8. The line l_1 has equation $\dfrac{x-2}{4} = \dfrac{y-4}{-2} = \dfrac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(7)

2. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2

(2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree.

(3)

7. The line l_1 has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where t is a scalar parameter.

(a) Show that l_1 and l_2 lie in the same plane.

(3)

(b) Write down a vector equation for the plane containing l_1 and l_2

(1)

(c) Find, to the nearest degree, the acute angle between l_1 and l_2

(3)

4. The plane Π_1 has equation

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Find a Cartesian equation for Π_1

(4)

The line l has equation

$$\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+2}{4}$$

(b) Find the coordinates of the point of intersection of l with Π_1

(3)

The plane Π_2 has equation

$$\mathbf{r}.(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 5$$

(c) Find, to the nearest degree, the acute angle between Π_1 and Π_2

(2)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ is perpendicular to Π.

(2)

(b) Hence find a Cartesian equation of Π.

(2)

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$$

where t is a scalar parameter.

The point A lies on l.

Given that the shortest distance between A and Π is $2\sqrt{29}$

(c) determine the possible coordinates of A.

(4)

8. Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

$$\mathbf{r}_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

and the equation for the flight path of the second bird is

$$\mathbf{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters and a is a constant.

In the model, the angle between the birds' flight paths is 120°

(a) Determine the value of a.

(4)

(b) Verify that, according to the model, there is a common point on the flight paths of the two birds and find the coordinates of this common point.

(5)

The position of the nest is modelled as being at this common point.

The tree containing the nest is in a park.

The ground level of the park is modelled by the plane with equation

$$2x - 3y + z = 2$$

(c) Hence determine the shortest distance from the nest to the ground level of the park.

(3)

(d) By considering the model, comment on whether your answer to part (c) is reliable, giving a reason for your answer.

(1)

5. The line l_1 has equation $\dfrac{x+5}{1} = \dfrac{y+4}{-3} = \dfrac{z-3}{5}$

The plane Π_1 has equation $2x + 3y - 2z = 6$

(a) Find the point of intersection of l_1 and Π_1

(2)

The line l_2 is the reflection of the line l_1 in the plane Π_1

(b) Show that a vector equation for the line l_2 is

$$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

(5)

The plane Π_2 contains the line l_1 and the line l_2

(c) Determine a vector equation for the line of intersection of Π_1 and Π_2

(2)

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b$ where a and b are constants.

Given that the planes Π_1, Π_2 and Π_3 form a sheaf,

(d) determine the value of a and the value of b.

(3)

4. Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface. The water pipes are modelled as straight line segments. One water pipe, W, is buried beneath a particular road. With respect to a fixed origin O, the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and W passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$. The units are in metres.

(a) Use the model to calculate the acute angle between W and the road surface.

(5)

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to W from C.

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect C to W.

(6)

4. The line l has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -7$$

Determine whether the line l intersects Π at a single point, or lies in Π, or is parallel to Π without intersecting it.

(5)

8. A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)

4. **All units in this question are in metres.**

A lawn is modelled as a plane that contains the points $L(-2, -3, -1)$, $M(6, -2, 0)$ and $N(2, 0, 0)$, relative to a fixed origin O.

(a) Determine a vector equation of the plane that models the lawn, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

(3)

(b) (i) Show that, according to the model, the lawn is perpendicular to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

(ii) Hence determine a Cartesian equation of the plane that models the lawn.

(4)

There are two posts set in the lawn.
There is a washing line between the two posts.
The washing line is modelled as a straight line through points at the top of each post with coordinates $P(-10, 8, 2)$ and $Q(6, 4, 3)$.

(c) Determine a vector equation of the line that models the washing line.

(2)

(d) State a limitation of one of the models.

(1)

The point $R(2, 5, 2.75)$ lies on the washing line.

(e) Determine, according to the model, the shortest distance from the point R to the lawn, giving your answer to the nearest cm.

(2)

Given that the shortest distance from the point R to the lawn is actually 1.5 m,

(f) use your answer to part (e) to evaluate the model, explaining your reasoning.

(1)

6. A mining company has identified a mineral layer below ground.

The mining company wishes to drill down to reach the mineral layer and models the situation as follows.

With respect to a fixed origin O,

- the ground is modelled as a horizontal plane with equation $z = 0$
- the mineral layer is modelled as part of the plane containing the points $A(10, 5, -50)$, $B(15, 30, -45)$ and $C(-5, 20, -60)$, where the units are in metres

(a) Determine an equation for the plane containing A, B and C, giving your answer in the form $\mathbf{r.n} = d$

(5)

(b) Determine, according to the model, the acute angle between the ground and the plane containing the mineral layer. Give your answer to the nearest degree.

(3)

The mining company plans to drill vertically downwards from the point (5, 12, 0) on the ground to reach the mineral layer.

(c) Using the model, determine, in metres to 1 decimal place, the distance the mining company will need to drill in order to reach the mineral layer.

(2)

(d) State a limitation of the assumption that the mineral layer can be modelled as a plane.

(1)

6. The surface of a horizontal tennis court is modelled as part of a horizontal plane, with the origin on the ground at the centre of the court, and

- **i** and **j** are unit vectors directed across the width and length of the court respectively
- **k** is a unit vector directed vertically upwards
- units are metres

After being hit, a tennis ball, modelled as a particle, moves along the path with equation

$$\mathbf{r} = (-4.1 + 9\lambda - 2.3\lambda^2)\mathbf{i} + (-10.25 + 15\lambda)\mathbf{j} + (0.84 + 0.8\lambda - \lambda^2)\mathbf{k}$$

where λ is a scalar parameter with $\lambda \geqslant 0$

Assuming that the tennis ball continues on this path until it hits the ground,

(a) find the value of λ at the point where the ball hits the ground.

(2)

The direction in which the tennis ball is moving at a general point on its path is given by

$$(9 - 4.6\lambda)\mathbf{i} + 15\mathbf{j} + (0.8 - 2\lambda)\mathbf{k}$$

(b) Write down the direction in which the tennis ball is moving as it hits the ground.

(1)

(c) Hence find the acute angle at which the tennis ball hits the ground, giving your answer in degrees to one decimal place.

(4)

The net of the tennis court lies in the plane $\mathbf{r}.\mathbf{j} = 0$

(d) Find the position of the tennis ball at the point where it is in the same plane as the net.

(3)

The maximum height above the court of the top of the net is 0.9 m.

Modelling the top of the net as a horizontal straight line,

(e) state whether the tennis ball will pass over the net according to the model, giving a reason for your answer.

(1)

With reference to the model,

(f) decide whether the tennis ball will actually pass over the net, giving a reason for your answer.

(2)

6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 is parallel to $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

(a) Show that l_1 and l_2 are perpendicular.

(2)

The plane Π contains the line l_1 and is perpendicular to $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

(b) Determine a Cartesian equation of Π

(2)

(c) Verify that the point $A\,(3, 1, 1)$ lies on Π

(1)

Given that

- the point of intersection of Π and l_2 has coordinates (2, 3, 2)
- the point $B\,(p, q, r)$ lies on l_2
- the distance AB is $2\sqrt{5}$
- p, q and r are positive integers

(d) determine the coordinates of B.

(6)

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P 7 2 8 0 6 A 0 1 8 3 6