Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(9k-k)-\frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$=\frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right)=\frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(4k-k)}+\frac{1}{(2k-k)}$	dM1	1.1b
	$=\frac{2}{3k}\quad\left(\propto\frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\mathrm{d}x = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k-k \rightarrow A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $=\frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\mathrm{d}x = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)}+\frac{1}{(k)}=\left(-\frac{1}{3}+1\right)\times\frac{1}{k}\propto\frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to *k*.

You may see substitution used but the mark is scored for the same result. See below

$u=2x-k\rightarrow\left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x=Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x\rightarrow x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

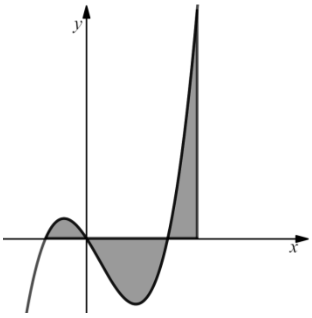
A1*: Proceeds to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**.

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

Question	Scheme	Marks	AOs
13	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right) \quad \{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = ...$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\} \frac{1}{2}x^2\ln x - \int \frac{1}{x}.\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question 13

M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } \mathrm{g}'(x))$, where $\mathrm{g}(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\mathrm{e}, \mathrm{e})$ meets the x-axis i.e. Sets $y = 0$ in $y - \mathrm{e} = m_N(x - \mathrm{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\mathrm{e}$, allowing un-simplified values for x such as $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$
Note:	Allow $x = \text{awrt } 8.15$
M1:	Scored for either • Area under curve $= \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ ...\ ]_1^{\mathrm{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • **or** Area under line $= \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{\mathrm{d}x\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \,\mathrm{d}x \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y\,\mathrm{d}x$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = \left(b+2\right)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = \frac{20}{3}$ oe for this mark

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)}\Rightarrow p-9=6\Rightarrow p=15*$	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\text{I}=\int\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5\frac{15-3x}{(2x-4)(x+3)}\,\mathrm{d}x$ Or $[\ldots\ldots\ldots\ldots]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

Question	Scheme	Marks	AOs
5	States $\left\{\lim\limits_{\delta x\to 0}\sum\limits_{x=4}^{9}\sqrt{x}\ \delta x \text{ is}\right\}$ $\int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b
	$= \frac{2}{3}\times 9^{\frac{3}{2}} - \frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda\times 9^{\frac{3}{2}} - \lambda\times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x - \int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}} + c\right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Question	Scheme	Marks	AOs
10 (a)	$x=u^2+1\Rightarrow \mathrm{d}x=2u\mathrm{d}u$ oe	B1	1.1b
	Full substitution $\int\frac{3\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)}=\int\frac{3\times 2u\ \mathrm{d}u}{\left(u^2+1-1\right)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p=2,q=3$	B1	1.1b
	$=\int\frac{3\times 2\cancel{u}\ \mathrm{d}u}{u^{\cancel{2}}(3+2u)}=\int\frac{6\ \mathrm{d}u}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)}=\frac{A}{u}+\frac{B}{3+2u}\Rightarrow A=...,B=...$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$	A1	1.1b
	$\int\frac{6\ \mathrm{d}u}{u(3+2u)}=2\ln u-2\ln(3+2u)\qquad(+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u="3",u="2"$ with some correct ln work leading to $k\ln b$ E.g. $(2\ln 3-2\ln 9)-(2\ln 2-2\ln 7)=2\ln\frac{7}{6}$	M1	1.1b
	$\ln\frac{49}{36}$	A1	2.1
		(6)	
			(10 marks)
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $\mathrm{d}x=2u\mathrm{d}u$ or exact equivalent. E.g. $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$, $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x=u^2+1$, including $\mathrm{d}x\to ...u\mathrm{d}u$ to form an integrand in terms of u. Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p=2,q=3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B.

A1: Correct PF $\frac{6}{u(3+2u)}=\frac{2}{u}-\frac{4}{3+2u}$ (Not scored for just the correct values of A and B)

dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns. Look for $P\ln u+Q\ln(3+2u)$

A1ft: Correct integration for their $\frac{A}{u}+\frac{B}{3+2u}\to A\ln u+\frac{B}{2}\ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k\ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u's back to x's and use limits of 5 and 10.

A1: Proceeds to $\ln\frac{49}{36}$. Answers without working please send to review.

Question	Scheme	Marks	AOs
6(a)	$x^2+8x-3=(Ax+B)(x+2)+C$ or $Ax(x+2)+B(x+2)+C$ $\Rightarrow A=\ldots,B=\ldots,C=\ldots$ **or** $x+6$ $x+2\overline{)x^2+8x-3}$ $\underline{x^2+2x}$ $6x-3$ $\underline{6x+12}$ -15	M1	1.1b
	Two of $A=1, B=6, C=-15$	A1	1.1b
	All three of $A=1, B=6, C=-15$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+8x-3}{x+2}\,\mathrm{d}x=\int x+6-\frac{15}{x+2}\,\mathrm{d}x=\ldots-15\ln(x+2)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-15\ln(x+2)\quad(+c)$	A1ft	1.1b
	$\int_0^6 \frac{x^2+8x-3}{x+2}\,\mathrm{d}x=\left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$ $=(18+36-15\ln 8)-(0+0-15\ln 2)$ $=18+36-(15-45)\ln 2$ or e.g. $18+36+15\ln\left(\frac{2}{8}\right)$	M1	2.1
	$=54-30\ln 2$	A1	1.1b
		(4)	
			(7 marks)

Question	Scheme	Marks	AOs
8	$\mathrm{f}'(x) = 6x^2 + ax - 23 \Rightarrow \mathrm{f}(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$\mathrm{f}(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots\ (6)$	dM1	1.1b
	$(\mathrm{f}(x)=)\,2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(\mathrm{f}(x)=)(x+4)(2x^2-5x-3)\quad(\mathrm{f}(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Question Number	Scheme	Marks
3(a)	$4x^3+2x^2+17x+8 \equiv (Ax+B)(x^2+4)+Cx+D$	
	Compare x^3 terms: $A=4$	B1
	Compare x^2 terms: $B=2$	B1
	Compare either x term or constant term: $4A+C=17$ or $4B+D=8$	
	$\Rightarrow C=..\text{or } D=..$	M1
	$\Rightarrow C=1, D=0$	A1
		(4)
(b)	$\int_1^4 \frac{4x^3+2x^2+17x+8}{x^2+4}\,\mathrm{d}x = \int_1^4 4x+2+\frac{x}{x^2+4}\,\mathrm{d}x$	M1
	$=\left[2x^2+2x,+\frac{1}{2}\ln(x^2+4)\right]_1^4$	M1, M1A1
	$=\left[2\times16+2\times4+\frac{1}{2}\ln(20)\right]-\left[2\times1+2\times1+\frac{1}{2}\ln(5)\right]$	
	$=36+\frac{1}{2}\ln\left(\frac{20}{5}\right)$	dM1
	$=36+\ln(2)$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
4. (a)	$\left\{\int (2x+3)^{12}\,\mathrm{d}x\right\} = \frac{(2x+3)^{13}}{(13)(2)}\,\{+c\}$ $\pm\lambda(2x+3)^{13}$	M1
	$\frac{(2x+3)^{13}}{(13)(2)}\,\{+c\}$ (Ignore '+ c')	A1
		[2]
(b)	$\left\{\int \frac{5x}{4x^2+1}\,\mathrm{d}t\right\} = \frac{5}{8}\ln(4x^2+1)\ \{+c\}$ or $\frac{5}{8}\ln(x^2+\frac{1}{4})\ \{+k\}$	M1
		A1
		[2] **4**

Notes

(a) **M1**: Gives $\pm\lambda(2x+3)^{13}$ where λ is a constant or $\pm\mu(x+\frac{3}{2})^{13}$

A1: Coefficient does not need to be simplified so is awarded for $\frac{(2x+3)^{13}}{(13)(2)}$ or for $\frac{2^{12}}{13}(x+\frac{3}{2})^{13}$ i.e. $\frac{4096}{13}(x+\frac{3}{2})^{13}$

Ignore subsequent errors and condone lack of constant c

N.B. If a binomial expansion is attempted, then it needs all thirteen terms to be correctly integrated for M1A1

(b) **M1**: Gives $\pm\mu\ln(4x^2+1)$ where μ is a constant or $\pm\mu\ln(x^2+\frac{1}{4})$ or indeed $\pm\mu\ln(k(4x^2+1))$

May also be awarded for $\frac{5}{8}\ln(4x+1)$ or $\frac{5}{8}\ln(x^2+1)$, where coefficient 5/8 is correct and there is a slip writing down the bracket.

It may also be given for $\pm\mu\ln(u)$ where u is clearly defined as $(4x^2+1)$ or equivalent substitutions such as $\pm\mu\ln(4u+1)$ where $u=x^2$

A1: $\frac{5}{8}\ln(4x^2+1)$ or $\frac{5}{8}\ln(x^2+\frac{1}{4})$ o.e. The modulus sign is not needed but allow $\frac{5}{8}\ln\left|4x^2+1\right|$

Also allow $0.625\ln(4x^2+1)$ and condone lack of constant c

N.B. $\frac{5}{8}\ln 4x^2+1$ with no bracket can be awarded M1A0

Question Number	Scheme	Marks
4	$x = 2\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 2\cos\theta$	B1
	$\displaystyle\int \frac{1}{\left(4-x^2\right)^{3/2}}dx = \int \frac{1}{\left(4-4\sin^2\theta\right)^{3/2}}\,2\cos\theta(d\theta)$	M1
	$\displaystyle= \int \frac{1}{4}\sec^2\theta(d\theta)\ OR \int \frac{1}{4}\times\frac{1}{\cos^2\theta}(d\theta)$	M1
	$= \dfrac{1}{4}\tan\theta$	dM1A1
	Uses limits 0 and $\dfrac{\pi}{3}$ in their integrated expression	
	$= \left[\dfrac{1}{4}\tan\theta\right]_0^{\frac{\pi}{3}} = \dfrac{\sqrt{3}}{4}$	M1A1
		(7 marks)

B1 States either $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$. Condone $x' = 2\cos\theta$

M1 Attempt to produce integral in just θ by substituting $x = 2\sin\theta$ and using $dx = \pm A\cos\theta\ (d\theta)$
You may condone a missing $d\theta$

M1 Uses $1-\sin^2\theta = \cos^2\theta$ and simplifies integral to $\displaystyle\int C\sec^2\theta(\,d\theta)$ or $\displaystyle\int \frac{C}{\cos^2\theta}(\,d\theta)$

Again you may condone a missing $d\theta$

dM1 Dependent upon previous M1 for $\displaystyle\int \sec^2\theta \to \tan\theta$

A1 $\dfrac{1}{4}\tan\theta(+c)$. No requirement for the $+c$

M1 Changes limits in x to limits in θ of 0 and $\dfrac{\pi}{3}$, then subtracts their integrated expression either way around. The subtraction of 0 can be implied if f (0) =0. If the candidate changes the limits to 0 and 60 (degrees) it scores M0, A0. Alternatively they could attempt to change their integrated expression in θ back to a function in x and use the original limits. Such a method would require seeing either $\cos\theta = \sqrt{1-\dfrac{x^2}{4}}$ or $\tan\theta = \dfrac{\frac{x}{2}}{\sqrt{1-\frac{x^2}{4}}}$

A1 $\dfrac{\sqrt{3}}{4}$.

Question Number	Scheme	Marks
5.(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)-x}{(x+1)^2}=\frac{1}{(x+1)^2}$ or $y=\frac{x}{x+1}=1-\frac{1}{x+1}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{(x+1)^2}$	M1
	(see notes for further methods)	
	$\frac{1}{(x+1)^2}=\frac{1}{4}$ *or* $(x+1)^2=4$ *or* $x^2+2x+1=4$	A1
	$x=1,-3$	M1 A1
		(4)
(ii)	$\int\frac{t+1}{t}\,\mathrm{d}t=\int 1+\frac{1}{t}\,\mathrm{d}t=t+\ln t\ \ (+c)$ see notes for integration by parts.	M1A1
	$[t+\ln t]_a^{2a}=\ln 7\Rightarrow 2a+\ln 2a-a-\ln a=\ln 7$	
	$a+\ln\left(\frac{2a}{a}\right)=\ln 7\Rightarrow a=\ln\left(\frac{7}{2}\right)$ or $a=\ln 7-\ln 2$	dM1A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
8(a)	**Either** $f(\theta)=9\cos^2\theta+\sin^2\theta=9\cos^2\theta+1-\cos^2\theta$	M1
	$=8\cos^2\theta+1=8\dfrac{(\cos 2\theta+1)}{2}+1$	M1
	$=5+4\cos 2\theta$	A1 [3]
	Or $f(\theta)=9\dfrac{(\cos 2\theta+1)}{2}+1\dfrac{(1-\cos 2\theta)}{2}$	M1 M1
	$=5+4\cos 2\theta$	A1 [3]
(b)	**Either :Way1** splits as $\int_0^{\frac{\pi}{2}} a\theta^2\,d\theta+\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta$	
	$\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta=\ ...\theta^2\sin 2\theta\pm\int ...\theta\sin 2\theta d\theta$	M1
	$= \ =..\theta^2\sin 2\theta\pm..\theta\cos 2\theta\pm\int ..\cos 2\theta d\theta$	dM1
	Integral $=\underline{\underline{\left[2\theta^2\sin 2\theta+2\theta\cos 2\theta-\sin 2\theta\right]}}+\frac{5}{3}\theta^3$	$\underline{\underline{\text{A1}}}$ **B1ft**
	Use limits to give $\left[\dfrac{5\left(\dfrac{\pi}{2}\right)^3}{3}-\pi\right]-[0]\quad=\quad\left[\dfrac{5\pi^3}{24}-\pi\right]$	ddM1 A1 [6]
		(9 marks)
1st 4 marks	**Or: Way 2** $\int_0^{\frac{\pi}{2}}\theta^2 f(\theta)d\theta=\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta\ =$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\int ..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(...\theta\pm..\sin 2\theta)-..\theta\left(..\theta^2\pm...\cos 2\theta\right)\pm\int .\left(..\theta^2\pm...\cos 2\theta\right)d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-2\theta\left(\dfrac{5\theta^2}{2}-\cos 2\theta\right)+\left(\dfrac{5\theta^3}{3}-\sin 2\theta\right)$	A1 **B1ft**
1st 4 marks	**Or: Way 3 Way 2 that goes back to Way One**	
	$\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta=\theta^2(..\theta\pm..\sin 2\theta)-\int ..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int ...\theta^2 d\theta\right)\pm\int ..\theta\sin 2\theta d\theta$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int ...\theta^2 d\theta\right)\pm..\theta\cos 2\theta\pm\int ..\cos 2\theta d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-\dfrac{10}{3}\theta^3+2\theta\cos 2\theta-\sin 2\theta$	A1 **B1ft**

Question Number	Scheme	Notes	Marks
	Note that 2^x can be replaced by $\mathrm{e}^{x\ln 2}$ throughout and allow omission of "d*x*" throughout		
5	$\int x2^x \mathrm{d}x = x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\mathrm{d}x$	M1: Integrates by parts the right way around to obtain an expression of the form $ax2^x - \int b2^x \mathrm{d}x$. Allow *a* = 1 and/or *b* = 1. A1: $x\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2}\mathrm{d}x$ (Does not need to be seen all on one line)	M1A1
	$\int x2^x \mathrm{d}x = x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	dM1: Completes to obtain an expression of the form $... - k2^x$ A1: $x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}$	dM1A1
	$\left[x\frac{2^x}{\ln 2} - \frac{2^x}{(\ln 2)^2}\right]_0^2 = \left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - \left(\frac{0\times 2^0}{\ln 2} - \frac{2^0}{(\ln 2)^2}\right)$ Uses the limits 0 and 2 and subtracts the right way round. F(0) may be implied by e.g. $\frac{1}{(\ln 2)^2}$ But $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right) - (0)$ or just $\left(\frac{2\times 2^2}{\ln 2} - \frac{2^2}{(\ln 2)^2}\right)$ is ddM0		ddM1
	$\left(= \frac{8}{\ln 2} - \frac{4}{(\ln 2)^2} + \frac{1}{(\ln 2)^2}\right)$		
	$= \frac{8\ln 2 - 3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256 - 3}{(\ln 2)^2}, \frac{\ln 2^8 - 3}{(\ln 2)^2}$ Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1
			(6 marks)

	Alternative by substitution:		
	$u=2^x \Rightarrow \int x2^x\,\mathrm{d}x=\int \frac{\ln u}{\ln 2}.u.\frac{1}{u\ln 2}\,\mathrm{d}u=\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u$		
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u=\frac{1}{(\ln 2)^2}\left(u\ln u-\int \,\mathrm{d}u\right)$	M1: Integrates by parts the right way around to obtain an expression of the form $au\ln u-\int b\mathrm{d}u$. Allow a = 1 and/or b = 1. A1: $\frac{1}{(\ln 2)^2}\left(u\ln u-\int \,\mathrm{d}u\right)$	M1A1
	$\int \frac{\ln u}{(\ln 2)^2}\,\mathrm{d}u=\frac{1}{(\ln 2)^2}(u\ln u-u)$	dM1: Completes to obtain an expression of the form $...-ku$ A1: $\frac{1}{(\ln 2)^2}(u\ln u-u)$	dM1A1
	$\left[\frac{1}{(\ln 2)^2}(u\ln u-u)\right]_1^4=\frac{1}{(\ln 2)^2}(4\ln 4-4)-(\ln 1-1)$ Uses the limits 1 and 4 and subtracts the right way round.		M1
	$=\frac{4\ln 4-3}{(\ln 2)^2}$	Correct simplified fraction. Allow equivalent simplified forms e.g. $\frac{\ln 256-3}{(\ln 2)^2},\frac{\ln 2^8-3}{(\ln 2)^2},$ Allow denominator as (ln2)(ln2) and $\ln^2 2$ but not as $\ln 2^2$	A1

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\mathrm{d}x = \int x^{-2}\ln x\,\mathrm{d}x = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$	M1A1
	$=\frac{x^{-1}}{-1}\ln x + \int x^{-2}\,\mathrm{d}x$	
	$=\frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}\left(+c\right)$	M1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{\mathrm{e}} = \left(\frac{-1}{\mathrm{e}}\ln \mathrm{e} - \frac{1}{\mathrm{e}}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	M1
	$=1-\frac{2}{\mathrm{e}}$	A1
		(6)
	Alternative by substitution:	
	$u=\ln x \Rightarrow \int \frac{\ln x}{x^2}\mathrm{d}x = \int \frac{u}{\mathrm{e}^{2u}}\mathrm{e}^u\,\mathrm{d}u = \int u\mathrm{e}^{-u}\,\mathrm{d}u$	
	$\int u\mathrm{e}^{-u}\,\mathrm{d}u = -u\mathrm{e}^{-u} - \int -\mathrm{e}^{-u}\,\mathrm{d}u$	M1A1
	$\int u\mathrm{e}^{-u}\,\mathrm{d}u = -u\mathrm{e}^{-u} - \mathrm{e}^{-u}\left(+c\right)$	M1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[-u\mathrm{e}^{-u} - \mathrm{e}^{-u}\right]_0^1 = \left(-\frac{1}{\mathrm{e}} - \frac{1}{\mathrm{e}}\right) - \left(0-1\right)$	M1
	$=1-\frac{2}{\mathrm{e}}$	A1

(Condone the lack of "dx" throughout)

M1: An application of integration by parts the right way around.

If the rule is quoted it must be correct. (A version appears in the formula booklet)

Must see an expression of the form $Ax^{-1}\ln x \pm B\int x^{-1}\times\frac{1}{x}\,\mathrm{d}x$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$, $\left[-\frac{1}{x}\ln x\right]_1^{\mathrm{e}} + \int \frac{1}{x^2}\,\mathrm{d}x$

M1: It is for 'combining' their two terms in x correctly and integrating their resulting term by adding one to the power.

A1: A completely correct integral (simplified or un-simplified)

For students who substitute in limits early, look for e.g. $\left(\frac{\mathrm{e}^{-1}}{-1}\ln \mathrm{e}\right) - \left(\frac{1^{-1}}{-1}\ln 1\right) + \left[\frac{x^{-1}}{-1}\right]_1^{\mathrm{e}}$

M1: It is for substituting in the limits 1 and e (either way round) and subtracting.

Question Number	Scheme	Marks
5(i)	$\int\left((3x+5)^9+\mathrm{e}^{5x}\right)\mathrm{d}x=\frac{(3x+5)^{10}}{30},+\frac{\mathrm{e}^{5x}}{5}(+c)$	M1A1, B1
		(3)
(ii)	$\int\frac{x}{x^2+5}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+5\right)$	M1A1
	$\int_2^b\frac{x}{x^2+5}\,\mathrm{d}x=\ln\left(\sqrt{6}\right)=\frac{1}{2}\ln\left\|b^2+5\right\|-\frac{1}{2}\ln\left\|2^2+5\right\|=\ln\left(\sqrt{6}\right)$	M1
	$\Rightarrow\ln\left(\frac{b^2+5}{9}\right)=\ln 6\Rightarrow b=7$	**dd**M1, A1
		(5)
		(8 marks)

(i)

M1: For an integral of the form $C(3x+5)^{10}$ or $C(3x+5)^{9+1}$ where C is a constant and no other powers of $(3x + 5)$

A1: $\frac{(3x+5)^{10}}{30}$. No need for + c. Allow un-simplified e.g. $\frac{\frac{1}{3}(3x+5)^{10}}{10}$.

B1: $\mathrm{e}^{5x}\to\frac{\mathrm{e}^{5x}}{5}$

Mark each integration independently i.e. there is no need to see everything all on one line.

(ii)

M1: For an answer of the form $C\ln k\left(x^2+5\right)$ where C and k are constants. Allow log for ln.

A1: $\frac{1}{2}\ln k\left(x^2+5\right)$ or $\ln k\left(x^2+5\right)^{\frac{1}{2}}$ or $\frac{1}{2}\ln k\left|x^2+5\right|$. Allow log for ln.

M1: Substitutes in both 2 and b for x correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b=7$ only. $b = \pm 7$ scores A0 unless the -7 is rejected.

<u>Note: May see integration by substitution in (ii)</u>

E.g. $u=x^2+5$

M1: $\int\frac{x}{x^2+5}\,\mathrm{d}x=\int\frac{x}{u}\frac{\mathrm{d}u}{2x}=\frac{1}{2}\ln u$

For an answer of the form $C\ln k(u)$ where C is a constant

Allow log for ln as above.

A1: $\frac{1}{2}\ln ku$

M1: $\left[\frac{1}{2}\ln u\right]_9^{b^2+5}=\frac{1}{2}\ln\left(b^2+5\right)-\frac{1}{2}\ln 9=\ln\sqrt{6}$

Substitutes in both 9 and b^2 + 5 correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b=7$ only. $b = \pm 7$ scores A0 unless the -7 is rejected.

Question Number	Scheme	Marks
8	$2+\ldots$	B1
	Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants	M1
	$\frac{3}{x}$ or $-\frac{1}{x-1}$ or $A = 3$ or $B = -1$	A1
	$\frac{3}{x}-\frac{1}{x-1}$	A1 **(B1 on Epen)**
	$\int_3^4 \frac{2x^2-3}{x(x-1)}\,\mathrm{d}x = \int_3^4 \left(2+\frac{3}{x}-\frac{1}{x-1}\right)\mathrm{d}x$	
	$=\left[2x+3\ln x-\ln(x-1)\right]_3^4$	M1 A1ft
	$=\left(8+3\ln 4-\ln 3\right)-\left(6+3\ln 3-\ln 2\right)=2+\ln\left(\frac{128}{81}\right)$	M1 A1cso
		(8 marks)

B1: 2 +…

M1: Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants

A1: $\frac{3}{x}$ or $-\frac{1}{x-1}$ or one correct constant

B1: $\frac{3}{x}-\frac{1}{x-1}$

M1: For $\int \frac{*}{x}+\frac{*}{x-1}\,\mathrm{d}x \to p\ln mx+q\ln n(x-1)$ where $*$, p, q, m and n are constants.

A1ft: $2x+3\ln x-\ln(x-1)$. Follow through their "2", A and B so look for "2"x+A$\ln x$+B$\ln(x-1)$. This mark can be withheld if the brackets are missing unless subsequent work suggests their intended presence.

M1: For substituting in 3 and 4, subtracting either way around and using correct addition or subtraction log laws at least once.

A1: cso $2+\ln\left(\frac{128}{81}\right)$ or $2+\ln\left(1\frac{47}{81}\right)$ (Do not allow $2+\ln\left(\frac{2^7}{3^4}\right)$) $2+\ln\left(\frac{128}{81}\right)+c$ is also A0

Question Number	Scheme		Marks
6. (i)	$\int xe^{4x}\ \mathrm{d}x = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\ \{\mathrm{d}x\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{\mathrm{d}x\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\ \{\mathrm{d}x\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\ \mathrm{d}x = \frac{8(2x-1)^{-2}}{(2)(-2)}\ \{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\ \{+c\}\}$	*{Ignore subsequent working}*.	**[2]**
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{x}\operatorname{cosec}2y\operatorname{cosec}y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$ **or** $\int \sin 2y \sin y\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$		B1 oe
	$\int 2\sin y\cos y\sin y\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \rightarrow 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = e^{x}\ \{+c\}$	$2\sin^2 y\cos y \rightarrow \frac{2}{3}\sin^3 y$	A1
		$e^{x} \rightarrow e^{x}$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^{0} + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^{x} - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^{x} - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$ **or** $\int \sin 2y \sin y\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\ \mathrm{d}y = \int e^{x}\ \mathrm{d}x$	$\sin 2y\sin y \rightarrow \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = e^{x}\ \{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$e^{x} \rightarrow e^{x}$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = e^{0} + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^{x} - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^{x} - \frac{11}{12}$	A1
			[7]
			12

Question	Scheme	Marks	AOs
2			

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

Question	Scheme	Marks	AOs
(a)	Uses an allowable method to estimate the area under the curve. E.g.	M1	3.1a
	Way 1: an attempt at the trapezium rule (see below)		
	Way 2: $\{s=\}\left(\frac{2+42}{2}\right)(25)\ \{=550\}$		
	Way 3: $42=2+25(a)\Rightarrow a=1.6\Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$		
	Way 4: $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \{=63(5)=315\}$		
	Way 5: $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \{=103(5)=515\}$		
	Way 6: $\{d=\}\ \frac{315+515}{2}\ \{=415\}$		
	Way 7: $\{d=\}\left(\frac{2+5+10+18+28+42}{6}\right)(25)\ \{=437.5\}$		
	$\frac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\frac{1}{2}\times["315"+"515"]$	M1	1.1b
	$=415\ \{\text{m}\}$	A1	1.1b
		(3)	
(b) **Alt 1**	**Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)**		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5)	B1ft	2.4
		(1)	
(b) **Alt 2**	**Uses a Way 4 method in (a)**		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
			(4 marks)

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42);\ \lambda>0$ for 1st M1
	Way 2: Uses $s=\left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed) × (time)

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\left\{0.5774+0.8452+2\left(0.7071+0.7746+0.8165\right)\right\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	3 × their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3 × 1.5
	If (a) is incorrect allow 3 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as 4.535 ≈ 4.50 • Very accurate as 4.535 to 2 sf is 4.5 • 4.51425 < 4.535 so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • 4.535 – 4.5 = 0.035 so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert\times100=0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert\times100=0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert\times100=0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert\times100=99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

Notes:

(a)

Question Number	Scheme	Marks
9. (a)	$\begin{array}{c\|cccccc} x & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline y & e^2 & e^{\sqrt{5}} & e^{\sqrt{6}} & e^{\sqrt{7}} & e^{\sqrt{8}} & e^3 \\ \hline & 7.389056... & 9.356469... & 11.582435... & 14.094030... & 16.918828... & 20.085536... \end{array}$	M1
	$\frac{1}{2}\times 1\times \underline{\{..........................\}}$	B1 oe
	$\frac{1}{2}\times 1\times \underline{\left\{e^2+e^3+2\left(e^{\sqrt{5}}+e^{\sqrt{6}}+e^{\sqrt{7}}+e^{\sqrt{8}}\right)\right\}}$ $\quad \left\{= \tfrac{1}{2}(27.47459302... + 103.903526...)\right\}$	M1
	$= 65.6890595... = 65.69$ (2 dp)	A1
	Special case (s.c.) Uses h = 5/4 with 5 ordinates giving answer 65.76 – award M0B0M1A1(s.c.)	**[4]**
	See note below	
(b)	$\left\{u=\sqrt{x}\Rightarrow\right\}\ \frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du}=2u$	B1
	$\left\{\int e^{\sqrt{x}}\,dx\right\} = \int e^{u}\,2u\,du$	M1 A1
	$= \{2\}\left(ue^{u}-\int e^{u}\,du\right)$	M1
	$= \{2\}\left(ue^{u}-e^{u}\right)$	A1
	$\left[2\left(ue^{u}-e^{u}\right)\right]_2^3 = 2\left(3e^3-e^3\right)-2\left(2e^2-e^2\right)$	ddM1
	$4e^3-2e^2$ or $2e^2(2e-1)$ etc.	A1
		[7] **11**

Question Number	Scheme	Marks
12(a)	0.9242 exactly	B1 **(1)**
(b)	Strip width =0.5	B1
	Area $\approx \frac{0.5}{2}((2+1.2958+2\times(1.3041+'0.9242'+0.9089))$	M1
	=2.393	A1 **(3)**
(c)	$\int \frac{x^2\ln x}{3} - 2x + 4\,\mathrm{d}x$	
	$= \frac{x^3}{9}\ln x - \int \frac{x^3}{9}\times\frac{1}{x}\mathrm{d}x, \quad -x^2+4x$	M1A1, B1
	$= \frac{x^3}{9}\ln x - \frac{x^3}{27} \quad (-x^2+4x)$	A1
	Area $= \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 4x\right]_1^3 = (3\ln 3 - 1 - 9 + 12) - \left(-\frac{1}{27} - 1 + 4\right)$	dM1
	$= \ln 27 - \frac{26}{27}$	A1 **(6)**
(d)	% error = $\pm\frac{\lvert real - approx\rvert}{real}\times 100$ =Accept awrt ±2.6%	M1A1 **(2)**
(e)	Increase the number of 'strips'	B1 **(1)** **(13 marks)**

(a)

B1 0.9242 exactly either in the table or within the trapezium rule in part (b)

(b)

B1 Uses a strip width of 0.5 or equivalent.

M1 Uses the correct form of the trapezium rule, a form of which appears in the formula booklet.

Look for $\frac{....}{2}((2+1.2958+2\times(1.3041+\text{their } 0.9242+0.9089))$

Accept for this the sum of four trapezia

A1 Awrt 2.393 (3dp)

Question Number	Scheme	Marks
13 (a)	awrt 0.3799 – may be seen in the table	B1
		(1)
(b)	Area = $\frac{1}{2}\times\left(\frac{e^2-e}{2}\right)(1+2\times'0.3799'+0)$ [The +0 is not required]	B1M1
	= awrt 2.055	A1
		(3)
(c) **Way 1**	$\int(\ln x)^2\,dx=\int 1\times(\ln x)^2\,dx=x(\ln x)^2-\int x\times\frac{2\ln x}{x}dx$	M1
	$=x(\ln x)^2-\int 2\ln x dx$	A1
	$=x(\ln x)^2-2x\ln x+\int 2dx$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1*
		(4)
(c) **Way 2**	$\int(\ln x)^2\,dx=\int(\ln x)\times(\ln x)\,dx=\ln x(x\ln x-x)-\int\frac{1}{x}\times(x\ln x-x)\,dx$	M1
	$=\ln x(x\ln x-x)-\int\ln x-1dx$	A1
	$=\ln x(x\ln x-x)-(x\ln x-x-x)$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1*
		(4)
(c) **Way3**	Use u = ln$x$ substitution to get to	
	$\int u^2e^u du=u^2e^u-\int 2ue^u du$	M1
	$=u^2e^u-\left[2ue^u-\int 2e^u du\right]$	A1
	$=u^2e^u-2ue^u+2e^u+k$	
	$=x(\ln x)^2-2x\ln x+2x(+c)$	dM1A1
		(4)
(d)	Volume = $\int\pi y^2 dx=\int\pi(2-\ln x)^2\,dx$	B1
	$\int(2-\ln x)^2\,dx=\int 4-4\ln x+(\ln x)^2\,dx$	M1
	Correct integration of at least two of their three terms (see notes)	M1
	$=4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\ (+c)$	A1
	Volume $=\pi\left[4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\right]_e^{e^2}$	ddM1
	$=2\pi e^2-5\pi e$	
	$=\pi e(2e-5)$	A1
		(6) **(14 marks)**

Question Number	Scheme	Marks
7.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+1)\times\frac{6x}{x^2+1}-6x\times\ln(x^2+1)}{(x^2+1)^2}$ Or $\frac{6x\left(1-\ln(x^2+1)\right)}{(x^2+1)^2}$	M1A1 [2]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow \cancel{(x^2+1)}3\frac{2x}{\cancel{(x^2+1)}}-3\ln(x^2+1)(2x)=0$	M1
	$\ln(x^2+1)=1 \quad so \;\; x=\sqrt{\mathrm{e}-1}$	M1A1
	$y=\frac{3}{\mathrm{e}}$	ddM1A1 [5]
(c)	$\frac{3}{2}\ln 2$ or 1.0397	B1 [1]
(d)		
	$\frac{1}{2}\times 1\times\underline{\{..........................\}}$	B1 oe
	$\underline{\frac{1}{2}\times 1\times\left\{0+\frac{3}{10}\ln 10+2\left(\frac{3}{2}\ln 2+\frac{3}{5}\ln 5\right)\right\}}$	M1
	$\left\{=\tfrac{1}{2}(0.6907755279..+4.010767..)\right\}$ = 2.351 (awrt 4 sf)	A1 [3] **(11 marks)**

(a)

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\frac{3\ln(x^2+1)}{(x^2+1)}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln\left(x^2+1\right), v=x^2+1, u'=.., v'=...$ followed by their $\frac{vu'-uv'}{v^2}$, then only accept answers of the form

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+1)A\frac{x}{x^2+1}-Bx\ln(x^2+1)}{(x^2+1)^2} or \frac{Ax-Bx\ln(x^2+1)}{(x^2+1)^2}$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=3\ln(x^2+1), v=\left(x^2+1\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln(x^2+1), v=\left(x^2+1\right)^{-1}, u'=.., v'=...$followed by their $vu'+uv'$, then only accept answers of the form

$\left(x^2+1\right)^{-1}\times A\frac{x}{x^2+1}+\left(x^2+1\right)^{-2}\times Bx\ln(x^2+1)$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $\mathrm{f}'(x)$. Remember to isw.

Question Number	Scheme	Notes	Marks
7 (a)	Strip width = 1	May be implied by their trapezium rule.	B1
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{15}}+2\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)\right)$ $\approx \frac{1}{2}(0.33...+0.25...+2(0.30...+0.27...))$	M1: Correct structure for the *y* values. Look for (*y* at *x* = 2) + (*y* at *x* = 5) + 2(sum of other *y* values). A1: Correct numerical expression. If decimals are used, look for awrt 1dp initially, however a correct final answer would imply this mark.	M1 A1
	Awrt 0.875		A1
			(4)
	May use separate trapezia:		
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{11}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{15}}\right)$		
	B1: Strip width = 1 M1: Correct structure for the *y* values as above A1: Correct expression as described above A1: Awrt 0.875		
(b)	$\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2x+5)^{\frac{1}{2}}$	M1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = k(2x+5)^{\frac{1}{2}}$ A1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2x+5)^{\frac{1}{2}}$	M1A1
	$\int_2^5 \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2(5)+5)^{\frac{1}{2}} - (2(2)+5)^{\frac{1}{2}}$	Substitutes 5 and 2 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… −3 unless the substitution of 5 and 2 is explicitly seen.	dM1
	$= \sqrt{15} - \sqrt{9}\ (= \sqrt{15} - 3)$	$\sqrt{15} - \sqrt{9}$ or $\sqrt{15} - 3$	A1
			(4)

Qu	Scheme	Marks
5.(a)	$\frac{7\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{7\pi\sqrt{2}}{8}$ AND $\frac{9\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{9\pi\sqrt{2}}{8}$	B1 **(1)**
(b)	$\frac{1}{2}\times\frac{\pi}{4}\times\{\text{..........................}\}$	B1 oe
	$\frac{1}{2}\times h\times\left\{0+0+2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)\right\}$	M1
	$= 11.91$ (only)	A1 **(3)**
(c)	$\int x\cos x\,dx \quad = [x\sin x]-\int\sin x\,dx$	M1
	$= x\sin x+\cos x\ (+c)$	dM1 A1 **(3)**
(d)	$[x\sin x+\cos x]_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \quad = \frac{5\pi}{2}+\frac{3\pi}{2}=4\pi$	M1 A1 **(2)** **(9 marks)**

(a)

B1: Both correct (as above) Must be exact and not decimal

(b)

B1: See $\frac{1}{2}\times\frac{\pi}{4}\times$ as part of trapezium rule or $h=\frac{\pi}{4}$ stated or used. This can be scored if '*h*' is in an unsimplified form.

M1: Correct structure of the bracket in the trapezium rule.

You may not see the zero's Eg $2\left(\text{"}\frac{7\pi}{4\sqrt{2}}\text{"}+2\pi+\text{"}\frac{9\pi}{4\sqrt{2}}\text{"}\right)$

A1: 11.91 only. This may be a result of using the decimal equivalents. Sight of 11.91 will score all 3 marks

(c)

M1: For a correct attempt at integration by parts to give an expression of the form $[\pm x\sin x]-\int\pm\sin x\,dx$

If you see such an expression you would only withhold the mark if there is evidence of an incorrect formula (seen or implied) Eg $\int udv = uv+\int vdu$

dM1: For $\pm\ x\sin x\ \pm\ \cos x$ following line one

A1: cso

Allow all three marks for candidate who just writes down the correct answer with no working

Watch for candidates who write down methods like this.

D	I
x	$\cos x$
1	$\sin x$
0	$-\cos x$

and then write $x\sin x-(-\cos x)$

This is a commonly taught algorithm (They differentiate down the lh column and integrate on the rh column. The answer is found by $D1\times I2-D2\times I3$ where *D*2 is the second entry in the *D* column. This can score full marks for the answer $x\sin x+\cos x$ but also pick up methods for slips.

If they attempt $D1\times I2+D2\times I3$ it is M0 as they are implying an incorrect formula. Ask your TL if unclear.

(d)

M1: Using limits $\frac{5\pi}{2}$ and $\frac{3\pi}{2}$ correctly in their answer to part (c) - substituting (seen correctly in all terms or implied in all terms) and subtracting either way around

A1: 4π or equivalent single term. CSO. It must have been derived from $x\sin x+\cos x$

Question Number	Scheme	Marks
8 (a)	Strip Width =1	B1
	$\text{Area} \approx \frac{1}{2}(0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027))$ $\left(=\frac{1}{2}\times 4.7547\right)$	M1
	Awrt =2.377	A1
		(3)
(b)	$\text{Volume}=(\pi)\int_2^7 \frac{x}{x^2+1}\mathrm{d}x=(\pi)\left[\frac{1}{2}\ln\left(x^2+1\right)\right]_2^7$	M1A1
	$=\frac{(\pi)}{2}(\ln 50-\ln 5)$	dM1
	$=\frac{\pi}{2}\ln 10$	A1
		(4)
		(7 marks)

(a)

B1: Strip width = 1 which may be implied by the $\frac{1}{2}$ in the trapezium rule

M1: For a correct attempt at using the trapezium rule.

Look for $\frac{1}{2}h((y \text{ at } x=2)+(y \text{ at } x=7)+2(\text{sum of other } y \text{ values}))$. Must be correct with no missing values and no extra values. (May be implied by a correct answer)

A1: Awrt = 2.377

Note: h = 5/6 gives Area 1.981125 and h = 5 gives 11.88675 and will probably just score the M1

Note that $\frac{1}{2}\times 1\times 0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027)$ scores B1 only unless the missing brackets are implied by a correct answer.

(b)

M1: Attempts to find $C\int\frac{x}{x^2+1}\mathrm{d}x$ to give an expression of the form $D\left[\ln k\left(x^2+1\right)\right]$

A1: $\text{Volume}=(\pi)\int\frac{x}{x^2+1}\mathrm{d}x=\frac{(\pi)}{2}\left[\ln\left(x^2+1\right)\right]$. Correct expression **with or without** π. **Ignore any limits.**

Do not allow the brackets around the $x^2 + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7$ and $x = 2$ and subtracting either way round. Following correct work, this mark may be implied by awrt 3.62.

A1: $V=\frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V=\frac{\pi}{2}\ln|10|$

By substitution 1:

M1: Uses $u=x^2$ Attempts to find $C\int\frac{x}{x^2+1}\mathrm{d}x$ to give an expression of the form $D[\ln k(u+1)]$

A1: $\text{Volume}=(\pi)\int\frac{x}{u+1}\frac{1}{2x}\mathrm{d}u=\frac{(\pi)}{2}[\ln(u+1)]$. Correct expression **with or without** π. **Ignore any limits.**

Do not allow the brackets around the $u + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7^2$ and $x = 2^2$ and subtracting either way round or changing back to x and substituting $x = 7$ and $x = 2$ and subtracting either way round.

Question Number	Scheme	Marks
6 (a)	0.41576	B1
		(1)
(b)	Strip width = $\frac{\pi}{4}$	B1
	Area $\approx \frac{1}{2}\times\frac{\pi}{4}\times\left\{0+2\left(0.76679+0.41576+0.15940\right)+0\right\}$ Or separate trapezia: $\frac{1}{2}\times\frac{\pi}{4}\times\{0+0.766792\}+\frac{1}{2}\times\frac{\pi}{4}\times\{0.766792+0.41576\}+$ $\frac{1}{2}\times\frac{\pi}{4}\times\{0.41576+0.15940\}+\frac{1}{2}\times\frac{\pi}{4}\times\{0.15940+0\}$	M1
	1.0540	A1
		(3)
(c)	Uses $vu'+uv'$: $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}$ **or** Uses $\frac{vu'-uv'}{v^2}$: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^{x}\times2\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{x}(\sin x)^{\frac{1}{2}}}{\mathrm{e}^{2x}}$	M1A1A1
		(3)
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}=0$	
	$\cos x=2\sin x$	M1
	$\tan x=\frac{1}{2}\Rightarrow x=0.464$	dM1A1
		(3)
		(10 marks)

(a)

B1: awrt 0.41576

(Note that degrees gives 0.068835….and scores B0)

(b)

B1: Strip width = $\frac{\pi}{4}$ or awrt 0.785. This may be implied by seeing $\frac{1}{2}\times\frac{\pi}{4}\times\{...\}$ or $\frac{\pi}{8}\times\{...\}$ within the trapezium formula

M1: Correct structure for the trapezium formula. Do not condone missing brackets unless they are implied by subsequent work. (Allow the 0's to be omitted in the brackets)

A1: awrt 1.0540 (Not 1.054) (note that this mark is still available even if (a) is not given to the required accuracy)

(Note that degrees gives 0.78149…)

(c)

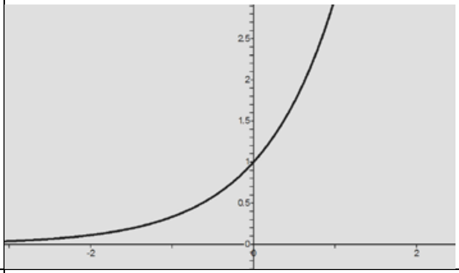
M1: Uses *vu*'+*uv*' with $u/v=2\mathrm{e}^{-x}, u/v=(\sin x)^{0.5}$ If the rule is quoted it must be correct.

It may be implied by, for example, $u=2\mathrm{e}^{-x}, v=\sqrt{\sin x}$ followed by their $u'=..,v'=..$and $vu'+uv'$

If it is not quoted nor implied then look for an expression of the form f(*x*) ± g(*x*) where f(*x*) or g(*x*) is of the form $A\mathrm{e}^{-x}\sqrt{\sin x}$ or $A\mathrm{e}^{-x}(\sin x)^{-0.5}\cos x$ with *A* non-zero.

A1: Either term of the derivative correct

A1: Completely correct derivative $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}$. Allow un-simplified and allow …+–… for … – …

Question	Scheme		Marks
9 (a)		Shape **or** *y* intercept at 1	B1
		Fully correct shape and intercept	B1
			[2]
(b)	State h = 2, or use of $\frac{1}{2}\times 2$;		B1
	$\underline{\{0.0625+16+2(\ 0.25+1+4\ )\}}$	For structure of $\{$................$\}$;	M1A1
	$\frac{1}{2}\times 2\times\underline{\{\ 26.5625\}}$ = *awrt* 26.56	Exact answer = $^{425}\!/_{16}$	A1cao
			[4]
(c)(i)	$4\times(b) = awrt\ 106$	Exact answer = $^{425}\!/_{4}$	M1A1ft
(ii)	$24+(b) = awrt\ 50.6$	Exact answer = $^{809}\!/_{16}$	M1A1ft
			[4] **(10 marks)**

(a)

B1 Score for either

- a correct shape for the curve. It must lie only in quadrants 1 and 2 and have a positive and increasing gradient from left to right. The gradient must be approximately 0 at the left hand end. Condone the curve appearing to be a straight line on the rhs. See Practice/Qualification items for clarification. Do not be concerned if it does not appear to be asymptotic to the *x*-axis at the LHS
- intercept at (0,1). Allow 1 being marked on the *y* - axis. Condone (1,0) on the correct axis.

B1 Fully correct. As a guide the gradient of the curve must appear to be 0 at the lh end and it must reach a level that is more than half way below the level of the intercept at (0,1). Allow $x=0, y=1$ in the text, it does not need to be on the sketch. Do not condone (1,0) even on the correct axis for this mark.

(b)

B1 For using a strip width of 2. This may appear in a trapezium rule as $\frac{1}{2}\times 2$ or 1 or equivalent

M1 Scored for the correct $\{......\}$ outer bracket structure. It needs to contain first *y* value plus last *y* value and the inner bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from inner bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are *x* values instead of *y* values

A1 For the correct bracket $\{......\}$

A1 For awrt 26.56. Accept 425/16

NB: Separate trapezia may be used: B1 for *h* =1, M1 for 1/2 *h*(*a* + *b*) used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Note: As *h* =1 the expression $1\times(16+0.0625)+2(0.25+1+4)$ will scores B1 M1 A1 with awrt 26.56 scoring the final A1.

(c)(i)

M1 For an attempt at finding $4\times(b)$. Also allow repeating the trapezium rule with each value ×4

A1ft For either awrt 106 or ft on the answer to $4\times(b)$ You may see 425/4 following 425/16 in (b)

(c)(ii)

M1 For an attempt at $24+(b)$ or $[3x]_{-4}^{4}+(b)$ Also allow repeating the trapezium rule with each value +3

A1ft For either awrt 50.6 or ft on the answer to 24+(b) You may see 809/16

Question	Scheme	Marks	AOs
10(a)	$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{H\cos 0.25t}{40} \Rightarrow \int \frac{1}{H}\mathrm{d}H = \int \frac{\cos 0.25t}{40}\mathrm{d}t$	M1	3.1a
	$\ln H = \frac{1}{10}\sin 0.25t\left(+c\right)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5 \Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right) = \frac{1}{10}\sin 0.25t \Rightarrow H = 5\mathrm{e}^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5\mathrm{e}^{0.1} = 5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t = \frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int \frac{1}{H}\mathrm{d}H = \int \frac{\cos 0.25t}{40}\mathrm{d}t$ or equivalent.

The integral signs need to be present on both sides and the $\mathrm{d}H$ AND $\mathrm{d}t$ need to be in the correct positions.

M1: Integrates both sides to reach $\ln H = A\sin 0.25t$ or equivalent with or without the + *c*

A1: $\ln H = \frac{1}{10}\sin 0.25t + c$ or equivalent with or without the + *c*. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H = 4\sin 0.25t + c$ or equivalent.

dM1: Substitutes $t=0, H=5 \Rightarrow c = ..$ There needs to have been a single " + *c* " to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H = 4\sin 0.25t + c \Rightarrow\Rightarrow H^{40} = \mathrm{e}^{4\sin 0.25t} + \mathrm{e}^{c} \Rightarrow 5^{40} = 1 + \mathrm{e}^{c} \Rightarrow c = ...$

Also many students will be attempting to get to the given answer so condone the method of finding c =...
These students will lose the A1* mark

A1*: Proceeds via $\ln H = \frac{1}{10}\sin 0.25t + \ln 5$ or equivalent to the given answer $H = 5\mathrm{e}^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone *c*'s going to *c*'s when they should be e^{c} or *A*

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="6">10 (a)</td><td colspan="2">$\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$ (for k or a numerical k)</td><td>M1</td><td>3.3</td></tr>
<tr><td colspan="2">$\int r^2\,dr = \int \pm k\,dt \Rightarrow ...$ (for k or a numerical k)</td><td>M1</td><td>2.1</td></tr>
<tr><td colspan="2">$\frac{1}{3}r^3 = \pm kt\ \{+c\}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td rowspan="2">$t=0,\ r=5$ and $t=4,\ r=3$
gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$,
where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth</td><td rowspan="2">$t=0,\ r=5$ and $t=240,\ r=3$
gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$,
where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth</td><td>M1</td><td>3.1a</td></tr>
<tr><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(5)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td colspan="2">$r=0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$</td><td>M1</td><td>3.4</td></tr>
<tr><td colspan="2">time = 5 minutes 6 seconds</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(c)</td><td colspan="2">Suggests a suitable limitation of the model. E.g.
• Model does not consider how the mint is sucked
• Model does not consider whether the mint is bitten
• Model is limited for times up to 5 minutes 6 seconds, o.e.
• Not valid for times greater than 5 minutes 6 seconds, o.e.
• Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked
• The model indicates that the radius of the mint is negative after it dissolves
• Model does not consider the temperature in the mouth
• Model does not consider rate of saliva production
• Mint could be swallowed before it dissolves in the mouth</td><td>B1</td><td>3.5b</td></tr>
<tr><td colspan="2"></td><td>(1)</td><td></td></tr>
<tr><td colspan="5" align="right">(8 marks)</td></tr>
</table>

Question	Scheme	Marks	AOs
14	$N = \frac{900}{3+7\mathrm{e}^{-0.25t}} = 900(3+7\mathrm{e}^{-0.25t})^{-1},\ t \in \mathbb{R},\ t \geq 0\ ;\quad \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$		
(a)	90	B1	3.4
		(1)	
(b) **Way 1**	$\frac{\mathrm{d}N}{\mathrm{d}t} = -900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{= \frac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1 A1	2.1 1.1b
	$\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}t} = \frac{900(0.25)\left(\left(\frac{900}{N}\right) - 3\right)}{\left(\frac{900}{N}\right)^2}$	dM1	2.1
	correct algebra leading to $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$ *	A1*	1.1b
		(4)	
(b) **Way 2**	$\frac{\mathrm{d}N}{\mathrm{d}t} = -900(3+7\mathrm{e}^{-0.25t})^{-2}\left(7(-0.25)\mathrm{e}^{-0.25t}\right)\ \left\{= \frac{900(0.25)(7)\mathrm{e}^{-0.25t}}{(3+7\mathrm{e}^{-0.25t})^2}\right\}$	M1 A1	2.1 1.1b
	$\frac{N(300-N)}{1200} = \frac{\left(\frac{900}{3+7\mathrm{e}^{-0.25t}}\right)\left(300 - \frac{900}{3+7\mathrm{e}^{-0.25t}}\right)}{1200}$	dM1	2.1
	$\text{LHS} = \frac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e., $\text{RHS} = \frac{900(300(3+7\mathrm{e}^{-0.25t})-900)}{1200(3+7\mathrm{e}^{-0.25t})^2} = \frac{1575\mathrm{e}^{-0..25t}}{(3+7\mathrm{e}^{-0.25t})^2}$ o.e. and states hence $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N(300-N)}{1200}$ (or LHS = RHS) *	A1*	1.1b
		(4)	
(c)	Deduces $N = 150$ (can be implied)	B1	2.2a
	so $150 = \frac{900}{3+7\mathrm{e}^{-0.25T}} \Rightarrow \mathrm{e}^{-0.25T} = \frac{3}{7}$	M1	3.4
	$T = -4\ln\left(\frac{3}{7}\right)$ or $T =$ awrt 3.4 (months)	dM1 A1	1.1b 1.1b
		(4)	
(d)	either one of 299 or 300	B1	3.4
		(1)	
			(10 marks)

Question	Scheme	Marks	AOs
14 (a)	$\{u=4-\sqrt{h}\Rightarrow\}\ \frac{\mathrm{d}u}{\mathrm{d}h}=-\frac{1}{2}h^{-\frac{1}{2}}$ or $\frac{\mathrm{d}h}{\mathrm{d}u}=-2(4-u)$ or $\frac{\mathrm{d}h}{\mathrm{d}u}=-2\sqrt{h}$	B1	1.1b
	$\left\{\int\frac{\mathrm{d}h}{4-\sqrt{h}}=\right\}\ \int\frac{-2(4-u)}{u}\,\mathrm{d}u$	M1	2.1
	$=\int\left(-\frac{8}{u}+2\right)\mathrm{d}u$	M1	1.1b
	$=-8\ln u+2u\ \{+c\}$	M1 A1	1.1b 1.1b
	$=-8\ln\left\|4-\sqrt{h}\right\|+2(4-\sqrt{h})+c\ =\ -8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h}\ +k\ \ *$	A1*	2.1
		(6)	
(b)	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{t^{0.25}(4-\sqrt{h})}{20}=0\Rightarrow\right\}\ 4-\sqrt{h}=0$	M1	3.4
	Deduces any of $0<h<16,\ 0\le h<16,\ 0<h\le16,\ 0\le h\le16,$ $h<16,\ h\le16$ or all values up to 16	A1	2.2a
		(2)	
(c) **Way 1**	$\int\frac{1}{(4-\sqrt{h})}\mathrm{d}h\ =\ \int\frac{1}{20}t^{0.25}\mathrm{d}t$	B1	1.1b
	$-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h}\ =\frac{1}{25}t^{1.25}\ \ \{+c\}$	M1 A1	1.1b 1.1b
	$\{t=0,\ h=1\Rightarrow\}\ -8\ln(4-1)-2\sqrt{(1)}\ =\frac{1}{25}(0)^{1.25}\ +c$	M1	3.4
	$\Rightarrow c=\ -8\ln(3)-2\ \Rightarrow\ \ -8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h}\ =\frac{1}{25}t^{1.25}-8\ln(3)-2$ $\{h=12\Rightarrow\}\ -8\ln\left\|4-\sqrt{12}\right\|-2\sqrt{12}\ =\frac{1}{25}t^{1.25}-8\ln(3)-2$	dM1	3.1a
	$t^{1.25}=221.2795202...\ \Rightarrow\ t=\sqrt[1.25]{221.2795...}$ or $t=(221.2795...)^{0.8}$	M1	1.1b
	$t=75.154...\Rightarrow t=$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) **Way 2**	$\int_1^{12}\frac{20}{(4-\sqrt{h})}\mathrm{d}h\ =\ \int_0^T t^{0.25}\mathrm{d}t$	B1	1.1b
	$\left[20(-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h})\right]_1^{12}=\left[\frac{4}{5}t^{1.25}\right]_0^T$	M1 A1	1.1b 1.1b
	$20(-8\ln(4-\sqrt{12})-2\sqrt{12})-20(-8\ln(4-1)-2\sqrt{1})=\frac{4}{5}T^{1.25}-0$	M1 dM1	3.4 3.1a
	$T^{1.25}=221.2795202...\ \Rightarrow\ T=\sqrt[1.25]{221.2795...}$ or $T=(221.2795...)^{0.8}$	M1	1.1b
	$T=75.154...\Rightarrow T=$ 75.2 (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = A\text{e}^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = \text{e}^{\pm t}$, $n = A\text{e}^{\pm t}$, $n = A\text{e}^{\pm kt}$ condoning $n = A + B\text{e}^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = A\text{e}^{kt}$, $n = \text{e}^{kt+c}$, $n = \text{e}^{kt}\text{e}^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = \text{e}^{kt+c}$ followed by $n = \text{e}^{kt} + \text{e}^{c}$ o.e. $n = \text{e}^{kt} + A$ such as is M1 A0

……

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{\text{d}n}{\text{d}t} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{\text{d}n}{\text{d}t} \propto n$

……

Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ (for positive constant c)	B1	3.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}=-c$ and $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	M1	2.1
	$-c=4\pi r^2\times\frac{\mathrm{d}r}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{c}{4\pi r^2}=-\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{k}{r^2}\Rightarrow\int r^2\mathrm{d}r=\int -k\mathrm{d}t$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3}=-kt(+\alpha)$	A1	1.1b
	Uses $t=0, r=40\Rightarrow\alpha=...$ $\alpha=\frac{64000}{3}$	M1	1.1b
	Uses $t=5, r=20$ & $\alpha=...\Rightarrow k=...$	M1	3.4
	$r^3=64000-11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. $"64000-11200t"...0\Rightarrow t...$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
		(10 marks)	

Notes:

Question Number	Scheme	Marks
9(a)	$u = 4-\sqrt{x} \Rightarrow x = (4-u)^2 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = -2(4-u)$	M1A1
	$\displaystyle\int \frac{\mathrm{d}x}{4-\sqrt{x}} = \int \frac{-2(4-u)\mathrm{d}u}{u} = \int -\frac{8}{u} + 2\mathrm{d}u$	M1A1
	$= -8\ln u + 2u \quad (+c)$	dM1
	$= -8\ln\left\|4-\sqrt{x}\right\| + 2\left(4-\sqrt{x}\right)(+c)$ oe	A1
		(6)
(b)	Height increases when $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{4-\sqrt{h}}{20} > 0 \Rightarrow (0<)\ h < 16$	M1A1
		(2)
(c)	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{4-\sqrt{h}}{20} \Rightarrow \displaystyle\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \int \frac{\mathrm{d}t}{20}$	B1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20} + c$	M1A1
	Substitute t=0, h=1 $\quad -8\ln 3 + 6 = c$	dM1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20} - 8\ln 3 + 6$ oe	A1
	Substitute h=10 into $\Rightarrow -8\ln\left(4-\sqrt{10}\right) + 2\left(4-\sqrt{10}\right) = \dfrac{t}{20} - 8\ln 3 + 6$	ddM1
	$\Rightarrow t = \text{awrt } 118 \text{ (years)}$	A1
		(7) **(15 marks)**
Alt (c)	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{4-\sqrt{h}}{20} \Rightarrow \displaystyle\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \int \frac{\mathrm{d}t}{20}$	B1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20}$	M1
	$\Rightarrow \left[-8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right)\right]_{h=1}^{h=10} = \left[\dfrac{t}{20}\right]_{t=0}^{T}$	A1
	$\Rightarrow \left(-8\ln\left(4-\sqrt{10}\right) + 2\left(4-\sqrt{10}\right)\right) - \left(-8\ln\left(4-\sqrt{1}\right) + 2\left(4-\sqrt{1}\right)\right) = \dfrac{T}{20}$	dM1,ddM1, A1
	$\Rightarrow T = \text{awrt } 118 \text{ (years)}$	A1

Question Number	Scheme	Marks
6. (a)	$\frac{5-4x}{(2x-1)(x+1)} \equiv \frac{A}{(2x-1)} + \frac{B}{(x+1)}$ so $5-4x \equiv A(x+1)+B(2x-1)$	B1
	Let $x=-1$, $9 = B(-3) \Rightarrow B =$ Let $x=\frac{1}{2}$, $3 = A\left(\frac{3}{2}\right) \Rightarrow A =$	M1
	A = 2 and B = -3 or $\left\{\frac{5-4x}{(2x-1)(x+1)} \equiv \frac{2}{(2x-1)} - \frac{3}{(x+1)}\right\}$	A1
		[3]
(b) (i), (ii)	$\int \frac{1}{y}\,\mathrm{d}y = \int \frac{5-4x}{(2x-1)(x+1)}\,\mathrm{d}x$	B1
	$= \int \frac{2}{(2x-1)} - \frac{3}{(x+1)}\,\mathrm{d}x = C\ \ln(2x-1) + D\ln(x+1)$	M1
	$= \frac{\text{"2"}}{2}\ln(2x-1) - \text{"3"}\ln(x+1)$	A1ft
	$\ln y = \ln(2x-1) - 3\ln(x+1) + c$	A1
Method 1 for (ii)	$\ln 4 = \ln(2(2)-1) - 3\ln(2+1) + c \Rightarrow c = \{\ln 36\}$	M1
	$\ln y = \ln(2x-1) - 3\ln(x+1) + \ln 36$ so $\ln y = \ln\left(\frac{36(2x-1)}{(x+1)^3}\right)$ So $y = \frac{36(2x-1)}{(x+1)^3}$	M1 A1 **[7]**
Method 2 for (ii)	Solution as Method 1 up to $\ln y = \ln(2x-1) - 3\ln(x+1) + c$ so first four marks as before	B1M1A1A1
	Writes $y = \frac{A(2x-1)}{(x+1)^3}$ as general solution which would earn the 3rd M1 mark.	M1
	Then may substitute to find their constant A, which would earn the 2nd M1 mark.	M1
	Then A1 for $y = \frac{36(2x-1)}{(x+1)^3}$ as before.	A1
		[7] **10**

Question Number	Scheme	Marks
8 (a)	$\frac{\mathrm{d}}{\mathrm{d}y}(\ln\tan 2y)=\frac{1}{\tan 2y}\times 2\sec^2 2y$	M1A1
	$=\frac{\cos 2y}{\sin 2y}\times\frac{2}{\cos^2 2y} \quad = =\frac{k}{\sin 2y\cos 2y}$	M1
	$=\frac{4}{2\sin 2y\cos 2y} \quad =\frac{4}{\sin 4y}$	A1* cso
		(4)
(b) **Way 1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos x\sin 4y\Rightarrow\int\frac{\mathrm{d}y}{\sin 4y}=\int 2\cos x\mathrm{d}x$	B1
	$\Rightarrow\frac{1}{4}\ln\tan 2y=2\sin x(+c)$	M1A1
Finds limits first	Put $x=0, y=\frac{\pi}{6}\Rightarrow\frac{1}{4}\ln\tan 2\frac{\pi}{6}=2\sin 0+c\Rightarrow c=...\left(\frac{1}{4}\ln\sqrt{3}\ or\ \frac{1}{8}\ln 3\right)$	M1
	Takes exponentials so $\tan 2y=\mathrm{e}^{8\sin x+c}$	M1
Finds limits after removing lns	$\tan 2y=\mathrm{e}^{8\sin x+c}$ (so $\tan 2y=A\mathrm{e}^{8\sin x}$)	M1 (bM3 on epen)
	Put $x=0, y=\frac{\pi}{6}$, so $A=$ or $\mathrm{e}^{c}=$	M1 (bM2 on epen)
	$\tan 2y=\sqrt{3}\ \mathrm{e}^{8\sin x}$	A1
		(6) **(10 marks)**
(b) **Way 2**	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos x\sin 4y\Rightarrow\int\frac{\mathrm{d}y}{\sin 4y}=\int 2\cos x\mathrm{d}x$	B1
	$\Rightarrow-\frac{1}{4}\ln(\mathrm{cosec}4y+\cot 4y)=2\sin x(+c)$	M1A1
	Sub $x=0, y=\frac{\pi}{6}$	
	$\Rightarrow-\frac{1}{4}\ln(\mathrm{cosec}\frac{2\pi}{3}+\cot\frac{2\pi}{3})=2\sin 0+c\Rightarrow c=...\left(-\frac{1}{4}\ln\frac{1}{\sqrt{3}}\text{ or }\frac{1}{4}\ln\sqrt{3}\right)$	M1
	$-\frac{1}{4}\ln(\frac{1+\cos 4y}{\sin 4y})=\frac{1}{4}\ln(\tan 2y)=2\sin x+\frac{1}{4}\ln\sqrt{3}$ so $\tan 2y=\sqrt{3}\ \mathrm{e}^{8\sin x}$	M1A1 **(6)**
(b) **Way 3**	Special case: Differentiates the answer. Marks available B0M1A0M1M1A1	
	$\tan 2y=A\ \mathrm{e}^{B\sin x}\rightarrow(2)\sec^2 2y\frac{\mathrm{d}y}{\mathrm{d}x}=AB\cos x\mathrm{e}^{B\sin x}$	**M1**
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{B\cos x\tan 2y\cos^2 2y}{(2)}$	**M1**
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{B\cos x 2\sin 2y\cos 2y}{(4)} \quad = \quad \frac{B\cos x\sin 4y}{(4)}$	**M1**
	$B=8$ and $A=\sqrt{3}$	**A1 4/6**

Question Number	Scheme	Marks
9(a)	$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x-2}$	
	$\frac{2}{x^2}, \frac{-6}{3x-2}$ $(B=2\ ,\ C=-6)$	B1, B1,
	$3x^2-4 \equiv Ax(3x-2)+B(3x-2)+Cx^2 \Rightarrow A=..$	M1
	$\frac{3}{x}$ $(A=3)$ is one of the fractions	A1
		[4]
(b)	$\int \frac{1}{y}\,\mathrm{d}y = \int \frac{3x^2-4}{x^2(3x-2)}\,\mathrm{d}x$	B1
	$\ln y = \int \frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x-2}\,\mathrm{d}x$	M1
	$= A\ln x - \frac{B}{x}+\frac{C}{3}\ln(3x-2)$ $(+k)$	M1A1ft
	$y = \mathrm{e}^{A\ln x-\frac{B}{x}+\frac{C}{3}\ln(3x-2)+D}$ or $y = D\mathrm{e}^{A\ln x-\frac{B}{x}+\frac{C}{3}\ln(3x-2)}$	M1
	$y = Kx^3(3x-2)^{-2}e^{-\frac{2}{x}}$ or $\frac{Kx^3e^{-\frac{2}{x}}}{(3x-2)^2}$ or $\frac{\mathrm{e}^k x^3\mathrm{e}^{-\frac{2}{x}}}{(3x-2)^2}$ oe	A1cso
		[6] **(10 marks)**

Question Number	Scheme	Marks
13(a)	$V=\frac{1}{3}\pi h^2(30-h)=10\pi h^2-\frac{1}{3}\pi h^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h}=20\pi h-\pi h^2$ or $V=\frac{1}{3}\pi h^2(30-h)\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h}=\frac{2}{3}\pi h(30-h)-\frac{1}{3}\pi h^2$	M1A1
	M1: Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$ either by multiplying out and differentiating each term to give a derivative of the form $\alpha h-\beta h^2$ or by the product rule to give a derivative of the form $\alpha h(30-h)\pm\beta h^2$. **A1:** Any correct (possibly un-simplified) form for $\frac{\mathrm{d}V}{\mathrm{d}h}$	
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow -\frac{1}{10}V=\left(20\pi h-\pi h^2\right)\times\frac{\mathrm{d}h}{\mathrm{d}t}$	M1
	Uses a **correct** form of the chain rule, e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ or uses $\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{1}{10}V$.	
	$\Rightarrow -\frac{1}{10}\times\frac{1}{3}\pi h^2(30-h)=\pi h(20-h)\times\frac{\mathrm{d}h}{\mathrm{d}t}\left(\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=...\right)$	M1
	Substitutes $V=\frac{1}{3}\pi h^2(30-h)$ and rearranges to obtain $\frac{\mathrm{d}h}{\mathrm{d}t}$ in terms of h	
	$\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{h(30-h)}{30(20-h)}*$ This is a given answer. There must have been intermediate lines and correct factorisation and no errors and "$\frac{\mathrm{d}h}{\mathrm{d}t}=$"must be seen at some point.	A1*
		(5)
(b)	$\frac{30(20-h)}{h(30-h)}\equiv\frac{A}{h}+\frac{B}{30-h}$ Correct form for the partial fractions	B1
	$30(20-h)\equiv A(30-h)+Bh$ $h=30\Rightarrow 30B=-300\Rightarrow B=-10$ and $h=0\Rightarrow 30A=600\Rightarrow A=20$ Attempts to get both constants by a correct method e.g. substituting, comparing coefficients, cover up rule	M1
	$\frac{30(20-h)}{h(30-h)}\equiv\frac{20}{h}-\frac{10}{30-h}$ Correct partial fractions (or states "A" = 20, "B" = -10)	A1
		(3)

(c)	**Way 1**		
	$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)}\mathrm{d}h = -1\int \mathrm{d}t$ A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other.		B1
	$20\ln h + 10\ln(30-h)$	M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$ A1: Correct integration for their partial fractions of the form $\frac{A}{h}+\frac{B}{30-h}$ following through their "*A*" and "*B*".	M1A1ft
	$t = 0, h = 10 \Rightarrow c = 20\ln 10 + 10\ln 20$	Substitutes *h* = 10 and *t* = 0 to find a value for *c*. NB *c* = 76.0…	M1
	$h = 5 \Rightarrow t = 20\ln 10 + 10\ln 20 - 10\ln 25 - 20\ln 5$ Substitutes *h* = 5 and uses their value of *c* to find a value for *t*.		ddM1
	$t = 11.63$ (secs)	Awrt 11.63 only	A1cso
			(6)
			(14 marks)
	(c) Way 2		
	$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h(30-h)}{30(20-h)} \Rightarrow \int \frac{30(20-h)}{h(30-h)}\mathrm{d}h = -1\int \mathrm{d}t$ A correct statement which may be implied by subsequent work. Condone the omission of "d*h*" and "d*t*" provided the intention is clear but the minus sign must be present on one side or the other.		B1
	$20\ln h + 10\ln(30-h)$	M1: Integrates their partial fractions to obtain $\pm P\ln h \pm Q\ln(30-h)$ A1: Correct integration for their partial fractions of the form $\frac{A}{h}+\frac{B}{30-h}$ following through their "*A*" and "*B*".	M1A1ft
	$(t =)[20\ln h + 10\ln(30-h)]_5^{10}$ or $(t =)[20\ln h + 10\ln(30-h)]_{10}^{5}$	Attempts the limits 5 and 10 for *h*. Either statement as shown is sufficient.	M1
	$(t =)[20\ln 10 + 10\ln 20] - [20\ln 5 + 10\ln 25]$	Substitutes *h* = 5 and *h* = 10 to find a value for *t*.	ddM1
	$t = 11.63$	Awrt 11.63 only	A1cso
			(6)

Qu	Scheme		Marks
12. (a)	Way 1: Uses $x = kt$ or $t = cx$ and $x = 1.5$ when $t = 2$ so $k =$ or $c =$ $t = \frac{4}{3}x$	Way 2: Uses $x = kt + c$ with $x = 0$, $t = 0$ and with $x = 1.5$ when $t = 2$ so $k =$ $t = \frac{4}{3}x$	M1 A1 **(2)**
(b)	$t = 4$		B1 **(1)**
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\lambda}{(2x+1)}$ so separate variables to give $\int (2x+1)\,\mathrm{d}x = \int \lambda\,\mathrm{d}t$		M1
	$x^2 + x = \lambda t(+c')$ or $\frac{(2x+1)^2}{4} = \lambda t(+c)$ so $t =$		M1
	(When $t = 0, x = 0$ so $c = ¼$ or c' =0) so $t = \frac{x^2 + x}{\lambda}$		A1 **(3)**
(d)	Uses $x = 1.5$ when $t = 2$ to give $\lambda = \frac{15}{8}$		B1 **(1)**
(e)	$t = \frac{x^2 + x}{\lambda} = \frac{12}{\lambda}$ = 6.4 hours later so 10.24pm or 22.24		M1 A1 **(2)** **(9 marks)**

Mark (a) and (b) together

(a)

M1: Uses correct $x = kt$ or $t = cx$ *and* $x = 1.5$ when $t = 2$ to find their constant (may not be *k or c*)
This may be the result of a differential equation $\mathrm{d}x/\mathrm{d}t = k$

A1: $t = \frac{4}{3}x$ oe such as $t = \frac{x}{0.75}$ or even $t = \frac{x}{3/4}$ Just this with no working is M1 A1

(b)

B1: $t = 4$

Mark (c),(d) and (e) together

(c)

M1: Correct separation but condone missing integral signs

M1: Correct form for both integrals- may not find *c* or even include a *c*

A1: Obtains a correct answer for *t* in terms of *x* and λ by using either $x = 0, t = 0 \Rightarrow t = \frac{x^2 + x}{\lambda}$ or

$t = \frac{(2x+1)^2 - 1}{4\lambda}$ oe. Alternatively uses $x = 1.5, t = 2 \Rightarrow t = \frac{4x^2 + 4x + 8\lambda - 15}{4\lambda}$ oe

Condone correct responses where '*c*' seems to have been either cancelled out or ignored

(d)

B1: $\lambda = \frac{15}{8}$ or decimal i.e. 1.875

(e)

M1: Substitutes $x = 3$ into their expression for t. Implied by $t = \frac{12}{"\lambda"}$

A1: 10.24pm or 22:24 only

Question Number	Scheme	Marks
12 (a)	States or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=0.4\pi-0.2\pi\sqrt{h}$	B1
	States or uses $V=4\pi h\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi$	M1A1
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow0.4\pi-0.2\pi\sqrt{h}=4\pi\times\frac{\mathrm{d}h}{\mathrm{d}t}$	M1
	$20\frac{dh}{dt}=2-\sqrt{h}$	A1*
		(5)
(b)	Separates the variables $\int20\frac{\mathrm{d}h}{2-\sqrt{h}}=\int1\,\mathrm{d}t$	M1
	$(t=)\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\mathrm{d}h$	A1*
		(2)
(c)	$h=(2-x)^2\Rightarrow\mathrm{d}h=-2(2-x)\mathrm{d}x$	B1
	$T=\int\frac{20}{2-\sqrt{h}}\mathrm{d}h=\int\frac{20}{2-(2-x)}\times-2(2-x)\mathrm{d}x$	M1
	$=\int-\frac{80}{x}+40\,\mathrm{d}x$ **(No need for limits here)**	**d**M1A1
	$T=\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\mathrm{d}h=\int_{1.6}^{0.5}-\frac{80}{x}+40\,\mathrm{d}x=\left[-80\ln x+40x\right]_{1.6}^{0.5}$	**dd**M1
	$=\left[-80\ln0.5+20\right]-\left[-80\ln1.6+64\right]=80\ln3.2-44=49\left(\text{minutes}\right)$	**ddd**M1A1
		(7)
		(14 marks)

(a)

B1: States or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=0.4\pi-0.2\pi\sqrt{h}$. This may be embedded within the chain rule but must be identifiable as $\frac{\mathrm{d}V}{\mathrm{d}t}$

M1: Attempts $\frac{\mathrm{d}V}{\mathrm{d}h}$ from an equation for the volume of a cylinder. Accept $V=c\times\pi h\rightarrow\frac{\mathrm{d}V}{\mathrm{d}h}=c\pi$

A1: $\frac{\mathrm{d}V}{\mathrm{d}h}=4\pi$ This may be embedded within the chain rule

M1: Uses a correct form of the chain rule: Eg $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}$

Also accept forms such as $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$

A1*: $20\frac{dh}{dt}=2-\sqrt{h}$ This is a given answer so no errors must be seen e.g. bracketing errors.

Question Number	Scheme	Marks
10 (a)	$t(t-4)=0\Rightarrow t=4$ Hence $x=\frac{20\times 4}{2\times 4+1}=\frac{80}{9}$	M1A1
		(2)
(b)	$x=\frac{20t}{2t+1}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{20(2t+1)-20t\times 2}{(2t+1)^2}=\left(\frac{20}{(2t+1)^2}\right)$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{(2t-4)}{20/(2t+1)^2},=\frac{(t-2)(2t+1)^2}{10}$	M1A1,A1
		(5)
	Mark c(i) and (ii) together:	
(c)(i)	$x=\frac{20t}{2t+1}\Rightarrow 2tx+x=20t\Rightarrow t(20-2x)=x\Rightarrow t=\frac{x}{20-2x}$ or $\frac{-x}{2x-20}$	M1A1
		(2)
(ii)	Sub $t=\frac{x}{20-2x}$ into $y=t(t-4)\Rightarrow y=\frac{x}{20-2x}\left(\frac{x}{20-2x}-4\right)$	M1
	$\Rightarrow y=\frac{x}{20-2x}\times\left(\frac{x}{20-2x}-\frac{4(20-2x)}{20-2x}\right)$	**d**M1
	$\Rightarrow y=\frac{x}{20-2x}\times\left(\frac{9x-80}{20-2x}\right)$	
	$\Rightarrow y=\frac{x(9x-80)}{(20-2x)^2}$, oe $\quad 0<x<10$ or $k=10$	A1, B1
		(4)
		(13 marks)

(a)

M1: Attempts to find x when $t=4$

A1: $\frac{80}{9}$ (Not 8.88… but isw if $\frac{80}{9}$ is seen)

(Ignore any attempts to find x when $t=0$)

(b)

M1: Attempts to apply the quotient rule on $\frac{20t}{2t+1}$ with $u=20t, v=2t+1$

Alternatively applies the product rule on $20t(2t+1)^{-1}$ OR writes $\frac{20t}{2t+1}$ as $A-\frac{B}{2t+1}$ and uses the chain rule

A1: $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{20(2t+1)-20t\times 2}{(2t+1)^2}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}=20(2t+1)^{-1}+20t\times -2(2t+1)^{-2}$

Question Number	Scheme	Notes	Marks
7.	$\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{(h-9)},\ \ 9 < h \le 200;\quad h = 130,\ \frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$		
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow\ k = ...$	Substitutes $h = 130$ and either $\frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$ or $\frac{\mathrm{d}h}{\mathrm{d}t} = 1.1$ into the printed equation and rearranges to give $k = ...$	M1
	so, $k = -\frac{1}{10}$ or -0.1	$k = -\frac{1}{10}$ or -0.1	A1
			[2]
(b) **Way 1**	$\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int k\,\mathrm{d}t$	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs.	B1
	$\int (h-9)^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t$		
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\,(+c)$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = (\text{their } k)t$, with/without $+c$, or equivalent, which can be un-simplified or simplified.	A1
	$\{t = 0, h = 200 \Rightarrow\}\ 2\sqrt{(200-9)} = k(0) + c$	Some evidence of applying both $t = 0$ and $h = 200$ to changed equation containing a constant of integration, e.g. c or A	M1
	$\Rightarrow\ c = 2\sqrt{191} \Rightarrow\ 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ \ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = ...$	**dependent on the previous M mark** Applies $h = 50$ and their value of c to their changed equation and rearranges to find the value of $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ isw or awrt 148	A1 cso
			[6]
(b) **Way 2**	$\int_{200}^{50} \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int_{0}^{T} k\ \mathrm{d}t$	Separates the variables correctly. $\mathrm{d}h$ and $\mathrm{d}t$ should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary.	B1
	$\int_{200}^{50} (h-9)^{-\frac{1}{2}}\mathrm{d}h = \int_{0}^{T} k\ \mathrm{d}t$		
	$\left[\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{200}^{50} = \left[kt\right]_0^T$	Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu \neq 0$	M1
		$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = (\text{their } k)t$, with/without limits, or equivalent, which can be un-simplified or simplified.	A1
	$2\sqrt{41} - 2\sqrt{191} = kt$ or kT	Attempts to apply limits of $h = 200, h = 50$ and (can be implied) $t = 0$ to their changed equation	M1
	$t = \frac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$	**dependent on the previous M mark** Then rearranges to find the value of $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	$t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148 **or** 2 hours and awrt 28 minutes	A1 cso
			[6]
			8

Question Number	Scheme	Marks
7. (a)	$\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$	
	$2 \equiv A(P-2) + BP$ Can be implied.	M1
	$A = -1,\ B = 1$ Either one.	A1
	giving $\frac{1}{(P-2)} - \frac{1}{P}$ See notes. **cao, aef**	A1
		[3]
(b)	$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{2}P(P-2)\cos 2t$	
	$\int \frac{2}{P(P-2)}\,\mathrm{d}P = \int \cos 2t\ \mathrm{d}t$ can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t\ \ (+c)$ $\pm\lambda\ln(P-2) \pm \mu\ln P,$ $\lambda \neq 0,\ \mu \neq 0$	M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\ \ \ln 1 - \ln 3 = 0 + c\ \ \{\Rightarrow\ c = -\ln 3 \text{ or } \ln(\tfrac{1}{3})\}$ See notes	M1
	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t - \ln 3$ $\ln\left(\frac{3(P-2)}{P}\right) = \frac{1}{2}\sin 2t$	
	$\frac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t}$ Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,$ $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t}$ gives $3P - P\mathrm{e}^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$ A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ * Correct proof.	A1 * cso
		[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$ States $P = 4$ or applies $P = 4$	M1
	$\frac{1}{2}\sin 2t = \ln\left(\frac{3(4-2)}{4}\right)\ \left\{= \ln\left(\frac{3}{2}\right)\right\}$ Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$ anything that rounds to 0.473 Do not apply isw here	A1
		[3] **13**

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
7. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \quad (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = A\mathrm{e}^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda,\mu,\beta,K,\delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = AP\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow P - AP\mathrm{e}^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - A\mathrm{e}^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - A\mathrm{e}^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0,\ P = 3 \Rightarrow\}\quad 3 = \frac{2}{(1 - A\mathrm{e}^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}\mathrm{e}^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

		Question 7 Notes
7. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1,\ B = 1$

Question Number	Scheme		Marks
8.	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = k\left(M - x\right)$, where M is a constant		
(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}$ is the <u>rate of increase</u> of the <u>mass of waste</u> products.	Any one correct explanation.	B1
	M is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u> (or the <u>initial mass</u> of <u>unburned fuel</u>)	Both explanations are correct.	B1
			[2]
(b)	$\displaystyle\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$ or $\displaystyle\int \frac{1}{k(M-x)}\,\mathrm{d}x = \int \mathrm{d}t$		B1
	$-\ln\left(M - x\right) = kt\ \{+c\}$ or $-\dfrac{1}{k}\ln\left(M - x\right) = t\ \{+c\}$	See notes	M1 A1
	$\{t = 0\ ,\ x = 0 \Rightarrow\}\ -\ln\left(M - 0\right) = k(0) + c$	See notes	M1
	$c = -\ln M \Rightarrow -\ln\left(M - x\right) = kt - \ln M$		
	then either... $-kt = \ln\left(M - x\right) - \ln M$ $-kt = \ln\left(\dfrac{M-x}{M}\right)$ $e^{-kt} = \dfrac{M-x}{M}$		ddM1
	or... $kt = \ln M - \ln\left(M - x\right)$ $kt = \ln\left(\dfrac{M}{M-x}\right)$ $e^{kt} = \dfrac{M}{M-x}$		
	$Me^{-kt} = M - x$ **or** $\left(M - x\right)e^{kt} = M$ $M - x = Me^{-kt}$		
	leading to $x = M - Me^{-kt}$ or $x = M(1 - e^{-kt})$ oe		A1 * **cso**
			[6]
(c)	$\left\{x = \dfrac{1}{2}M\ ,\ t = \ln 4 \Rightarrow\right\}\ \dfrac{1}{2}M = M(1 - e^{-k\ln 4})$		M1
	$\Rightarrow \dfrac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \dfrac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$		
	So $k = \dfrac{1}{2}$		A1
	$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$		dM1
	$x = \dfrac{2}{3}M$	$x = \dfrac{2}{3}M$	A1 **cso**
			[4] **12**