Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(3y^2\right)=6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(qxy\right)=qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy$	M1	2.1
	$3px^2+qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$\left(qx+6y\right)\frac{\mathrm{d}y}{\mathrm{d}x}=-3px^2-qy\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=...$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p\left(-1\right)^3+q\left(-1\right)\left(-4\right)+3\left(-4\right)^2=26$	M1	1.1b
	$19x+26y+123=0\Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p\left(-1\right)^2-q\left(-4\right)}{q\left(-1\right)+6\left(-4\right)}=\frac{26}{19}$ or $\frac{q\left(-1\right)+6\left(-4\right)}{3p\left(-1\right)^2+q\left(-4\right)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624\Rightarrow\ p=...,q=...$	dM1	1.1b
	$p=2,\ \ q=-5$	A1	1.1b
		(5)	
			(9 marks)

Notes

(a)

M1: For selecting the appropriate method of differentiating:

Allow this mark for either $3y^2\to\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $qxy\to\alpha x\frac{\mathrm{d}y}{\mathrm{d}x}+\beta y$

A1: Fully correct differentiation. Ignore any spurious $\frac{\mathrm{d}y}{\mathrm{d}x}=...$

dM1: A valid attempt to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject with 2 terms only in $\frac{\mathrm{d}y}{\mathrm{d}x}$ coming from qxy and $3y^2$

Depends on the first method mark.

A1: Fully correct expression

(b)

M1: Uses $x=-1$ and $y=-4$ in the equation of C to obtain an equation in p and q

B1: Deduces the correct gradient of the given normal.

This may be implied by e.g.

$$19x+26y+123=0\Rightarrow y=-\frac{19}{26}x+...\Rightarrow\text{Tangent equation is }y=\frac{26}{19}x+...$$

M1: Fully correct strategy to establish an equation connecting p and q using $x=-1$ and $y=-4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the gradient of the normal. E.g. $(a)=-1\div\text{their}-\frac{19}{26}$ or $-1\div(a)=\text{their}-\frac{19}{26}$

dM1: Solves simultaneously to obtain values for p and q.

Depends on both previous method marks.

A1: Correct values

Note that in (b), attempts to form the equation of the normal in terms of *p* and *q* and then compare coefficients with 19*x* + 26*y* + 123 = 0 score no marks. If there is any doubt use Review.

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:
(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\frac{dy}{dx}+2y$ unless you see evidence that they have used the incorrect law *vu*'-*uv*'

A1: Fully correct derivative $2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

Allow attempts where candidates write $2x\mathrm{d}x-2x\mathrm{d}y-2y\mathrm{d}x+6y\mathrm{d}y=0$

but watch for students who write $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x=4\sin 2y$ and inverts $\frac{dx}{dy}=8\cos 2y \Rightarrow \frac{dy}{dx}=\frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx}=\frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx}=k\frac{1}{\cos 2y}=..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy}=8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx}=\frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x=4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy}=k\cos 2y \Rightarrow \frac{dy}{dx}=\frac{1}{k\cos 2y}$ or $1=k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x=4\sin 2y \rightarrow y=...\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx}=\frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x=8\sin y\cos y \Rightarrow \frac{dx}{dy}=\pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx}=\frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy}=8\cos 2x \Rightarrow \frac{dy}{dx}=\frac{1}{8\cos 2x}$ This is M0 A0

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x \tan y + x^2 \sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \mathrm{f}(x)$	M1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2x \times \dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{-18\times\left(x^4+81\right)-\left(-18x\right)\left(4x^3\right)}{\left(x^4+81\right)^2} = \dfrac{54\left(x^4-27\right)}{\left(x^4+81\right)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $\cot y \to -\mathrm{cosec}^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = ...$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x \tan y + x^2 \sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

Allow also $\sec^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{18}{x^3}$ or $2x = -9\mathrm{cosec}^2 y \dfrac{\mathrm{d}y}{\mathrm{d}x}$ amongst others

M1: Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \mathrm{f}(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-18x}{x^4+81}$

Question Number	Scheme	Marks
5 (a)	Sets $y = 2^x$ and takes ln of both sides to get $\ln y = x\ln 2$	M1
	Differentiates wrt x to get $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = ..$	dM1
	Rearranges to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x \ln 2$ cao	A1*
		(3)
(b)	Differentiates wrt x $\underline{\underline{2 + 6y\frac{\mathrm{d}y}{\mathrm{d}x}}} + \underline{6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}} = 4\times 2^x \ln 2$ oe	$\underline{\text{M1}}$, $\underline{\underline{\text{B1}}}$, A1
	Substitutes (2, 0) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow 2 + 12\frac{\mathrm{d}y}{\mathrm{d}x} = 16\ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16\ln 2 - 2}{12}$ $(= 0.758)$	M1
	Find equation of tangent using (2, 0) and their numerical $\frac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$y = \frac{(16\ln 2 - 2)(x-2)}{12}$ oe Accept $y = 0.76x - 1.52$	A1
		(6)
		(9 marks)
Alt 1 **5 (a)**	Writes $2^x = \mathrm{e}^{x\ln 2}$	M1
	Differentiates wrt x to get $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x\ln 2}\right) = \mathrm{e}^{x\ln 2}\ln 2 = 2^x \ln 2$ cao	dM1 A1*
		(3)
Alt 2 **5 (a)**	Sets $y = 2^x$ and takes $\ln_2$ of both sides to get $\ln_2 y = x \Rightarrow \frac{\ln y}{\ln 2} = x \Rightarrow \ln y = x\ln 2$	M1
		(3)

Question Number	Scheme	Marks
2.	$\underline{3x^2} - \left(\underline{\underline{3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}}}\right) \underline{- 1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 - 3y - 1}{3x - 3y^2}\right\}$ *not necessarily required.*	
	At $(2, -1)$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(2)^2 - 3(-1) - 1}{3(2) - 3(-1)^2}$ $\left\{= \frac{14}{3}\right\}$	M1
	$\mathbf{T}$: $y - -1 = \frac{14}{3}(x - 2)$	dM1
	$\mathbf{T}$: $14x - 3y - 31 = 0$ or equivalent	A1
		[6] **6**

Notes

1st M1: Differentiates implicitly to include either $\pm ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $\pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$.

(Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right)$ at start and omission of = 0 at end.)

1st A1: $x^3 \rightarrow \underline{3x^2}$ **and** $-x + y^3 - 11 \rightarrow \underline{-1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}}$ (so the -11 should have gone) **and** = 0 needed **here or implied by further work**. Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right)$ at start.

2nd M1: An attempt to apply the product rule: $-3xy \rightarrow -\left(3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ or $\pm 3y \pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ o.e.

3rd M1: Correct method to collect **two (not three)** dy/dx terms and to evaluate the gradient at $x = 2$ $y = -1$ (This stage may imply the earlier "=0")

4th dM1: This is dependent on all previous method marks

Uses line equation with their $\frac{14}{3}$. May use $y = \frac{14}{3}x + c$ and attempt to evaluate c by substituting $x = 2$ and $y = -1$. (May be implied by correct answer)

2nd A1: Any positive or negative whole number multiple of $14x - 3y - 31 = 0$ is acceptable. Must have = 0.

N.B. If anyone attempts the question using $\frac{\mathrm{d}x}{\mathrm{d}y}$ instead of $\frac{\mathrm{d}y}{\mathrm{d}x}$, please send to review

Question Number	Scheme	Marks
1. (a)	$\cancel{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)} = \underline{8x - 2y\frac{\mathrm{d}y}{\mathrm{d}x}} + \underline{\underline{2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y}} = 0$	$\underline{\text{M1}}\underline{\underline{\text{B1}}}\text{A1}$
	Either Way 1: Sets $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ in each term in their differentiated expression $\Rightarrow 8x - 4y + 4x + 2y = 0, \Rightarrow y - 6x = 0$ *	dM1 ddM1,A1*
	Or Way 2: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{8x+2y}{2y-2x}\right)$ (ft their differentiated expression) $\frac{8x+2y}{2y-2x} = 2$, so $y - 6x = 0$*	dM1 ddM1,A1*
		(6)
(b)	Put $y = 6x$ or $x = \frac{y}{6}$ into $4x^2 - y^2 + 2xy + 5 = 0$ and obtains $Ay^2 = B$ or $Ax^2 = B$ where A and B are constants	M1
	$x = \pm\frac{1}{2}$ **or** $y = \pm 3$ **or** $\left(\frac{1}{2}, 3\right)$ **or** $\left(-\frac{1}{2}, -3\right)$	A1
	both $\left(\frac{1}{2}, 3\right)$ and $\left(-\frac{1}{2}, -3\right)$ and no extra solutions	A1
		(3) **(9 marks)**

Question Number	Scheme	Marks
3	Differentiates wrt x $\quad 3^x\ln 3+\underline{\underline{6\frac{\mathrm{d}y}{\mathrm{d}x}}}=\underline{\tfrac{3}{2}y^2+3xy\frac{\mathrm{d}y}{\mathrm{d}x}}$ Substitutes (2, 3) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow 9\ln 3+6\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{27}{2}+18\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{9\ln 3-\frac{27}{2}}{12}=\frac{6\ln 3-9}{8},=\frac{-9+\ln 729}{8}$	B1 $\underline{\underline{\text{B1}}}$, $\underline{\text{M1, A1}}$ M1 A1, A1 **(7)** **(7 marks)**

B1 Differentiates $3^x\to 3^x\ln 3$ or $\mathrm{e}^{x\ln 3}\to\mathrm{e}^{x\ln 3}\ln 3$

B1 Differentiates $6y\to 6\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Uses the product rule to differentiate $\frac{3}{2}xy^2$. Evidence could be sight of $\underline{\tfrac{3}{2}y^2+kxy\frac{\mathrm{d}y}{\mathrm{d}x}}$
If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v =.., v'=.. followed by their vu'+uv'. For this M to be scored y^2 must differentiate to $ky\frac{\mathrm{d}y}{\mathrm{d}x}$, it cannot differentiate to $2y$.

A1 A completely correct differential of $\frac{3}{2}xy^2$. It need not be simplified.

M1 Substitutes $x=2,\ y=3$ into their expression containing a derivative to find a 'numerical' value for $\frac{\mathrm{d}y}{\mathrm{d}x}$
The candidate may well have attempted to change the subject. Do not penalise accuracy errors on this method mark

A1 Any correct numerical answer in the form $\frac{p\ln q-r}{s}$ where p, q, r and s are constants e.g. $\frac{9\ln 3-\frac{27}{2}}{12}$

A1 Exact answer. Accept either $\frac{-9+\ln 729}{8}$ or $\frac{\ln 729-9}{8}$

……

Note: There may be candidates who multiply by 2 first and start with $2\times 3^x+12y=3xy^2$

This is perfectly acceptable and the mark scheme can be applied in a similar way.

……

Question Number	Scheme	Notes	Marks
2	$\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=4x\cos x+4\sin x$	Applies product rule to $4x\sin x$ to give $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=\pm 4x\cos x+4\sin x$	M1
	$\frac{\mathrm{d}\left(\pi y^2\right)}{\mathrm{d}y}=2\pi y\frac{\mathrm{d}y}{\mathrm{d}x}$	Applies chain rule to πy^2 to give $\frac{\mathrm{d}\left(\pi y^2\right)}{\mathrm{d}y}=Ay\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$4x\sin x=\pi y^2+2x\Rightarrow 4x\cos x+4\sin x=2\pi y\frac{\mathrm{d}y}{\mathrm{d}x}+2$ Fully correct differentiation. oe Accept $4x\cos x\mathrm{d}x+4\sin x\mathrm{d}x=2\pi y\mathrm{d}y+2\mathrm{d}x$		A1
	For the differentiation ignore any spurious $"\frac{\mathrm{d}y}{\mathrm{d}x}="$		
	Alternative for first 3 marks using explicit differentiation: $y=\left(\frac{1}{\sqrt{\pi}}\right)(4x\sin x-2x)^{\frac{1}{2}}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{1}{2\sqrt{\pi}}\right)(4x\sin x-2x)^{-\frac{1}{2}}(4x\cos x+4\sin x-2)$ M1: $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x}=\pm 4x\cos x+4\sin x$ (as before) M1: $(4x\sin x-2x)^{\frac{1}{2}}\rightarrow k(4x\sin x-2x)^{-\frac{1}{2}}$		M1 M1
	Allow omission of π and sign errors when rearranging for the M marks		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\pi}}(4x\sin x-2x)^{-\frac{1}{2}}(4x\cos x+4\sin x-2)$ oe		A1
	$x=\frac{\pi}{2},y=1$ $\Rightarrow 4=2\pi\frac{\mathrm{d}y}{\mathrm{d}x}+2\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots\left(\frac{1}{\pi}\right)$	Uses $x=\frac{\pi}{2}$ and $y=1$ to obtain a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (may be implied). For implicit differentiation, there must be a d*y*/d*x* and there must be *x*'s and *y*'s. Explicit differentiation just requires use of $x=\frac{\pi}{2}$.	M1
	$y-1="-\pi"\left(x-\frac{\pi}{2}\right)$ or $y="-\pi"x+c\Rightarrow c=1+\frac{\pi^2}{2}$ Uses normal gradient $-1\Big/\frac{\mathrm{d}y}{\mathrm{d}x}$ and $x=\frac{\pi}{2},y=1$ to find equation of normal. Must use $-1\Big/\left(\text{their }\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ and $x=\frac{\pi}{2}$ and $y=1$ must be correctly placed. If using *y* = *mx* + *c* must reach as far as *c* = …		M1
	$y-1=-\pi\left(x-\frac{\pi}{2}\right)$ oe	Allow 3sf or more decimal equivalent answers e.g. $y=-3.14x+5.93$, $y-1=-3.14(x-1.57)$ etc.	A1cso
			(6 marks)

Qu	Scheme	Marks
1	Differentiate wrt x $\underline{3x^2} + \underline{\underline{6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}} + \underline{3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$ Substitutes (1, 3) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}\left(=-\frac{7}{10}\right)$ $(y-3) = -\frac{7}{10}(x-1)$ so $7x + 10y - 37 = 0$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$ M1 M1A1 **(6)** **(6 marks)**

M1 : Differentiates implicitly to include either $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **term**

Accept $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ appearing as $3x^2y'$ or $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as $3y^2y'$

A1: Differentiates $y^3 \rightarrow 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $x^3 \rightarrow 3x^2$ **and** $37 \rightarrow 0$

B1: Uses the product rule to differentiate $3x^2y$ giving $\underline{\underline{6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}$

Do not penalise students who write $3x^2\mathrm{d}x + 6xy\mathrm{d}x + 3x^2\mathrm{d}y + 3y^2\mathrm{d}y = 0$

M1: Substitutes $x = 1,\ y = 3$ into their expression (correctly each at least once) to find a 'numerical' value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (may be incorrect). Note that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$

M1: Use of $(y-3) = m(x-1)$ where m is their numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$

Alternatively uses $y = mx + c$ with $(1,3)$ and their m as far as c =..

A1: Accept integer multiples of the answer i.e. $7kx + 10ky - 37k = 0$ for example $21x + 30y - 111 = 0$

Note: If the gradient $-\frac{7}{10}$ just appears (from a graphical calculator) only M3 may be awarded

Question Number	Scheme	Marks
1	$3x^2+2xy-2y^2+4=0 \Rightarrow 6x+\overline{\overline{2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y}}\underline{-4y\frac{\mathrm{d}y}{\mathrm{d}x}}=0$	$\overline{\overline{\text{B1}}}$ $\underline{\text{M1}}$ A1
	Sets $x=2, y=4 \Rightarrow 12+4\frac{\mathrm{d}y}{\mathrm{d}x}+8-16\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}$	M1
	Uses $x=2, y=4$ and their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3} \Rightarrow (y-4)=\frac{5}{3}(x-2)$	M1
	$5x-3y+2=0$	A1
		(6 marks)

B1: $2xy$ differentiated correctly to give $2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ or any equivalent correct expression.

M1: Attempts to apply the chain rule to $-2y^2$ to give an expression of the form $Ay\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Fully correct differentiation of $3x^2-2y^2+4$ to give $6x-4y\frac{\mathrm{d}y}{\mathrm{d}x}$ and "= 0" which may be implied by subsequent work. "= 0" may also be implied if the candidate rearranges the given equation first.
Allow the candidate to start with $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ for all the above marks but if **this** $\frac{\mathrm{d}y}{\mathrm{d}x}$ is used to find the gradient, the next mark would be withheld as the two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms must come from the $2xy$ and $2y^2$ terms – see below.

Note: If $6x\mathrm{d}x+2x\mathrm{d}y+2y\mathrm{d}x-4y\mathrm{d}y=0 \Rightarrow 6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ is seen, score B1 for $2x\mathrm{d}y+2y\mathrm{d}x$ and M1 for $6x\mathrm{d}x-4y\mathrm{d}y=0$ then A1 for $6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$

M1: Substitutes $x=2$ and $y=4$ and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ (this may be implied e.g. they may rearrange their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find $-\frac{\mathrm{d}x}{\mathrm{d}y}$ and then substitute). This is not formally dependent on the first M but is dependent upon them having two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their derivative. One coming from $2xy$ and one coming from $2y^2$.

M1: Uses $x=2$ and $y=4$ and their numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}\left(=\frac{20}{12}=\frac{5}{3}\right)$ to find an equation of a <u>tangent</u> (not a normal). If $y = mx + c$ is used they much reach as far as finding a value for c.
A1: Accept $5x-3y+2=0$ or any integer multiple of this equation.

Question Number	Scheme	Marks
2.(a)	$y^3+x^2y-6x=0\Rightarrow \underline{3y^2\frac{\mathrm{d}y}{\mathrm{d}x}}+\overline{\overline{x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy}}-6=0$	$\overline{\overline{\text{B1}}}$ $\underline{\text{M1}}$ A1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2xy}{x^2+3y^2}$	M1A1
		(5)
(b)	$6-2xy=0\Rightarrow y=\frac{3}{x}$	M1
	Substitute $y=\frac{3}{x}$ into $y^3+x^2y-6x=0\Rightarrow\frac{27}{x^3}+\frac{3x^2}{x}-6x=0$	**d**M1
	$\Rightarrow x^4=9$	**dd**M1A1
	Points $\left(\sqrt{3},\sqrt{3}\right)\left(-\sqrt{3},-\sqrt{3}\right)$	A1A1
		(6)
		(11 marks)
Alt(b)	$6-2xy=0\Rightarrow x=\frac{3}{y}$	M1
	Substitute $x=\frac{3}{y}$ into $y^3+x^2y-6x=0\Rightarrow y^3+\frac{9}{y^2}y-6\times\frac{3}{y}=0$	**d**M1
	$\Rightarrow y^4=9$	**dd**M1A1
	Points $\left(\sqrt{3},\sqrt{3}\right)\left(-\sqrt{3},-\sqrt{3}\right)$	A1A1
		(6)

(a)

B1: Applies the product rule to x^2y to obtain $x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy$

M1: Applies the chain rule to y^3 to obtain $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: $y^3-6x=0\Rightarrow 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}-6=0$. i.e. y^3 differentiated correctly **and** $-6x\rightarrow-6$ **and** "= 0" seen or implied.

M1: Attempts to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. This is dependent upon them having two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their derivative. One coming from their differentiation of x^2y and the other from their differentiation of y^3

A1: Accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2xy}{x^2+3y^2}$ or equivalent.

Ignore a spurious "$\frac{\mathrm{d}y}{\mathrm{d}x}=$" at the start but see the note above regarding where the $\frac{\mathrm{d}y}{\mathrm{d}x}$'s must come from for the second method mark.

<u>If the candidate differentiates with respect to *y*, the same scheme can be applied:</u>

B1: $x^2y\rightarrow x^2+2xy\frac{\mathrm{d}x}{\mathrm{d}y}$. M1: $-6x\rightarrow A\frac{\mathrm{d}x}{\mathrm{d}y}$ A1: $y^3-6x=0\Rightarrow 3y^2-6\frac{\mathrm{d}x}{\mathrm{d}y}=0$

M1: Attempts to make $\frac{\mathrm{d}x}{\mathrm{d}y}$ the subject. This is dependent upon them having two $\frac{\mathrm{d}x}{\mathrm{d}y}$ terms in their derivative. One coming from their differentiation of x^2y and the other from their differentiation of $-6x$

Question Number	Scheme	Notes	Marks
4.	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) **Way 1**	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\cancel{=}\right\}\underline{8x - 3y^2\frac{dy}{dx}}\ \underline{\underline{-4y - 4x\frac{dy}{dx}}}\ \overline{\overline{+2^y\ln 2\frac{dy}{dx}}}\ \underline{=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4\ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 **cso**
	NOTE: You can recover work for part (a) in part (b)		**[6]**
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N (^{1} m_T), either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	y (or c) $= \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ **or** $\frac{13}{2} - \ln 2$ **or** $-\ln 2 + \frac{13}{2}$	A1 **cso isw**
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		**[3]**
			9
(a) **Way 2**	$\left\{\frac{\cancel{dx}}{\cancel{dy}}\cancel{=}\right\}\underline{8x\frac{dx}{dy} - 3y^2}\ \underline{\underline{-4y\frac{dx}{dy} - 4x}} + \overline{\overline{2^y\ln 2}}\ \underline{=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2)\frac{dx}{dy} - 3(4)^2 - 4(4)\frac{dx}{dy} - 4(-2) + 2^4\ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$	or exact equivalent	A1 **cso**
	Note: You must be clear that Way 2 is being applied before you use this scheme		**[6]**
	Question 4 Notes		
4. (a)	**Note**	<u>**For the first four marks**</u>	
		Writing down ***from no working*** • $\frac{dy}{dx} = \frac{4y - 8x}{-3y^2 - 4x + 2^y\ln 2}$ or $\frac{8x - 4y}{3y^2 + 4x - 2^y\ln 2}$ scores M1A1M1B1 • $\frac{dy}{dx} = \frac{8x - 4y}{-3y^2 - 4x + 2^y\ln 2}$ or $\frac{4y - 8x}{3y^2 + 4x - 2^y\ln 2}$ scores M1A0M1B1	
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y\ln 2\,dy = 0$ scores M1A1M1B1	

Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) **Way 1**	$\left\{\frac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}} \cancel{\times}\right\}\ \left(\underline{\underline{4xy + 2x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}\right)\ \underline{+ 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} + \pi\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$		$\text{M1}\ \underline{\text{A1}}\ \underline{\underline{\text{B1}}}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} =\right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)}\ \left\{= \frac{-8}{22 + \pi}\right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical $m_N\ (\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\}\ x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) **Way 2**	$\left\{\frac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}y}} \cancel{\times}\right\}\ \left(\underline{\underline{4xy\frac{\mathrm{d}x}{\mathrm{d}y} + 2x^2}}\right)\ \underline{+ 2\frac{\mathrm{d}x}{\mathrm{d}y} + 4 + \pi\sin(\pi y) = 0}$		$\text{M1}\ \underline{\text{A1}}\ \underline{\underline{\text{B1}}}$
	$\frac{\mathrm{d}x}{\mathrm{d}y}\left(4xy + 2\right) + 2x^2 + 4 + \pi\sin(\pi y) = 0$		dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
	Question 3 Notes		
3. (a)	**Note** Writing down ***from no working*** • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4xy + 2}{2x^2 + 4 + \pi\sin(\pi y)}$ scores M1A0B1M1A0		
	Note Few candidates will write $4xy\,\mathrm{d}x + 2x^2\mathrm{d}y + 2\,\mathrm{d}x + 4\mathrm{d}y + \pi\sin(\pi y)\mathrm{d}y = 0$ leading to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or equivalent. This should get full marks.		

Question Number	Scheme		Marks
2.	$3^{x-1}+xy-y^2+5=0$		
		$3^{x-1}\to 3^{x-1}\ln 3$	B1 oe
	$\left\{\frac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}}\cancel{=}\right\}$ $3^{x-1}\ln 3+\left(y+x\frac{\mathrm{d}y}{\mathrm{d}x}\right)-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$.	M1*
		$xy\to +y+x\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
		$\ldots+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1
	$\{(1,3)\Rightarrow\}$ $3^{(1-1)}\ln 3+3+(1)\frac{\mathrm{d}y}{\mathrm{d}x}-2(3)\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Substitutes $x=1,\ y=3$ into their differentiated equation or expression.	dM1*
	$\ln 3+3+\frac{\mathrm{d}y}{\mathrm{d}x}-6\frac{\mathrm{d}y}{\mathrm{d}x}=0\ \Rightarrow\ 3+\ln 3=5\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3+\ln 3}{5}$		dM1*
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\left(\ln \mathrm{e}^3+\ln 3\right)=\frac{1}{5}\ln\left(3\mathrm{e}^3\right)$	**Uses** $3=\ln \mathrm{e}^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\ln\left(3\mathrm{e}^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 2

B1: Correct differentiation of 3^{x-1} . I.e. $3^{x-1}\to 3^{x-1}\ln 3$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)\to\frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1}=\mathrm{e}^{(x-1)\ln 3}\to \ln 3\,\mathrm{e}^{(x-1)\ln 3}$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)=\frac{1}{3}\mathrm{e}^{x\ln 3}\to\frac{1}{3}(\ln 3)\mathrm{e}^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$).

B1: $xy\to +y+x\frac{\mathrm{d}y}{\mathrm{d}x}$

1st A1: $\ldots+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$=0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3+y+x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}y}{\mathrm{d}x}$ leading to $3^{x-1}\ln 3+y=2y\frac{\mathrm{d}y}{\mathrm{d}x}-x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x=1,\ y=3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{\mathrm{d}y}{\mathrm{d}x}$ and rearranges to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y+3^{x-1}\ln 3}{2y-x}$ before substituting in $x=1$ and $y=3$

2nd A1: **cso.** Uses $3=\ln \mathrm{e}^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}\ln\left(3\mathrm{e}^3\right)$, $\left(=\frac{1}{\lambda}\ln\left(\mu\mathrm{e}^3\right),\ \lambda=5\text{ and }\mu=3\right)$

Note: $3=\ln \mathrm{e}^3$ needs to be seen in their proof.

Notes for Question 2 Continued