Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \sin x \cosh x + \cos x \sinh x$	M1	1.1a
	$\frac{d^2y}{dx^2} = \cos x \cosh x + \sin x \sinh x + \cos x \cosh x - \sin x \sinh x$ $(= 2\cos x \cosh x)$	M1	1.1b
	$\frac{d^3y}{dx^3} = 2\cos x \sinh x - 2\sin x \cosh x$	M1	1.1b
	$\frac{d^4y}{dx^4} = -4\sinh x \sin x = -4y$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{d^2y}{dx^2}\right)_0 = 2,\ \left(\frac{d^6y}{dx^6}\right)_0 = -8,\ \left(\frac{d^{10}y}{dx^{10}}\right)_0 = 32$	B1	3.1a
	Uses $y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \ldots$ with their values	M1	1.1b
	$= \frac{x^2}{2!}(2) + \frac{x^6}{6!}(-8) + \frac{x^{10}}{10!}(32)$	A1	1.1b
	$= x^2 - \frac{x^6}{90} + \frac{x^{10}}{113400}$	A1	1.1b
		(4)	
(c)	$2(-4)^{n-1}\frac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
		(10 marks)	

Notes:

(a)

M1: Realises the need to use the product rule and attempts first derivative

M1: Realises the need to use a second application of the product rule and attempts the second derivative

M1: Correct method for the third derivative

A1*: Obtains the correct 4th derivative and links this back to y

(b)

B1: Makes the connection with part **(a)** to establish the general pattern of derivatives and finds the correct non-zero values

M1: Correct attempt at Maclaurin series with their values

A1: Correct expression un-simplified

A1: Correct expression and simplified

(c)

M1: Generalising, dealing with signs, powers and factorials

A1: Correct expression

Question	Scheme	Marks	AOs
2 (a)	$\cos^2\frac{x}{3}=\left(1-\frac{\left(\frac{x}{3}\right)^2}{2}+\frac{\left(\frac{x}{3}\right)^4}{24}-\ldots\right)^2=\ldots$ or $\left(1-\frac{x^2}{18}+\frac{x^4}{1944}-\ldots\right)^2=\ldots$ or $\frac{1}{2}\left(1\pm\cos\frac{2x}{3}\right)=\frac{1}{2}\left(1\pm\left(1-\frac{1}{2}\left(\frac{2x}{3}\right)^2+\frac{1}{4!}\left(\frac{2x}{3}\right)^4-\right)\ldots\right)$	M1	2.2a
	$=1-\frac{x^2}{9}+\frac{1}{243}x^4$	A1	1.1b
		(2)	
(b)	$\int\frac{1-\frac{x^2}{9}+\frac{1}{243}x^4}{x}=\int\frac{1}{x}-\frac{x}{9}+\frac{1}{243}x^3=A\ln x+Bx^2+Cx^4$ where A, B and $C\neq 0$	M1	3.1a
	$\ln x-\frac{x^2}{18}+\frac{1}{972}x^4$	A1ft	1.1b
	= awrt 0.98295	A1	2.2a
		(3)	
(c)	Calculator = awrt 0.98280	B1	1.1b
		(1)	
(d)	E.g. the approximation is correct to 3 d.p.	B1	3.2b
		(1)	
			(7 marks)

Notes:

(a)

M1: Deduces the required series by using the Maclaurin series for cos x, replacing x with $\frac{x}{3}$ and squares, or first applying the double angle identity (allow sign error) and then applying the series for cos x with $\frac{2x}{3}$. Attempts at finding from differentiation score M0 as the cosine series is required.

A1: Correct series

(b)

M1: Divides their series in part (a) by x and integrates to the form $A\ln x+Bx^2+Cx^4$

A1ft: Correct integration, follow through on their coefficients and need not be simplified.

A1: Deduces the definite integral awrt 0.98295

(c)

B1: Correct value.

(d)

B1: Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just “underestimate” or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.

Question	Scheme	Marks	AOs
3(a)	$f'(x)=A(1-x^2)^{-\frac{1}{2}}$ $\quad f''(x)=Bx(1-x^2)^{-\frac{3}{2}}$ and $f'''(x)=C(1-x^2)^{-\frac{3}{2}}+Dx^2(1-x^2)^{-\frac{5}{2}}$ or $\dfrac{C(1-x^2)^{\frac{3}{2}}+Dx^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	M1	2.1
	$f'(x)=(1-x^2)^{-\frac{1}{2}}$ or $\dfrac{1}{\sqrt{1-x^2}}$ $\quad f''(x)=x(1-x^2)^{-\frac{3}{2}}$ or $\dfrac{x}{(1-x^2)^{\frac{3}{2}}}$ and $f'''(x)=(1-x^2)^{-\frac{3}{2}}+3x^2(1-x^2)^{-\frac{5}{2}}$ or $\dfrac{1}{(1-x^2)^{\frac{3}{2}}}+\dfrac{3x^2}{(1-x^2)^{\frac{5}{2}}}$ from quotient rule $\dfrac{(1-x^2)^{\frac{3}{2}}+3x^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	A1	1.1b
	Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies the formula $f(x)=f(0)+f'(0)x+f''(0)\dfrac{x^2}{2}+f'''(0)\dfrac{x^3}{6}$ $\{f(0)=0,\ f'(0)=1,\ f''(0)=0,\ f'''(0)=1\}$	M1	1.1b
	$f(x)=x+\dfrac{x^3}{6}$ cso	A1	1.1b
		(4)	
(b)	$\arcsin\left(\dfrac{1}{2}\right)=\dfrac{1}{2}+\dfrac{\left(\frac{1}{2}\right)^3}{6}=\dfrac{\pi}{6}\Rightarrow\pi=...$	M1	1.1b
	$\pi=\dfrac{25}{8}$ o.e.	A1ft	2.2b
		(2)	
		(6 marks)	

Notes:

(a)

M1: Finds the correct form of the first three derivatives, may be unsimplified – the third may come later.

A1: Correct first three derivatives, may be unsimplified – the third may come later.

M1: Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies to the correct formula, needs to go up to x^3.

A1: $x+\dfrac{x^3}{6}$ cso ignore any higher terms whether correct or not

Special case: If they think that their $f''(0)\neq 0$ then maximum score M1 A0 M1 A0

M1 for correct form of the first two derivatives

M1 Correctly uses their $f(0)$, $f'(0)$, $f''(0)$ and applies to the correct formula

Question	Scheme	Marks	AOs
9(a)(i)	$\frac{dy}{dx} = \ldots cosh^{n-1} x \, sinh\, x$ $\frac{d^2y}{dx^2} = \ldots cosh^{n-2} x\, sinh^2 x + \ldots cosh^{n-1} x\, cosh\, x$ **Alternatively** $y = \left(\frac{e^x+e^{-x}}{2}\right)^n$ leading to $\frac{dy}{dx} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2} \left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots \left(\frac{e^x+e^{-x}}{2}\right)^n$	M1	1.1b
	$\frac{dy}{dx} = n\, cosh^{n-1} x\, sinh\, x$ $\frac{d^2y}{dx^2} = n(n-1)\, cosh^{n-2} x\, sinh^2 x + n\, cosh^n x$ **Alternatively** $\frac{dy}{dx} = n\left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2} \left(\frac{e^x-e^{-x}}{2}\right)^2 + n\left(\frac{e^x+e^{-x}}{2}\right)^n$	A1	2.1
	$\frac{d^2y}{dx^2} = n(n-1)\, cosh^{n-2} x\, (cosh^2 x - 1) + n\, cosh^n x$	M1	2.1
	$\frac{d^2y}{dx^2} = n^2\, cosh^n x - n(n-1)\, cosh^{n-2} x$* cso	A1*	1.1b
		(4)	
(a)(ii)	$\frac{d^3y}{dx^3} = \ldots cosh^{n-1} x\, sinh\, x - \ldots cosh^{n-3} x\, sinh\, x$ $\frac{d^4y}{dx^4}$ $= \ldots cosh^{n-2} x\, sinh^2 x + \ldots cosh^n x - \ldots cosh^{n-4} x\, sinh^2 x - \ldots cos$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\, cosh^{n-1} x\, sinh\, x - n(n-1)(n-2)\, cosh^{n-3} x\, sinh\, x$ $\frac{d^4y}{dx^4} = n^3(n-1)\, cosh^{n-2} x\, sinh^2 x + n^3\, cosh^n x$ $-n(n-1)(n-2)(n-3)\, cosh^{n-4} x\, sinh^2 x - n(n-1)(n$ $-2)\, cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 1 using $\frac{d^2y}{dx^2} = n^2 y - n(n-1)\, cosh^{n-2} x$ leading to $\frac{d^3y}{dx^3} = n^2 \frac{dy}{dx} - \ldots cosh^{n-3} x\, sinh\, x$ $\frac{d^4y}{dx^4} = n^2 \frac{d^2y}{dx^2} - \ldots cosh^{n-4} x\, sinh^2 x - \ldots cosh^{n-2} x$	M1	1.1b

	$\frac{d^3y}{dx^3} = n^2\frac{dy}{dx} - n(n-1)(n-2)\cosh^{n-3}x\sinh x$ $\frac{d^4y}{dx^4} = n^2\frac{d^2y}{dx^2} - n(n-1)(n-2)(n-3)\cosh^{n-4}x\sinh^2 x - n(n-1)(n-2)\cosh^{n-2}x$	A1	1.1b
		(2)	
	Alternative 2 $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = \ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2[n^2\cosh^n x - n(n-1)\cosh^{n-2}x] - n(n-1)[\ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x]$	M1	1.1b
	$y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = (n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2[n^2\cosh^n x - n(n-1)\cosh^{n-2}x] - n(n-1)[(n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x]$	A1	1.1b
		(2)	
	Alternative 3 Using $\frac{d^2y}{dx^2} = n^2\left(\frac{e^x+e^{-x}}{2}\right)^n - n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ leading to $\frac{d^3y}{dx^3} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^4y}{dx^4} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-\ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$	A1	1.1b

	$\frac{d^4y}{dx^4} = n^3(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-n(n-1)(n-2)(n-3)\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$		
		(2)	
(b)	When $x = 0$ $y = 1,\quad y' = 0,\quad y'' = n^2 - n(n-1),\quad y^{(3)} = 0,$ $y^{(4)} = n^3 - n(n-1)(n-2)$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \ldots$	M1	1.1b
	$y = 1 + \frac{nx^2}{2} + \frac{(3n^2-2n)x^4}{24} + \ldots$ cso	A1	2.5
		(2)	
			(8 marks)

Notes:

(a)(i)

M1: Uses the chain rule and product rule to find the first and second derivatives which must be of the required form, condone sign slips

Alternatively uses the exponential definition and uses the chain rule and product rule to find the first and second derivatives which must be of the required form.

A1: Correct unsimplified first and second derivatives, may be in exponential form.

M1: Uses the identity $\pm cosh^2 x \pm sinh^2 x = 1$

A1*: Achieves the printed answer with no errors or omissions e.g. missing x's

(a)(ii)

M1: Uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 1

M1: Using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\,cosh^{n-2} x$ to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 2

M1: Using $y = cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 cosh^n x - n(n-1)\,cosh^{n-2} x$

$y = cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = \ldots cosh^{n-2} x - \ldots cosh^{n-4} x$ leading to

Question	Scheme	Marks	AOs
2(a)	$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$	B1	1.1b
		(1)	
(b)	**Version 1** $e^{(e^x - 1)} = e^{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots - 1}$ **or** **Version 2** $e^{(e^x-1)} = 1 + (e^x - 1) + \frac{(e^x-1)^2}{2!} + \frac{(e^x-1)^3}{3!} + \ldots$	M1	1.1b
	Version 1.1 $= 1 + \left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) + \frac{1}{2}\left(x + \frac{x^2}{2!} + \ldots\right)^2 + \frac{1}{6}(x + \ldots)^3 + \ldots$ **Or** **Version 1.2** $e^{x + \frac{x^2}{2} + \frac{x^3}{6}} = e^x \times e^{\frac{x^2}{2}} \times e^{\frac{x^3}{6}}$ $= \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)\left(1 + \frac{x^2}{2} + \ldots\right)\left(1 + \frac{x^3}{6} + \ldots\right)$ **Or** **Version 2.1** $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \ldots$ $= \frac{1}{3} + \frac{1}{6}e^{3x} + \frac{1}{2}e^x$ $= \frac{1}{3} + \frac{1}{6}\left(1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{6}\right) + \frac{1}{2}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$ **Or** **Version 2.2** $= 1 + (e^x - 1) + \frac{(e^{2x} - 2e^x + 1)}{2} + \frac{(e^{3x} - 3e^{2x} + 3e^x - 1)}{6} + \ldots$	M1	3.1a

	$=1+\left(x+\frac{x^2}{2}+\frac{x^3}{6}\right)+\frac{1}{2}\left[\left(1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}\right)-2\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)+1\right]$ $+\frac{1}{6}\left[\left(1+3x+\frac{(3x)^2}{2}+\frac{(3x)^3}{6}\right)-3\left(1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}\right)+3\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)-1\right]$		
	$=1+x+\left(\frac{1}{2}+\frac{1}{2}\right)x^2+\left(\frac{1}{6}+\frac{1}{2}\times 2\times\frac{1}{2}+\frac{1}{6}\right)x^3+\ldots$	**dM1**	1.1b
	$=1+x+x^2+\frac{5}{6}x^3+\ldots$	**A1** **A1**	1.1b 2.1
		(5)	
			(6 marks)

Notes:

(a)

B1: Correct series (ignore terms beyond x^3).

(b)

M1: Correctly applies the exponential Maclaurin expansion at least once, either to the base exponent or in the index. Allow 2 for 2! and 6 for 3! in the cube term. Follow through on their series seen in (a)

M1: A complete attempt to use the exponential Maclaurin series to produce a cubic expression in terms of x only. Allow if the 3! is incorrect for this mark, but a polynomial in x must have been achieved. Condone a slip with one term. Follow through on their series seen in (a)

dM1: Dependent on previous method mark only**.** Expands the brackets and gathers terms (not necessarily fully simplified, but should have a single term for each power).

A1: Any two correct from coefficients of x, x^2 and x^3, need not be simplified.

A1: Fully correct answer with simplified terms.

NB: Question instructs to use standard Maclaurin series, so use of differentiation scores no mark.

Special case: Using $e^{\left(e^x-1\right)}=e^{e^x}\times e^{-1}$ can score M1M1M0A0A0 for using Maclaurin series e^{e^x} and then on e^x, e^{2x}, e^{3x}