{x^2}{2}(+c)$	dM1: $x^n\to x^{n+1}$ on any term. **Dependent** on the first M. A1: At least one term correct, simplified or un-simplified. Allow powers **and** coefficients to be un-simplified e.g. $2\times\frac{x^{-2+1}}{-1}$, $+4\times\frac{x^{1+1}}{2}$	dM1A1
	$=-\frac{2}{x}+2x^2+c$	All correct and simplified including the $+c$. Accept equivalents such as $-2x^{-1}+2x^2+c$	A1
			(4)
	There are no marks in (ii) for use of the trapezium rule – must use integration		
(ii)	$\int\left(\frac{4}{\sqrt{x}}+k\right)dx$ $=\int(4x^{-0.5}+k)dx=4\frac{x^{0.5}}{0.5}+kx(+c)$	M1: Integrates to obtain either $\alpha x^{0.5}$ or kx A1: Correct integration (simplified or un-simplified). Allow powers **and** coefficients to be un-simplified e.g. $4\frac{x^{-0.5+1}}{0.5}$. There is no need for $+c$	M1A1
	$\left[4\frac{x^{0.5}}{0.5}+kx\right]_2^4=30\Rightarrow(8\sqrt{4}+4k)-(8\sqrt{2}+2k)=30$ Substitutes both $x=4$ and $x=2$ into **<u>changed</u>** expression involving k, subtracts either way round and sets equal to 30 Condone poor use or omission of brackets when subtracting.		M1
	$2k+16-8\sqrt{2}=30\Rightarrow k=7+4\sqrt{2}$	**dd**M1: Attempts to solve for k from a **linear** equation in k. **Dependent upon both M's and need to have seen** $\int k\,dx\to kx$. A1: $7+4\sqrt{2}$ or exact equivalent e.g. $7+2^{2.5}$, $7+4\times2^{0.5}$	**dd**M1A1
			(5)
			(9 marks)

Question Number	Scheme		Marks
12(a)	$y=\frac{3}{4}x^2-4\sqrt{x}+7 \Rightarrow \frac{dy}{dx}=\frac{3}{2}x-2x^{-0.5}$	M1: Differentiates to obtain at least one correct power for one of the terms in x. (may be un-simplified) e.g. $x^2 \to x^{2-1}$ or $\sqrt{x} \to x^{\frac{1}{2}-1}$ A1: Correct derivative. Allow un-simplified e.g. $2\times\frac{3}{4}x^{2-1}$ or $-4\times\frac{1}{2}x^{\frac{1}{2}-1}$	M1A1
	At $x=4$ $\frac{dy}{dx}=\frac{3}{2}(4)-2(4)^{-0.5}=...$	Substitutes $x = 4$ into a changed function in an attempt to find the gradient.	M1
	$y-11="5"(x-4)$ or $y=mx+c \Rightarrow 11="5"\times 4+c \Rightarrow c=...$	Correct straight line method using (4, 11) correctly placed and their dy/dx at x = 4 for the tangent **not the normal**. If using $y = mx + c$, must reach as far as finding a value for c. **Dependent on the previous M.**	**d**M1
	$y=5x-9$	Correct printed equation with no errors seen. **Beware of the "5" appearing from wrong working.**	A1*
	<u>Important Note:</u> **Following a correct derivative, if candidate states x = 4 so dy/dx = 5, this is fine if they then complete correctly – allow full marks.** **However, following a correct derivative, if the candidate <u>just</u> states dy/dx = 5 and then proceeds to obtain the correct straight line equation, the final mark can be withheld. Some evidence is needed that the candidate is considering the gradient at x = 4.**		
			(5)

For part (b), in all cases, look to apply the appropriate scheme that gives the candidate the best mark

	Finds area under curve between 1 and 4 and subtracts triangle C (see diagram at end)		
(b) Way 1	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7 \, dx = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3} \times \frac{3}{4}x^{2+1}$, $-\frac{2}{3} \times 4x^{0.5+1}$, $7x^1$ and $+ c$ is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle $= \frac{1}{2} \times (4 - '1.8') \times 11 = (12.1)$ Correct method for the area of a triangle - look for $\frac{1}{2} \times (4 - '1.8') \times 11$ This may be implied by the evaluation of $\int_{'1.8'}^{4} 5x - 9 \, dx = \left[5\frac{x^2}{2} - 9x\right]_{'1.8'}^{4}$		M1
	Correct method for area = Area A + Area B + Area C – Area C $\left(\frac{1}{4}4^3 - \frac{8}{3} \times 4^{1.5} + 7 \times 4\right) - \left(\frac{1}{4}1^3 - \frac{8}{3} \times 1^{1.5} + 7 \times 1\right) - '12.1'$ Correct combination of areas. **Dependent on both previous method marks.**		**dd**M1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)
			(11 marks)

	Finds area under curve between 1 and "1.8" and adds "line – curve" or "curve – line" between "1.8" and 4		
(b) Way 2	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7 \, dx = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3}\times\frac{3}{4}x^{2+1}$, $-\frac{2}{3}\times 4x^{0.5+1}$, $7x^1$ and $+c$ is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be seen on a diagram.	B1
	Area between "1.8" and 4 = $\pm\int_{'1.8'}^{4}\left(\frac{3}{4}x^2 - 4\sqrt{x} + 7\right) - (5x-9)\, dx = \pm\left[\frac{1}{4}x^3 - \frac{8}{3}x^{1.5} - \frac{5x^2}{2} + 16x\right]_{'1.8'}^{4}$ $= \frac{56}{3} - 15.7182...(= 2.9485...)$ Attempts to integrate "curve – line" or "line – curve", substitute the limits "1.8" and 4 and subtracts.		M1
	Correct method for area = Area A + Area B $\left(\left(\frac{1}{4}"1.8"^3 - \frac{8}{3}"1.8"^{1.5} + 7\times"1.8"\right) - \left(\frac{1}{4}1^3 - \frac{8}{3}1^{1.5} + 7\times 1\right) + '2.9485...'\right)$ Correct combination of areas. **Dependent on both previous method marks.**		**dd**M1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)

Question	Scheme	Marks
1.	$\mathrm{f}(x) = 3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}$	
	$\int\left(3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}\right)\mathrm{d}x = \frac{3x^3}{3} + \frac{x^2}{2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-2}}{-2} (+c)$	M1 A1A1A1
	$= x^3 + \frac{x^2}{2} - 8x^{\frac{1}{2}} - 3x^{-2} + c$	A1
		[5]
		5 marks

Notes

M1: Attempt to integrate original f(x)– one power increased $x^n \to x^{n+1}$

A1: Two of the four terms in x correct un simplified or simplified– (ignore no constant here). They may be listed.

$3x^2 \to 3\frac{x^3}{3}$ is acceptable for an un simplified term BUT $3x^2 \to 3\frac{x^{2+1}}{2+1}$ isn't

A1: **Three terms** correct **(may be) unsimplified.** They may be listed separately

A1: All four terms correct (may be) unsimplified on a single line.

A1 **cao: All four** terms **correct simplified** with **constant** of integration on a single line. You may isw after sight of correct answer.

Question	Scheme	Marks
14.	$y=-x^2+6x-8$	
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2x+6$ and substitutes $x=5$ to give gradient $= m = -4$	M1 A1
	Normal has gradient $\frac{-1}{m}=\left(\frac{1}{4}\right)$	M1
	Equation of normal is $(y+3)="\frac{1}{4}"(x-5)$ so $x-4y-17=0$	dM1 A1 **[5]**
(b)	$\int -x^2+6x-8\,\mathrm{d}x=-\frac{x^3}{3}+6\frac{x^2}{2}-8x$	M1
	The Line meets the x-axis at 17	B1
	The Curve meets the x-axis at 4	B1
	Uses correct limits correctly for their integral i.e. $\left[-\frac{x^3}{3}+6\frac{x^2}{2}-8x\right]_4^5=-\frac{5^3}{3}+6\frac{5^2}{2}-8\times5-(-\frac{4^3}{3}+6\frac{4^2}{2}-8\times4)$	M1
	Finds area above line, using area of triangle or integration $=\frac{1}{2}\times3\times("17"-5)$	M1
	Area of R = $18+1\frac{1}{3}=19\frac{1}{3}$	A1
		[6]
		11 marks

Question Number	Scheme	Marks
6(a)	$\dfrac{x^2-4}{2\sqrt{x}}=\dfrac{x^2}{2\sqrt{x}}-\dfrac{4}{2\sqrt{x}}=\dfrac{1}{2}x^{\frac{3}{2}}-2x^{-\frac{1}{2}}$	M1A1A1
		(3)
(b)	$\displaystyle\int\frac{x^{\frac{3}{2}}}{2}-2x^{-\frac{1}{2}}\mathrm{d}x=\frac{x^{\frac{5}{2}}}{2\times2.5}-2\frac{x^{\frac{1}{2}}}{0.5}(+c)$	M1 A1ft A1
	$=\dfrac{x^{\frac{5}{2}}}{5}-4x^{\frac{1}{2}}+c$	B1
		(4)
		(7 marks)

(a)

M1 Attempt to divide by $2\sqrt{x}$ to get exactly two terms (not three)
This can be implied by any of A, B, p or q being correct.

A1 Two of A, B, p or q correct. Look for two of the four numbers $\dfrac{x^{\mathbf{\frac{3}{2}}}}{\mathbf{2}}-\mathbf{2}x^{-\mathbf{\frac{1}{2}}}$ oe

Allow the power as $\dfrac{3}{2}$ appearing as $2-\dfrac{1}{2}$

A1 Completely correct expression $\dfrac{x^{\frac{3}{2}}}{2}-2x^{-\frac{1}{2}}$ or equivalent. Eg accept $0.5x^{1.5}-2x^{-0.5}$
The powers must now be simplified.

(b)

M1 Increases a fractional index by one. Do not allow if the candidate integrates the numerator and denominator of the original function

A1ft One of the fractional terms correct unsimplified.
You may follow through on any term with a fractional index.

A1 Both terms correct unsimplified
Allow the powers and coefficients such as $\dfrac{5}{2}$ appearing as $\left(\dfrac{3}{2}+1\right)$ for this mark

B1 $=\dfrac{x^{\frac{5}{2}}}{5}-4x^{\frac{1}{2}}+c$ or exact equivalent such as $0.2x^{2.5}-4x^{0.5}+c$