Question	Scheme	Marks	AOs
2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\text{f}(1.8) = 7 - 2\times 1.8^2 = 0.52 \Rightarrow \text{gf}(1.8) = \text{g}(0.52) = \dfrac{3\times 0.52}{5\times 0.52 - 1} = \ldots$	M1	1.1b
	$\text{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\text{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
			(5 marks)

Notes

(a)

B1: Correct range. Allow f (x) or f for y. Allow e.g. $\{y \in \mathbb{R}:\ y \leqslant 7\}$, $-\infty < y \leqslant 7$, $(-\infty, 7]$

(b)

M1: Full method to find f (1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute x = 1.8 into an attempt at gf (x).

E.g. $\text{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3\left(7-2(1.8)^2\right)}{5\left(7-2\times(1.8)^2\right)-1} = \ldots$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}$, $\dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

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Question	Scheme	Marks	AOs
1	$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = ...$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

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Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a = 3$, $f(x) = 3x^3 + 10x^2 - 9x - 4$ and shows that $(x-1)$ is a factor of this $f(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3 + 10x^2 - 9x - 4 = (x-1)(3x^2 + 13x + 4)$ Hence $a = 3$

E.g. 2: $f(x) = 3x^3 + 10x^2 - 9x - 4$, $f(1) = 3 + 10 - 9 - 4 = 0$ Hence $a = 3$

The solutions via this method must end with the value for a to score the A1

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Question	Scheme	Marks	AOs
1	$\mathrm{g}(x)=\dfrac{2x+5}{x-3},\ x\geq 5$		
(a) Way 1	$\mathrm{g}(5)=\dfrac{2(5)+5}{5-3}=7.5\Rightarrow \mathrm{gg}(5)=\dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) Way 2	$\mathrm{gg}(x)=\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}\Rightarrow \mathrm{gg}(5)=\dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2<y\leq\dfrac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y=\dfrac{2x+5}{x-3}\Rightarrow\ yx-3y=2x+5\Rightarrow yx-2x=3y+5$	M1	1.1b
	$x(y-2)=3y+5\ \Rightarrow\ x=\dfrac{3y+5}{y-2}\ \left\{\text{or } y=\dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{3x+5}{x-2},\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y=\dfrac{2x-6+11}{x-3}\ \Rightarrow\ y=2+\dfrac{11}{x-3}\ \Rightarrow y-2=\dfrac{11}{x-3}$	M1	1.1b
	$x-3=\dfrac{11}{y-2}\ \Rightarrow x=\dfrac{11}{y-2}+3\ \left\{\text{or } y=\dfrac{11}{x-2}+3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{11}{x-2}+3,\quad 2<x\leq\dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question 1

(a)	
M1:	Full method of attempting $\mathrm{g}(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x=5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $\mathrm{gg}(x)=\dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Question	Scheme	Marks	AOs
1	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

$$\begin{array}{r|l} & 3x^2+(2a-9)x+23-6a \\ x+3 & 3x^3+2ax^2-4x+5a \\ & \underline{3x^3+9x^2} \\ & (2a-9)x^2 -4x \\ & \underline{(2a-9)x^2 +(6a-27)x} \\ & (23-6a)x+5a \\ & (23-6a)x+69-18a \end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

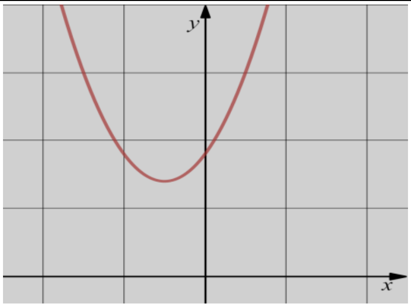
M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm 3)=0$. Allow for a linear equation in a leading to $a=...$. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{)3x^3+2ax^2-4x+5a}$ with quotient $3x^2...$ followed by a remainder in a set $=0\Rightarrow a=...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

Question	Scheme	Marks	AOs
6 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x = ...$ or $4x-7=28 \Rightarrow x = ...$	M1	1.1b
	At least one critical value $x = 2-3\sqrt{3}$ or $x = \frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x = ...$ and $4x-7=28 \Rightarrow x = ...$	M1	1.1b
	Correct final answer of '$x < 2-3\sqrt{3}$, $x > \frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	(4)	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x = \left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x = \left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $\text{h}^{-1}(x)$} $= \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 6

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x = 0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x = 0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0) = 13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x = -1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R} : x < 2-3\sqrt{3} \cup x > 8.75\}$ is fine for the final A mark

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
	(5 marks)		
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*\_*}{*\_*}\right)-a}{\left(\frac{*\_*}{*\_*}\right)-b}$ where *a* and *b* are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

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Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)

Notes:

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line l.

E.g. Attempt using two points: Finds $m = \pm\frac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find c. Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form $y = \ldots$ but the correct equation can be implied from an inequality. E.g. $\ldots < y < 6x + 25$

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The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant a in $y = a(x \pm 2)^2 + 13$ or the constants a, b in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in a and b using the point (-2, 13) the gradient being 0 at $x = -2$. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25$ $(2a - b = -6)$ and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $\mathrm{f}(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25 \, \mathrm{d}x$

B1ft: Fully defines the region R. Follow through on their equations for l and C.

Allow strict or non -strict inequalities as long as they are used consistently.

E.g. Allow for example "$3(x+2)^2 + 13 < y < 6x + 25$" "$3(x+2)^2 + 13 \leqslant y \leqslant 6x + 25$"

Allow the inequalities to be given separately, e.g. $y < 6x + 25$, $y > 3(x+2)^2 + 13$. Set notation may be used so $\{(x,y) : y > 3(x+2)^2 + 13\} \cap \{(x,y) : y < 6x + 25\}$ is fine but condone with or without any of $(x,y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include "$y < 6x + 25$ **or** $y > 3(x+2)^2 + 13$", $\{(x,y) : y > 3(x+2)^2 + 13\} \cup \{(x,y) : y < 6x + 25\}$

Values of x could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25, x < 0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs
4 (a)	$\mathrm{gf}(x) = 3\ln \mathrm{e}^x$	M1	1.1b
	$= 3x,\ \left(x \in \mathbb{R}\right)$	A1	1.1b
		(2)	
(b)	$\mathrm{gf}(x) = \mathrm{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln \mathrm{e}^x$
An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $\mathrm{e}^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$
or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question Number	Scheme		Marks
7. (a)	Method 1	Method 2	
	$y = \dfrac{3x-5}{x+1}$	$y = 3 - \dfrac{8}{x+1}$	
	$y(x+1) = 3x-5 \Rightarrow xy + y = 3x - 5$	$\dfrac{8}{x+1} = 3 - y$ so $x + 1 = \dfrac{8}{3-y}$	M1
	$y + 5 = 3x - xy \Rightarrow y + 5 = x(3-y)$ $\Rightarrow \dfrac{y+5}{3-y} = x$	$x = \dfrac{8}{3-y} - 1$	M1
	Hence $\left(f^{-1}(x)\right) = \dfrac{x+5}{3-x}$ $(x \in \mathbb{R},\ x \neq 3)$	Hence $\left(f^{-1}(x)\right) = \dfrac{8}{3-x} - 1$ $(x \in \mathbb{R},\ x \neq 3)$	A1 oe
			[3]
(b)	$ff(x) = \dfrac{3\left(\dfrac{3x-5}{x+1}\right) - 5}{\left(\dfrac{3x-5}{x+1}\right) + 1}$	$ff(x) = 3 - \dfrac{8}{3 - \dfrac{8}{x+1} + 1}$	M1 A1
	$= \dfrac{\dfrac{3(3x-5) - 5(x+1)}{x+1}}{\dfrac{(3x-5) + (x+1)}{x+1}}$	$ff(x) = 3 - \dfrac{8(x+1)}{4x-4}$	M1
	$= \dfrac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \dfrac{4x - 20}{4x - 4}$		
	$= \dfrac{x-5}{x-1}$ (note that $a = -5$.)	$= \dfrac{x-5}{x-1}$	A1
			[4]
(c)	$fg(2) = f(4-6) = f(-2) = \dfrac{3(-2) - "5"}{-2+1}$;$= 11$ or substitute 2 into $fg(x) = \dfrac{3(x^2-3x)-5}{x^2-3x+1}$;$= 11$		M1; A1
			[2]
(d)	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$		M1
	Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$		B1
	$\underline{-2.25 \leqslant g(x) \leqslant 10}$ or $\underline{-2.25 \leqslant y \leqslant 10}$		A1
			[3] **12**

Qu	Scheme	Marks
4.(a)	$0 < \text{f}(x) < \frac{4}{5}$	M1A1 **(2)**
(b)	$y = \dfrac{4}{3x+5} \qquad \Rightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \dfrac{4-5y}{3y}$	dM1
	$\text{f}^{-1}(x) = \dfrac{4-5x}{3x} \qquad \left(0 < x < \frac{4}{5}\right)$	A1o.e. **(3)**
(c)	$\text{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$	B1 (1)
(d)	$\dfrac{3x+5}{4} = \dfrac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4\times15\times15$ and so deduce no real roots	M1 A1 **(4)** **(10 marks)**

(a)
M1: One limit such as $y > 0$ or $y < 0.8$. Condone for this mark both limits but with x (not y) or with the boundary included. For example $[0, 0.8], 0 < x < 0.8, 0 \leqslant y \leqslant 0.8$

A1: Fully correct so accept $0 < \text{f}(x) < \frac{4}{5}$ and exact equivalents $0 < y < \frac{4}{5}$ $(0, 0.8)$

(b)
M1: Set $y = \text{f}(x)$ or $x = \text{f}(y)$ and multiply both sides by denominator.
dM1:Make x (or a swapped y) the subject of the formula. Condone arithmetic slips

A1: o.e for example $y/\text{f}^{-1}(x) = \frac{1}{3}\left(\frac{4}{x} - 5\right)$ or $y = \left(\frac{4}{x} - 5\right)\Big/3$ - do not need domain for this mark. ISW after a correct answer.

(c) Mark parts c and d together
B1: $\text{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$ - allow any correct form then isw

(d)
M1: Sets fg(x) = gf(x) with **both sides correct** (but may be unsimplified) and forms a quadratic in x. Do not withhold this mark if fg or gf was originally correct but was "simplified" incorrectly and set equal to a correct gf
A1: Correct 3TQ. It need not be all on one side of the equation. The =0 can be implied by later work
M1: Attempts the discriminant or attempts the formula or attempts to complete the square.
A1: Completely correct work (cso) and conclusion. If $b^2 - 4ac$ has been found it must be correct (-576)

Question Number	Scheme	Marks
3 (a)	$0 < g < 3$	M1A1
		(2)
(b)	$y = \dfrac{6x}{2x+3} \Rightarrow 2xy + 3y = 6x \Rightarrow (6-2y)x = 3y \Rightarrow x = \dfrac{3y}{(6-2y)}$	M1A1
	$\Rightarrow g^{-1}(x) = \dfrac{3x}{(6-2x)} \qquad 0 < x < 3$	A1ft
		(3)
(c)	$gg(x) = g\left(\dfrac{6x}{2x+3}\right) = \dfrac{6 \times \dfrac{6x}{2x+3}}{2 \times \dfrac{6x}{2x+3} + 3}$	M1
	$= \dfrac{6 \times 6x}{2 \times 6x + 3(2x+3)}$	dM1
	$= \dfrac{36x}{18x+9} = \dfrac{4x}{2x+1}$	A1
		(3)
		(8 marks)

(a)

M1: For one 'end' fully correct $g(x) > 0$ (**not** $x > 0$) or $g(x) < 3$ (**not** $x < 3$) or both ends (incorrect) eg. accept $0 \leqslant g \leqslant 3$. Accept incorrect notation such as $0 < x < 3$ for this mark but **not** $x > 0$ or $x < 3$ **on their own**.
Allow use of f rather than g for the M mark but not the A mark.

A1: Accept $0 < g < 3$, $0 < y < 3$, $g(x) > 0$ and $g(x) < 3$, $(0,3)$

(b)

M1: An attempt to make x or a replaced y the subject of the formula. The minimum expectation is that there is an attempt to cross multiply, expand and collect/factorise terms in x or a replaced y and obtain $x = \dfrac{\pm 3y}{(\pm 6 \pm 2y)}$ or equivalent i.e. sign errors only on their algebra.

A1: $x = \dfrac{3y}{(6-2y)}$ or $\dfrac{-3y}{(2y-6)}$ or $y = \dfrac{3x}{(6-2x)}$ or $\dfrac{-3x}{(2x-6)}$ or $-\dfrac{3}{2} - \dfrac{9}{2(x-3)}$ etc. Allow $2(x-3)$ for $(2x-6)$.

A1ft: $g^{-1}(x) = \dfrac{3x}{(6-2x)}\left(\text{or } \dfrac{-3x}{(2x-6)}\right)$ **and** $0 < x < 3$. You can follow through on any range from part (a) but the domain must be in terms of x not in terms of e.g. $g(x)$ or $g^{-1}(x)$. Do not allow $x \in \mathbb{R}$

Accept $y = \dfrac{3x}{(6-2x)}\left(\text{or } \dfrac{-3x}{(2x-6)}\right) \qquad 0 < x < 3$. Allow $2(x-3)$ for $(2x-6)$.

(c)

M1: Attempts to find $gg(x)$ by finding $g\left(\dfrac{6x}{2x+3}\right)$

dM1: Correct processing to obtain a single fraction of the form $\dfrac{a}{b}$. Achieved by,

- multiplying both numerator and denominator by $(2x+3)$ (must multiply both terms in the denominator)

Question Number	Scheme	Marks
3.(a)	$y..3$	B1
		(1)
(b)	$y=3+\sqrt{x+2} \Rightarrow y-3=\sqrt{x+2} \Rightarrow x=(y-3)^2-2$	M1 A1
	$\Rightarrow g^{-1}(x)=(x-3)^2-2, \quad \text{with } x..3$	A1
		(3)
(c)	$g(x)=x \Rightarrow 3+\sqrt{x+2}=x$	
	$\Rightarrow x+2=(x-3)^2 \Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a=\dfrac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x)=x \Rightarrow (x-3)^2-2=x$	
	$\Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	dM1, A1
		(4)

(a)

B1 States the correct range for g Accept $g(x)..3, g..3, \text{Range}..3, [3,\infty)$ Range is greater than or equal to 3

Condone $f..3$ Do not accept $g(x)>3, x..3, (3,\infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2}=y\pm3 \Rightarrow x+2=y^2\pm9$

A1 Achieves $x=(y-3)^2-2$ or if swapped $y=(x-3)^2-2$ or equivalent such as $x=y^2-6y+7$

A1 Requires a correct function in x + correct domain **or** a correct function in x with a correct follow through on the range in (a) but do not follow through on $x\in\mathbb{R}$

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>1(a)

(b)</td><td colspan="2">$\text{fg}(x) = \frac{28}{x-2} - 1 \quad \left(= \frac{30-x}{x-2}\right)$
Sets $\text{fg}(x) = x \Rightarrow \frac{28}{x-2} - 1 = x$
$\Rightarrow 28 = (x+1)(x-2)$
$\Rightarrow x^2 - x - 30 = 0$
$\Rightarrow (x-6)(x+5) = 0$
$\Rightarrow x = 6,\ x = -5$

$a = 6$</td><td>M1

M1

dM1 A1
(4)
B1 ft
(1)
5 marks</td></tr>
<tr><td>Alt 1(a)</td><td colspan="2">$\text{fg}(x) = x \Rightarrow \text{g}(x) = \text{f}^{-1}(x)$
$\frac{4}{x-2} = \frac{x+1}{7}$
$\Rightarrow x^2 - x - 30 = 0$
$\Rightarrow (x-6)(x+5) = 0$
$\Rightarrow x = 6,\ x = -5$</td><td>M1

M1

dM1 A1
4 marks</td></tr>
<tr><td>S. Case</td><td>Uses $\text{gf}(x)$ instead $\text{fg}(x)$
$\frac{4}{7x-1-2} = x$
$\Rightarrow 7x^2 - 3x - 4 = 0$
$\Rightarrow (7x+4)(x-1) = 0$
$\Rightarrow x = -\frac{4}{7},\quad x = 1$</td><td>Makes an error on $\text{fg}(x)$
Sets $\text{fg}(x) = x \Rightarrow \frac{7\times 4}{7\times(x-2)} - 1 = x$
$\Rightarrow x^2 - x - 6 = 0$
$\Rightarrow (x+2)(x-3) = 0$
$\Rightarrow x = -2,\quad x = 3$</td><td>M0

M1

dM1 A0
2 out of 4 marks</td></tr>
</table>

(a)

M1 Sets or implies that $\text{fg}(x) = \frac{28}{x-2} - 1$ Eg accept $\text{fg}(x) = 7\left(\frac{4}{x-2}\right) - 1$ followed by $\text{fg}(x) = \frac{7\times 4}{x-2} - 1$

Alternatively sets $\text{g}(x) = \text{f}^{-1}(x)$ where $\text{f}^{-1}(x) = \frac{x \pm 1}{7}$

Note that $\text{fg}(x) = 7\left(\frac{4}{x-2}\right) - 1 = \frac{28}{7(x-2)} - 1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\text{fg}(x) = x$ or $\text{g}(x) = \text{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for x. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x = 6$ and $x = -5$

(b)

B1ft For $a = 6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a = 6$ and even $x = 6$
Do not award marks for part (a) for work in part (b).

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times -2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0\Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0\Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)e^{-2x}\Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e},\quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1
		(6)
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times\pm Ae^{-2x}+\left(Bx\pm Cx^2\right)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times Ce^{2x}}{\left(e^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3\Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$
Condone errors on coefficients and signs

Question Number	Scheme	Marks
6.(a)	$\mathrm{f}(x) > k^2$	B1
		(1)
(b)	$y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow \mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = \mathrm{e}^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{\mathrm{e}}{\sqrt[6]{8}}\right) = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	A1
		(4)
(d)	$\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2$	M1
	$\Rightarrow \mathrm{fg}(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$\mathrm{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1
		(2)
		12 marks
(alt c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 2 + \ln x + \ln 2 + 2\ln x + \ln 2 + 3\ln x = 6$	
	$\Rightarrow 3\ln 2 + 6\ln x = 6$	
	$\Rightarrow \ln x = 1 - \frac{1}{2}\ln 2$	M1
	$\Rightarrow x = e^{1-\frac{1}{2}\ln 2}, = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	M1, A1
(alt e)		(4)
	$\Rightarrow 2\ln(2x) = \ln(2k^2 - k^2)$	
	$\Rightarrow \ln(2x)^2 = \ln(k^2), \Rightarrow 4x^2 = k^2 \Rightarrow x = \frac{k}{2}$	M1, A1

Question Number	Scheme	Marks
5.(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\dfrac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\dfrac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\dfrac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\dfrac{(x+1)}{(x-2)}=x$ $\dfrac{x+1}{x-2}=\dfrac{2x+1}{x-1}$ $\dfrac{2x+1}{x-1}=x$ $\dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=\ldots$	A1, dM1
	$a=\dfrac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
7(a)	$0 \leqslant \mathrm{f}(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$\mathrm{g}^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$\mathrm{gf}(x) = 16 \Rightarrow \mathrm{f}(x) = \mathrm{g}^{-1}(16) = 4$ oe	M1A1
	$\mathrm{f}(x) = 4 \Rightarrow x = 6$	B1
	$\mathrm{f}(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$\mathrm{gf}(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4$ oe	A1 (5)