

- 1. A disease occurs in 3% of a population.**
 - (a) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution. **(2)**
 - (b) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people. **(3)**
 - (c) Find the mean and variance of the number of people with the disease in a random sample of 100 people. **(2)**

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

- (d) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination. **(3)**



5. A continuous random variable X has the probability density function $f(x)$ shown in Figure 1.

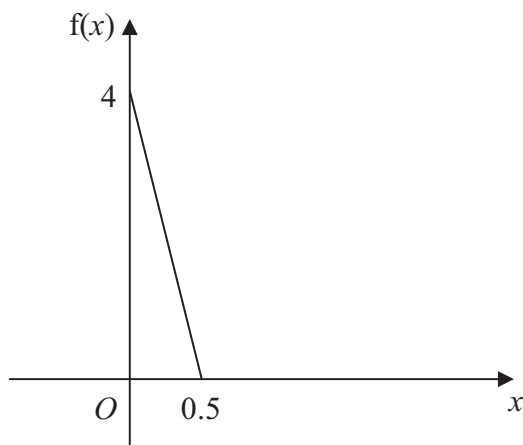


Figure 1

- (a) Show that $f(x) = 4 - 8x$ for $0 \leq x \leq 0.5$ and specify $f(x)$ for all real values of x . (4)
- (b) Find the cumulative distribution function $F(x)$. (4)
- (c) Find the median of X . (3)
- (d) Write down the mode of X . (1)
- (e) State, with a reason, the skewness of X . (1)



Question 5 continued

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6. Cars arrive at a motorway toll booth at an average rate of 150 per hour.
(a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X , per minute. (2)

(b) State clearly any assumptions you have made by suggesting this model. (2)

Using your model,

(c) find the probability that in any given minute
(i) no cars arrive,
(ii) more than 3 cars arrive. (3)

(d) In any given 4 minute period, find m such that $P(X > m) = 0.0487$ (3)

(e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period. (6)



Question 6 continued

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7. The queuing time in minutes, X , of a customer at a post office is modelled by the probability density function

$$f(x) = \begin{cases} kx(81-x^2) & 0 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{4}{6561}$. **(3)**

Using integration, find

(b) the mean queuing time of a customer, **(4)**

(c) the probability that a customer will queue for more than 5 minutes. **(3)**

Three independent customers shop at the post office.

(d) Find the probability that at least 2 of the customers queue for more than 5 minutes. **(3)**



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