

Numerical MethodsKey FormulaeNewton Raphson

- The Newton Raphson iteration for solving  $f(x)=0$  is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Key ConceptsLocating Roots

- If the function  $f(x)$  is continuous on the interval  $[a,b]$  &  $f(a)$  &  $f(b)$  have opposite signs, the  $f(x)$  has at least one root on  $(a,b)$ .
- $f(a)$  &  $f(b)$  have different signs  $\Rightarrow$  odd number of roots
- $f(a)$  &  $f(b)$  have the same sign  $\Rightarrow$  no roots or an even number of roots

Iteration

- The roots to  $f(x)=0$  could be estimated by rearranging to the form  $x_{n+1} = g(x_n)$ .
- Convergent Iterative Formula -  $x_{n+1}$  approaches a limit as  $n \rightarrow \infty$ .
- Divergent Iterative Formula -  $x_{n+1}$  is unbounded as  $n \rightarrow \infty$ .

Newton Raphson Iterative Formula

- $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$  may approximate the solution to  $f(x)=0$ .
- The formula does not work if  $f'(x_n)=0$ .
- The formula takes a long time to converge if  $f'(x_n)$  is close to 0.