6.

$$f(x) = \frac{x + 2}{x^2 + 9}$$

(a) Show that

$$\int f(x)\,dx = A\ln(x^2 + 9) + B\arctan\left(\frac{x}{3}\right) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence show that the mean value of $f(x)$ over the interval $[0, 3]$ is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval $[0, 3]$, of

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, where p and q are constants and q is in terms of k.

(2)

2. Show that

$$\int_0^{\infty} \frac{8x - 12}{(2x^2 + 3)(x + 1)}\,dx = \ln k$$

where k is a rational number to be found.

(7)

3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval [0, 3].

(2)

2. (a) Explain why $\int_1^{\infty} \frac{1}{x(2x+5)}\,dx$ is an improper integral.

(1)

(b) Prove that

$$\int_1^{\infty} \frac{1}{x(2x+5)}\,dx = a\ln b$$

where a and b are rational numbers to be determined.

(6)

5. (a)

$$y = \tan^{-1} x$$

Assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{1}{1 + x^2}$$

(3)

$$f(x) = x \tan^{-1} 4x$$

(b) Show that

$$\int f(x)\,dx = Ax^2 \tan^{-1} 4x + Bx + C \tan^{-1} 4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5)

(c) Hence find, in exact form, the mean value of $f(x)$ over the interval $\left[0, \frac{\sqrt{3}}{4}\right]$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 6 2 6 7 1 A 0 1 6 2 8

5. (i) Evaluate the improper integral

$$\int_{1}^{\infty} 2\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$$

(3)

(ii) The air temperature, θ °C, on a particular day in London is modelled by the equation

$$\theta = 8 - 5\sin\left(\frac{\pi}{12}t\right) - \cos\left(\frac{\pi}{6}t\right) \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

(a) Use calculus to show that the mean air temperature on this day is 8 °C, according to the model.

(3)

Given that the actual mean air temperature recorded on this day was higher than 8 °C,

(b) explain how the model could be refined.

(1)

9. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2-1}}\,dx = \frac{1}{2}\left[x\sqrt{x^2-1} + \operatorname{arcosh} x\right] + k$$

where k is an arbitrary constant.

(6)

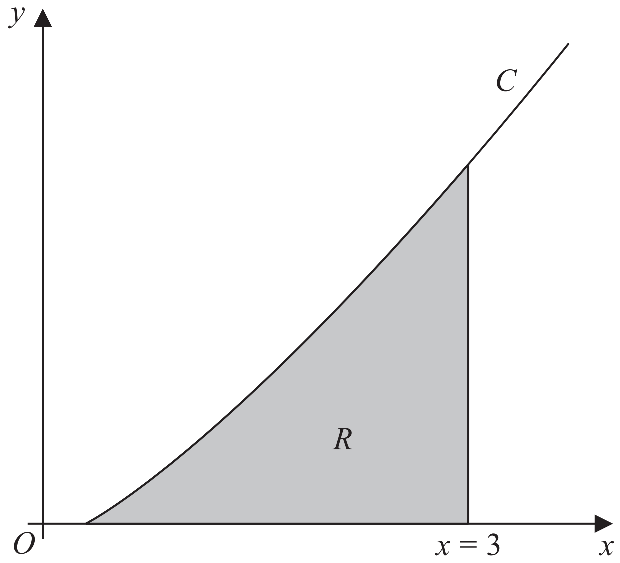

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15}x \operatorname{arcosh} x \qquad x \geqslant 1$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15}\left[17\ln\left(3+2\sqrt{2}\right) - 6\sqrt{2}\right]$$

(5)

5. The curve C has equation

$$y = \arccos\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2$$

(a) Show that C has no stationary points.

(3)

The normal to C, at the point where $x = 1$, crosses the x-axis at the point A and crosses the y-axis at the point B.

Given that O is the origin,

(b) show that the area of the triangle OAB is $\frac{1}{54}\left(p\sqrt{3} + q\pi + r\sqrt{3}\pi^2\right)$ where p, q and r are integers to be determined.

(5)

6. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}\,\mathrm{d}x = \ln\left(a\sqrt{2}\right) + b\pi$$

where a and b are constants to be determined.

(4)

9. (i) (a) Explain why $\int_0^{\infty} \cosh x \, dx$ is an improper integral.

(1)

(b) Show that $\int_0^{\infty} \cosh x \, dx$ is divergent.

(3)

(ii) $4\sinh x = p\cosh x$ where p is a real constant

Given that this equation has real solutions, determine the range of possible values for p

(2)

5. (a) Given that

$$y = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

(3)

(b)

$$f(x) = \arcsin(e^x) \qquad x \leqslant 0$$

Prove that f(x) has no stationary points.

(3)

2. (a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1)

(b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}}\, dx$$

(2)

(c) Determine the mean value of the function

$$f(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \qquad 3 \leqslant x \leqslant 13$$

giving your answer in the form $A \ln B$ where A and B are constants in simplest form.

(3)