



Oxford Cambridge and RSA

## Wednesday 14 October 2020 – Afternoon

### AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

#### ADVICE

- Read each question carefully before you start your answer.

## Formulae AS Level Mathematics A (H230)

### Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

### Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

### The binomial distribution

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

### Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

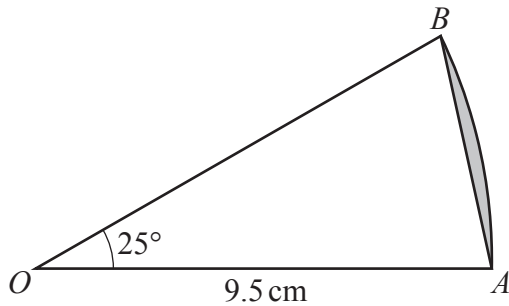
$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

## Section A: Pure Mathematics

Answer **all** the questions.

1



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius  $9.5$  cm. The angle  $AOB$  is  $25^\circ$ .

- (a) Calculate the length of the straight line  $AB$ . [2]
- (b) Find the area of the segment shaded in the diagram. [3]

2 Two curves have equations  $y = \ln x$  and  $y = \frac{k}{x}$ , where  $k$  is a positive constant.

- (a) Sketch the curves on a **single** diagram. [3]
- (b) Explain how your diagram shows that the equation  $x \ln x - k = 0$  has exactly one real root. [2]

3 **In this question you must show detailed reasoning.**

Find the equation of the normal to the curve  $y = 4\sqrt{x} - 3x + 1$  at the point on the curve where  $x = 4$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. [7]

**4 In this question you must show detailed reasoning.**

The cubic polynomial  $6x^3 + kx^2 + 57x - 20$  is denoted by  $f(x)$ . It is given that  $(2x - 1)$  is a factor of  $f(x)$ .

(a) Use the factor theorem to show that  $k = -37$ . [2]

(b) Using this value of  $k$ , factorise  $f(x)$  completely. [3]

(c) (i) Hence find the three values of  $t$  satisfying the equation  $6e^{-3t} - 37e^{-2t} + 57e^{-t} - 20 = 0$ . [2]

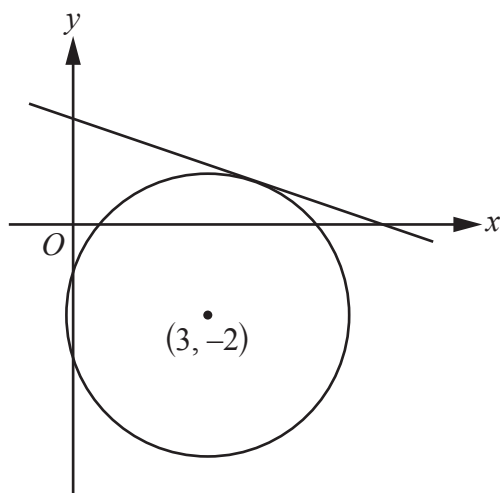
(ii) Express the sum of the three values found in part (c)(i) as a single logarithm. [2]

**5 A curve has equation  $y = a(x+b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. The curve has a stationary point at  $(-3, 2)$ .**

(a) State the values of  $b$  and  $c$ . [2]

When the curve is translated by  $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$  the transformed curve passes through the point  $(3, -18)$ .

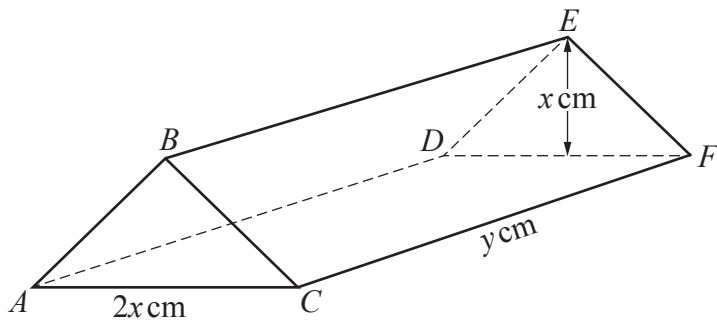
(b) Determine the value of  $a$ . [3]

**6 In this question you must show detailed reasoning.**

The diagram shows the line  $3y + x = 7$  which is a tangent to a circle with centre  $(3, -2)$ .

Find an equation for the circle. [6]

7



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism  $ABCDEF$ . The base  $ACFD$  of the roof is a horizontal rectangle, and the cross-section  $ABC$  of the roof is an isosceles triangle with  $AB = BC$ .

The lengths of  $AC$  and  $CF$  are  $2x$  cm and  $y$  cm respectively, and the height of  $BE$  above the base of the roof is  $x$  cm.

The total surface area of the **five** faces of the roof is  $600 \text{ cm}^2$  and the volume of the roof is  $V \text{ cm}^3$ .

- (a) Show that  $V = kx(300 - x^2)$ , where  $k = \sqrt{a} + b$  and  $a$  and  $b$  are integers to be determined. [6]
- (b) Use differentiation to determine the value of  $x$  for which the volume of the roof is a maximum. [4]
- (c) Find the maximum volume of the roof. Give your answer in  $\text{cm}^3$ , correct to the nearest integer. [1]
- (d) Explain why, for this roof,  $x$  must be less than a certain value, which you should state. [2]

**Section B: Mechanics**Answer **all** the questions.

- 8 A particle is in equilibrium under the action of the following three forces:  
 $(2\mathbf{i} - 4\mathbf{j})$  N,  $(-3q\mathbf{i} + 5p\mathbf{j})$  N and  $(-13\mathbf{i} - 6\mathbf{j})$  N.  
Find the values of  $p$  and  $q$ . [3]
- 9 A crane lifts a car vertically. The car is inside a crate which is raised by the crane by means of a strong cable. The cable can withstand a maximum tension of 9500 N without breaking. The crate has a mass of 55 kg and the car has a mass of 830 kg.
- (a) Find the maximum acceleration with which the crate and car can be raised. [2]
- (b) Show on a clearly labelled diagram the forces acting on the **crate** while it is in motion. [1]
- (c) Determine the magnitude of the reaction force between the crate and the car when they are ascending with maximum acceleration. [3]
- 10 A particle  $P$  is moving in a straight line. At time  $t$  seconds  $P$  has velocity  $v$  m s<sup>-1</sup> where  $v = (2t + 1)(3 - t)$ .
- (a) Find the deceleration of  $P$  when  $t = 4$ . [2]
- (b) State the positive value of  $t$  for which  $P$  is instantaneously at rest. [1]
- (c) Find the total distance that  $P$  travels between times  $t = 0$  and  $t = 4$ . [3]

**11** A car starts from rest at a set of traffic lights and moves along a straight road with constant acceleration  $4 \text{ m s}^{-2}$ . A motorcycle, travelling parallel to the car with constant speed  $16 \text{ m s}^{-1}$ , passes the same traffic lights exactly 1.5 seconds after the car starts to move. The time after the car starts to move is denoted by  $t$  seconds.

- (a) Determine the two values of  $t$  at which the car and motorcycle are the same distance from the traffic lights. [6]

These two values of  $t$  are denoted by  $t_1$  and  $t_2$ , where  $t_1 < t_2$ .

- (b) Describe the relative positions of the car and the motorcycle when  $t_1 < t < t_2$ . [1]

- (c) Determine the maximum distance between the car and the motorcycle when  $t_1 < t < t_2$ . [3]

**END OF QUESTION PAPER**

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