

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6664/01)

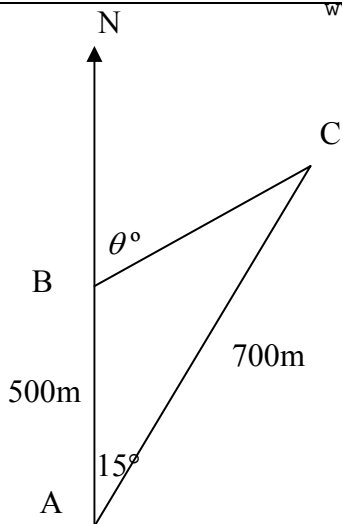
January 2008
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1.	<p>a)i) $f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8 \quad ; = 5$</p> <p>ii) $f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact) M1 is for attempt at either $f(3)$ or $f(-3)$ in (i) or $f(-2)$ or $f(2)$ in (ii).</p> <p>(b) $[(x+2)(x^2 - 4x + 4) \quad (= 0 \text{ not required}) \text{ [must be seen or used in (b)]}$ $(x+2)(x-2)^2 \quad (= 0) \quad (\text{can imply previous 2 marks})$</p> <p>Solutions: $x = 2$ or -2 (both) or $(-2, 2, 2)$ A1 (4)</p>	<p>M1; A1</p> <p>A1 (3)</p> <p>M1 A1 M1</p> <p style="text-align: right;">[7]</p>
Notes: (a)	<p>No working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate.</p> <p><u>Alternative (Long division)</u> Divide by $(x-3)$ OR $(x+2)$ to get $x^2 + ax + b$, a may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]</p> <p>(b) First M1 requires division by a found factor ; e.g $(x+2)$, $(x-2)$ or what candidate thinks is a factor to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x+2)(x^2 - 4x + 4)]$ or $(x-2)(x^2 - 4)$ Second M1: attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x+c)(x+d)$, where $cd = b$.] N.B. Second A1 is for solutions, not factors <u>Alternative (first two marks)</u> $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1] $b = -4, c = 4$ [A1]</p> <p><u>Method of grouping</u> $x^3 - 2x^2 - 4x + 8 = x^2(x-2) + 4(x-2)$ M1; $= x^2(x-2) - 4(x-2)$ A1 $[= (x^2 - 4)(x-2)] = (x+2)(x-2)^2$ M1 Solutions: $x=2, x=-2$ both A1</p>	
2.	<p>(a) Complete method, using terms of form ar^k, to find r [e.g. Dividing $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$</p> <p>(b) Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.]</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p>

<p>(c)</p>	<p style="text-align: right; font-size: small;">www.yesterdaymathsexam.com</p> <p>$(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)</p> <p>Substituting their values of a and r into correct formula for sum.</p> <p>$S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)</p>	<p>A1 (2)</p> <p>M1</p> <p>A1 (2) [6]</p>
<p>Notes:</p>	<p>(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly)</p> <p>(b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$</p> <p>In (a) and (b) correct answer, with no working, allow both marks.</p> <p>(c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.</p>	
<p>3.</p>	<p>(a)</p> $\left(1 + \frac{1}{2}x\right)^{10} = 1 + \frac{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}{\underline{\hspace{10em}}}$ <p style="text-align: right;">M1 A1</p> <p>$= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified) A1; A1 (4)</p> <p>[Allow A1A0, if totally correct with unsimplified, single fraction coefficients)</p> <p>(b)</p> $\left(1 + \frac{1}{2} \times 0.01\right)^{10} = 1 + 5(0.01) + \frac{45}{4} \text{ or } 11.25(0.01)^2 + 15(0.01)^3$ $= 1 + 0.05 + 0.001125 + 0.000015$ $= 1.05114 \quad \text{cao}$	<p>A1 A1</p> <p>A1; A1 (4)</p> <p>M1 A1✓</p> <p>A1 (3) [7]</p>
<p>Notes:</p>	<p>(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x, (iii) May be listed, need not be added; <i>this applies for all marks.</i></p> <p>First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1 for $1 + 5x$</p> <p>(b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)</p>	

<p>4.</p>	<p>(a) $3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (M1: Use of $\sin^2 \theta + \cos^2 \theta = 1$) $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3$ cso AG</p> <p>(b) $\sin^2 \theta = \frac{3}{5}$, so $\sin \theta = (\pm)\sqrt{0.6}$ Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -$ (may be implied by later work) M1 $\theta = 50.7685^\circ$ awrt $\theta = 50.8^\circ$ (dependent on first M1 only) A1 $\theta (= 180^\circ - 50.7685^\circ)$; = $129.23\dots^\circ$ awrt 129.2° [f.t. dependent on first M and 3rd M] $\sin \theta = - \sqrt{0.6}$ $\theta = 230.785^\circ$ and 309.23152° awrt $230.8^\circ, 309.2^\circ$ (both) M1A1 (7)</p>	<p>M1 A1 (2) M1 A1 M1; A1 ✓ M1A1 (7) [9]</p>
<p>Notes:</p>	<p>(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$.</p> <p>(b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ Second M1: Considering the $-$ value for $\sin \theta$. (usually later) First A1: Given for awrt 50.8°. Not dependent on second M. Third M1: For $(180 - 50.8_c)^\circ$, need not see written down Final M1: Dependent on second M (but may be implied by answers) For $(180 + \text{candidate}'s 50.8)^\circ$ or $(360 - 50.8_c)^\circ$ or equiv. Final A1: Requires both values. (no follow through) [Finds $\cos^2 \theta = k$ ($k = 2/5$) and so $\cos \theta = (\pm)\dots$M1, then mark equivalently]</p>	

6.



$$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$$

$$(\text{ = } 63851.92\dots)$$

$$BC = 253 \text{ awrt}$$

(a) $\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$

$\sin B = \sin 15 \times 700 / 253_c = 0.716\dots$ and giving an **obtuse** B (134.2°) dep

(b) $\theta = 180^\circ - \text{candidate's angle } B$ (Dep. on first M only, B can be acute) M1
 $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) A1 (4) [7]
 [46 needs to be from correct working]

M1 A1
A1 (3)

M1

M1

M1

A1 (4) [7]

Notes:

(a) If use $\cos 15^\circ = \dots$, then A1 not scored until written as $BC^2 = \dots$ correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras M1

$$BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2 \quad \text{A1}$$

$$BC = 253 \text{ awrt} \quad \text{A1}$$

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)

(i) $\cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$ or $700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$ M1

Finding angle B M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$$\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX} \quad \text{M1}$$

Finding value for $\angle CBX$ ($\approx 59^\circ$) M1

$$\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)] \quad \text{M1}$$

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$ M1

(iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ M2 {first two Ms earned in this case}

Solving for θ ; $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9 M1; A1

7	<p style="text-align: center;"><small>www.yesterdaymathsexam.com</small></p> <p>(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]</p> <p>(b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = ..$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,</p> <p>(c) (Area) $= \int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3} (+c)$ Correct use of correct limits on their result above (see notes on limits) [" $3x^2 - \frac{x^3}{3}$ "]⁴ - [" $3x^2 - \frac{x^3}{3}$ "]₀ with limits substituted [= $48 - 21\frac{1}{3} = 26\frac{2}{3}$] Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = \pm (area under curve - area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)</p>	<p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 M1 A1 M1 A1 (6)[10]</p>
Notes	<p>(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned ; accept " clear from diagram"</p> <p>(c) Alternative Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach</p> <p>(i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3} (+c)]$ A1, Correct use of correct limits on their result second M1, Totally correct, unsimplified \pm expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given $-10\frac{2}{3}$, they correct it] M1 for correct use of correct limits. Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy.</p> <p>Use of trapezium rule: M0A0MA0, possible A1 for triangle M1 (if correct application of trap. rule from $x = 0$ to $x = 4$) A0</p>	

8	<p>(a) $(x-6)^2 + (y-4)^2 = 3^2$</p> <p>(b) Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$ $= \sqrt{40}$ (= 6.325)</p> <p>[These first two marks can be scored if seen as part of solution for (c)]</p> <p>Complete method for $\cos \theta$, $\sin \theta$ or $\tan \theta$ e.g. $\cos \theta = \frac{MT}{MP} = \frac{3}{\text{candidate's } \sqrt{40}}$ (= 0.4743) ($\theta = 61.6835^\circ$) [If $TP = 6$ is used, then M0] $\theta = 1.0766$ rad AG</p> <p>(c) Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ $= \frac{3}{2} \sqrt{31}$ (= 8.3516..) allow awrt 8.35</p> <p>Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766$ (= 4.8446...)</p> <p>Area $TPQ = \text{candidate's } (8.3516.. - 4.8446..)$ $= 3.507$ awrt [Note: 3.51 is A0]</p>	<p>B1; B1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p> <p>[11]</p>
Notes	<p>(a) Allow 9 for 3^2.</p> <p>(b) First M1 can be implied by $\sqrt{40}$</p> <p>For second M1: May find $TP = \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin \theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) or $\tan \theta = \frac{\sqrt{31}}{3}$ (1.8859..) or cos rule</p> <p>NB. Answer is given, but allow final A1 if all previous work is correct.</p> <p>(c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40 - 9}$</p>	

9	<p>(a) (Total area) = $3xy + 2x^2$ (Vol:) $x^2y = 100$ ($y = \frac{100}{x^2}, xy = \frac{100}{x}$) Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area) (Area =) $\frac{300}{x} + 2x^2$ AG</p> <p>(b) $\frac{dA}{dx} = -\frac{300}{x^2} + 4x$ Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)</p> <p>(c) $\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}$ therefore minimum</p> <p>(d) Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) [$y = \frac{100}{4.2172^2} = 5.6228$] Area = 106.707 awrt 107</p>	<p>B1 B1 M1 A1 cso (4) M1A1 A1 (4) M1A1 (2) M1 A1 (2) [12]</p>
Notes	<p>(a) First B1: Earned for correct unsimplified expression, isw.</p> <p>(c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive” A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, \checkmark). (may be wrong x, or even no value of x found)</p> <p><u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum.</p> <p>OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with”107” A1: Both values greater than “$x = 107$” and conclude minimum.</p> <p>Allow marks for (c) and (d) where seen; even if part labelling confused.</p>	

