Question	Scheme	Marks	AOs
12 (i)	Uses $\cos^2\theta = 1 - \sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \dfrac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	
			(9 marks)

Notes

(i)

M1: Uses $\cos^2\theta = 1 - \sin^2\theta$ to form a 3TQ in $\sin\theta$

A1: Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

dM1: Solves their 3TQ in $\sin\theta$ to produce one value for θ. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

A1: Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40° = 666.9°$ o.e.

A1: awrt $\alpha = 125°$

B1: For 512
A1: For $-144x$

A1: For + $18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks

(b)

M1: For setting their $512a = 128$ and proceeding to find a value for *a*. Alternatively they could substitute $x = 0$ into both sides of the identity and proceed to find a value for *a*.

A1 ft: $a = \frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)

M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of "a" to find a value for "b"

A1: $b = \frac{9}{64}$ oe

Question	Scheme	Marks	AOs
12 (a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2

	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1-\cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
			(8 marks)

Notes

(a)

M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Note that it cannot just be stated.**

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2\left(1-\cos^2\theta\right)$ or equivalent

M1: Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$ " where y could be $\cos 3x$, $\cos x$, or even just y. When factorising look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$) , an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even ± the correct roots.

B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\frac{2}{3}, -\frac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $p = 10^{4.8}$ or $q = 10^{0.05}$	M1	1.1b

Question	Scheme	Marks	AOs
12(a)	$\frac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\frac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow \tan x=-\frac{5}{3}$	M1	2.1
	$x=$ awrt $121°, 301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\frac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2 \leftrightarrow \cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\frac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\frac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow \tan x=b\Rightarrow x=...$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A,-1<A<1$ or $\cos x=B,-1<B<1$

A1: Either $x=$ awrt $121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=$ awrt $121°$ **or** $301°$

Question	Scheme	Marks	AOs
9 (a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \dfrac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(o)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \dfrac{1}{3}$ Accept sight of $\dfrac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

Question	Scheme	Marks	AOs
9	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x = 430.5°,\ 435.5°$	A1	1.1b
			(5 marks)

Notes:

M1: Uses correct identity

A1: Correct three term quadratic

M1: Solves their three term quadratic to give values for cos x. (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

A1: Two correct answers in the given domain

Question Number	Scheme		Marks
8. (a)	Way 1	Way 2	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$	$2=(3\sin x-1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$	B1
	E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$	so $8\sin^2 x - 6\sin x = 1-\sin^2 x$	M1
	So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x-1)^2 = 2$ or $2=(3\sin x-1)^2$ *	$8\sin^2 x - 6\sin x = \cos^2 x$ *	A1cso*
			(3)
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x-1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \dfrac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt -0.1381		A1
	x = 53.58 , 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1
			(5) **[8]**

	Notes
(a)	**Way 1** B1: Uses $\cos^2 x = 1-\sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2=(3\sin x-1)^2$. If sin is used throughout instead of sin*x* it is A0. **Way 2** B1: **Needs correct expansion** and **split** M1: Collects $1-\sin^2 x$ together A1*: Conclusion and no errors seen
(b)	M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0. A1: **Both** correct answers for sin*x* (need plus and minus). Need not be simplified. dM1: Uses inverse sin to give one of the given correct answers 1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28

Question Number	Scheme	Marks
13(a)	$5\cos x+1=\sin x\tan x \Rightarrow 5\cos x+1=\sin x\times\dfrac{\sin x}{\cos x}$	M1
	$5\cos^2 x+\cos x=\sin^2 x$	A1
	$5\cos^2 x+\cos x=1-\cos^2 x$	M1
	$6\cos^2 x+\cos x-1=0$	A1*
		(4)
(b)	$6\cos^2 k+\cos k-1=0 \Rightarrow (3\cos k\text{-}1)(2\cos k+1)=0$	M1
	$\Rightarrow \cos k=\dfrac{1}{3},-\dfrac{1}{2}$	A1
	Either $\cos 2\theta=\dfrac{1}{3}\Rightarrow 2\theta=70.53, 289.47$ **Or** $\cos 2\theta=-\dfrac{1}{2}\Rightarrow 2\theta=120, 240$	
	$\Rightarrow \theta=35.3°, 144.7°, 60°, 120°$	M1A1M1A1
		(6)
		(10 marks)

(a)

M1 Uses $\tan x=\dfrac{\sin x}{\cos x}$ in the given equation. You may see it used in the form $\tan x\cos x=\sin x$

A1 A correct equation (not involving fractions) in both sin and cos. Eg $5\cos^2 x+\cos x=\sin^2 x$ oe

M1 Replaces $\sin^2 x$ by $1-\cos^2 x$ to produce a quadratic equation/expression in just $\cos x$

A1* Proceeds correctly to the given answer $6\cos^2 x+\cos x-1=0$

All notation should be consistent and correct including $\cos x$ instead of $\cos$ and $\sin^2 x$ instead of $\sin x^2$

Question	Scheme	Marks
10. (a)	Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$	M1
	Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$	M1
	So $8\sin x = -3 + 3\sin^2 x$ and $3\sin^2 x - 8\sin x - 3 = 0$ *	A1 * **[3]**
(b)	Solves the three term quadratic "$3\sin^2 x - 8\sin x - 3 = 0$"	M1
	So $(\sin x) = -\frac{1}{3}$ (or 3)	A1
	$(2\theta) = -19.47$ or 199.47 or 340.53	dM1
	$\theta = 99.7, 170.3, 279.7$ or 350.3	A1, A1 **[5]**
		8 marks
	Notes	

(a)

M1: Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$ or equivalent

M1: Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$

May also be seen $8\tan x = -3\cos x \Rightarrow 8\tan x = -3\sqrt{1-\sin^2 x}$

A1: Proceeds to given answer with no errors.
(This is a given answer so do not tolerate bracketing or notation errors such as $\cos^2 x$ written as $\cos x^2$ or $\sin x$ appearing as sin)

(b)

M1: Solving quadratic by usual methods (see notes).
If the formula is quoted it must be correct but allow solutions from calculators.

A1: You only need to see $-\frac{1}{3}$.

This is an intermediate answer so condone $-\frac{1}{3}$ appearing as awrt -0.333

Condone errors on the lhs so accept for this mark $x / a / \theta = -\frac{1}{3}$, $\sin x = -\frac{1}{3}$, $\sin 2x = -\frac{1}{3}$

dM1: Uses inverse sine to obtain an answer for 2θ.
This may appear as answers for x. The only stipulation is that $\text{invsin } k, |k| < 1$
It is dependent upon seeing a correct method of solving their quadratic
Accept answers rounding to 1 dp for 2θ e.g. awrt -19.5 or 199.5 or 340.5.
It may also be implied by a correct answer for θ e.g. awrt -9.7 or 99.7 or 170.2

A1: Two correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or 350.3

A1: All four correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or 350.3

Question Number	Scheme	Marks
8(a)	$\left(\dfrac{5+\sin\theta}{3\cos\theta}=2\cos\theta\right)\Rightarrow 5+\sin\theta=6\cos^2\theta$	M1
	$\Rightarrow 5+\sin\theta=6(1-\sin^2\theta)\Rightarrow 6\sin^2\theta+\sin\theta-1=0$	dM1A1*
		(3)
(b)	$6\sin^2\theta+\sin\theta-1=0\Rightarrow(3\sin\theta-1)(2\sin\theta+1)=0$	M1
	$(\sin\theta)=+\dfrac{1}{3},-\dfrac{1}{2}$	A1
	$\theta=19.5°,-30°$	dM1,A1
		(4)
		(7 marks)

(a)

M1 Attempts to cross multiply to form an equation in the form $5+\sin\theta=A\cos^2\theta$

dM1 Dependent upon previous M. For using $\cos^2\theta=\pm1\pm\sin^2\theta$ to get an equation in just $\sin\theta$

A1* This is a given answer. All aspects must be correct.
Mixed variables, say x's and θ's would lose this mark.
An otherwise correct solution with $\cos^2\theta$ or $(\cos\theta)^2$ written as $\cos\theta^2$ would also be M1 dM1 A0

(b)

M1 Attempts to factorise, usual rules. Accept answers by formula or just written down from a calculator. Accept answers/factors given in terms of x for this mark

A1 For the values $+\dfrac{1}{3},-\dfrac{1}{2}$
These do not have to be simplified and can be implied by correct answers for θ

dM1 Calculates at least one value of θ from their '$\sin\theta$'.
It is dependent upon the previous M. You may have to check with a calculator.
This may be implied by either $\theta=19.5°,-30°$ from a correct quadratic

A1 Both $\theta=19.5°$ and $-30°$ with this accuracy and no additional solutions inside the range.
Condone the answer $\theta=19.5°$ and $-30.0°$
Ignore any solutions outside the range.

Question Number	Scheme	Marks
	$7\sin x = 3\cos x$	
8(a)	$(\tan x =)\dfrac{3}{7}$	B1
		[1]
(b)	$\tan(2\theta + 30) = \dfrac{3}{7}$	B1ft
	$\tan^{-1}"\dfrac{3}{7}"\ (\alpha)$	M1
	One of $\theta =$ awrt 87 or awrt 177 or awrt 267 or awrt 357	A1
	Follow through any of their final θ's for $\theta \pm 90n$ within range	A1ft
	All of $\theta =$ 86.6, 176.6, 266.6, 356.6	A1
		[5]
		6 marks
	Notes	

(a)	**B1: (** $\tan x =)\dfrac{3}{7}$ or exact equivalent so accept recurring decimal ($0.428571...$) but not rounded answer
(b)	**B1ft:** Correct equation as shown or follow through their value for tan x from part (a). Must be $\tan(2\theta+30)=\ldots$ but $2\theta+30$ may be implied later by an attempt to subtract 30 and then divide by 2. If the processing is unclear or incorrect and $2\theta+30$ is never seen, score B0 here. **M1:** Finds arctan of their $\dfrac{3}{7}$. Could be implied by their value e.g. 23.19.. or just $\tan^{-1}"\dfrac{3}{7}"$ **A1:** For **one** of either $\theta =$ awrt 87 or awrt 177 or awrt 267 or awrt 357 **A1ft:** Follow through any of their **final** answers to which an integer multiple of 90 has been added or subtracted to give another solution in range **but not for adding a multiple of 90 to just α.** **A1: For all 4 correct answers to the required accuracy as stated in the scheme**. Ignore extra answers outside range but lose last A mark for extra answers inside range.

Question Number	Scheme	Marks
13. (i)	$(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin x+\cos x-\sin^2 x\cos x-\cos^2 x\sin x$ $(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin x+\cos x-(1-\cos^2 x)\cos x-(1-\sin^2 x)\sin x$ $(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin^3 x+\cos^3 x$ *	1st M1 2nd M1 A1 *
Alt I (i)	Use LHS = $(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)$ $\equiv \sin^3 x+\sin x\cos^2 x-\sin^2 x\cos x+\sin^2 x\cos x-\cos^2 x\sin x+\cos^3 x$ $(\sin x+\cos x)(1-\sin x\cos x)\equiv \sin^3 x+\cos^3 x$	**[3]** 2nd M1 1st M1 A1 * **[3]**
Alt II (i)	Use RHS $\equiv \sin^3 x+\cos^3 x$ = $(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)$ = $(\sin x+\cos x)(1-\sin x\cos x)$	M1 M1 A1 **[3]**
(ii)	Use $\tan\theta=\frac{\sin\theta}{\cos\theta}$ to give $3\sin\theta=\frac{\sin\theta}{\cos\theta}$ $\cos\theta=\frac{1}{3}$ and $\sin\theta=0$ So θ = 70.5, 289.5, or <u>0 and 180</u> (do not require degrees symbol)	M1 A1 A1 M1 A1, <u>B1</u> **[6]** **9 marks**

(i)

M1 Expands bracket to form 4 terms - condone sign slips but terms must be correct.

Allow $\sin x\sin x\cos x$ for $\sin^2 x\cos x$ and condone $\cos x^2$ for $\cos^2 x$

M1 Replaces $\sin^2 x$ by $(1-\cos^2 x)$ **and** $\cos^2 x$ by $(1-\sin^2 x)$

Condone $\cos x^2$ for $\cos^2 x$. This mark could be seen proceeding the line $\sin x(1-\cos^2 x)+\cos x(1-\sin^2 x)$.

A1* Completes proof with no errors. This is a given answer. Withhold this mark if poor notation or mixed variables are seen. Examples would be $\sin x+\cos x-(1-\mathbf{cos^2})\cos x-(1-\sin^2\boldsymbol{\theta})\sin x$

Don't accept for the A1* $\cos x^2$ for $\cos^2 x$ unless it is clearly bracketed.

(ii)

M1 Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ or equivalent to give $3\sin\theta=\frac{\sin\theta}{\cos\theta}$ *or* $3\cos\theta=1$

A1 Achieves $\cos\theta=\frac{1}{3}$

A1 Achieves $\sin\theta=0$

M1 For $\arccos\left(\mathit{their}\,\frac{1}{3}\right)$ leading to one value of θ to the nearest degree. (You may need a calculator to check this)

A1 For θ = awrt $70.5°, 289.5°$. This mark is withheld for extra solutions of $\arccos\left(\frac{1}{3}\right)$ in the range.

Ignore extra solutions outside the range.

B1 $\theta = 0, 180°$ This mark is withheld for extra solutions arising from $\sin\theta=0$ in the range.

Ignore extra solutions outside the range.

Note: Students who proceed from $\frac{\sin\theta}{\tan\theta}=\cos\theta$ can score M1A1A0M1A1B0 for 4 out of 6

Radian solutions, withhold only the final A1 mark. For your information solutions are awrt 2dp 1.23, 5.05, 0, 3.14 = (pi)

Question Number	Scheme	Marks
11 (a)	$\left(0, -\frac{\sqrt{3}}{2}\right)$	B1
	and (60°, 0) and (240°, 0) and (-120°, 0) and (-300°, 0)	B1 B1 **[3]**
(b)	$\sin(x-60^{\circ}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **(**= 0.2588)	M1
	$x - 60° = 15°$ (or 165° or – 195° or – 345°) or 0.262 or $\frac{\pi}{12}$ radians	A1
	So x = 75° or 225° or – 135° or –285° (allow awrt)	M1 A1 A1 **[5]**
		8 marks
	Notes	

(a) **B1** : Correct exact y intercept (not decimal) – allow on the diagram or in the text. Allow $y = -\frac{\sqrt{3}}{2}$

B1 for 2 correct x intercepts then **third B1** for all 4 correct x intercepts (may or may not be given as coordinates – may be given on graph) Must be in degrees. (Extra answers in the range lose the **third B1**)

(b) **M1**: Divides by 4 first giving correct statement $\sin(x-60^{\circ}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ but $(x-60^{\circ}) = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is M0 and** $\sin x - \sin 60^{\circ} = \frac{\sqrt{6}-\sqrt{2}}{4}$ **is also M0 and** $\sin(x-60^{\circ}) = \frac{\sqrt{4}}{4}$ **is M0** if not preceded by correct statement

A1: Obtains 15° (or 165° or – 195° or – 345°)

M1: Adds 60° to their previous answer which should have been in degrees and obtained by using inverse sine

A1: Two correct answers second **A1:** All four correct answers Extra answers outside range are ignored. Lose final A mark for extra wrong answers in the range.

If they approximate too early allow awrt answers given for full marks. (e.g. 75.01 etc)

Answers in mixture, degrees and radians: Allow first M A1 only so M1A1M0A0A0 for 60.262 for example

Question Number	Scheme	Marks
13(a)	$h = 3.7 + 2.5\cos(30t - 40)°, \quad 0 \le t < 24$	
	Max = 3.7+2.5=6.2m	B1
	Occurs when $30t - 40 = 0 \Rightarrow t = \frac{40}{30}, = 1:20am\left(01:20\right)$	M1A1,A1
		(4)
(b)	$3.7 + 2.5\cos(30t-40)° = 3 \Rightarrow \cos(30t-40)° = -\frac{0.7}{2.5} = \left(-0.28\right)$	M1
	$30t - 40 = 106.3, (253.7)$	A1
	$t =$ awrt 4.9 or 9.8	A1
	2nd Value $30t - 40 = 253.7 \Rightarrow t = ...$	M1
	$t =$ awrt 4.9 and 9.8	A1
	Boat cannot enter the harbour between 04:53 and 09:47	A1
		(6)
		(10 marks)

(a)

B1 $h_{max} = 6.2\ (m)$. The units are not important.

M1 Solves either $30t - 40 = 0 \Rightarrow t = ..$ or $30t - 40 = 360 \Rightarrow t = ...$ It may be implied by $t = \frac{40}{30}, \frac{400}{30}$ oe

A1 $t = \frac{40}{30}$, or exact equivalents like $\frac{4}{3}, 1.\dot{3}$.

A1 1:20am or $\left(01:20\right)$ The exact time of day is required. 1:20 or 1:20pm is incorrect.

Special case: Candidates who solve this part by differentiation can be allowed full marks

B1 $h_{max} = 6.2\ (m)$. The units are not important

M1 For $h' = A\sin(30t - 40)° = 0 \Rightarrow t = ..$

A1 $t = \frac{40}{30}$, or exact equivalents like $\frac{4}{3}, 1.\dot{3}$.

A1 1:20am Accept 01:20 or 0120 The exact time of day is required. 1:20 or 1:20pm is incorrect

(b)

M1 Sets h=3 and proceeds to $\cos(30t - 40)° = ..$ Accept inequalities in place of = sign

A1 Proceeds by taking invcos to reach either $30t - 40 =$ awrt 106.3, or $30t - 40 =$ awrt 253.7
Accept inequalities in place of = sign. This may be implied by a correct answer for $t = awrt$ 4.9, (9.8)

A1 One value for t correct. Accept either awrt 1 dp $t = 4.9$ or 9.8

M1 For the correct method of finding a second value of t.
Accept $30t - 40 = (360 - \alpha) \Rightarrow t = ...$ where α is their principal value

A1 Both values of t correct awrt 1dp. $t = 4.9$ and 9.8 . Ignore any values where $t \geqslant 12$ but withhold this mark for extra values in the range. These may be implied by 293 minutes and 587 minutes

A1 cso Both 04:53 and 09:47. Accept both 0453 and 0947
Accept 4:53 and 9:47 without the am as the question requires morning times.
Accept 293 and 587 minutes
If they state between 0 and 293 minutes and 587 and … minutes it is A0

Question Number	Scheme	Marks
7. (a)	$12\sin^2 x - \cos x - 11 = 0$ $12(1-\cos^2 x) - \cos x - 11 = 0$ and so $12\cos^2 x + \cos x - 1 = 0$ *	B1 * [1]
(b)	Solve quadratic to obtain ($\cos x$) $= \frac{1}{4}$ or $-\frac{1}{3}$ x = 75.5, 109.5, 250.5, 284.5 Answers in radians (see notes)	M1 A1 M1 A1cao **[4]**
		5 marks
	Notes	
(a) **(b)**	**B1:** Replaces $\sin^2 x$ by $(1-\cos^2 x)$ - or replace 11 by $11(\sin^2 x + \cos^2 x)$ and no errors seen to give printed answer including = 0 **M1**: Solving the correct quadratic equation (allow sign errors), by the usual methods (see notes) – implied by correct answers **A1**: Both answers needed – allow 0.25 and awrt -0.33 **M1** Uses inverse cosine to obtain two correct values for x for their values of cos$x$ e.g. (75.5 and 109.4 or 109.5) or (75.5 and 284.5) or (109.5 and 250.5) – allow truncated answers or awrt here. **A1:** All four correct – allow awrt. Ignore extra answers outside range but lose last A mark for extra answers inside range Answers in radians are 1.3, 5.0, 1.9 and 4.4 Allow M1A0 for two or more correct asnwers	