Question	Scheme	Marks	AOs
15 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=$...	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as c = ... to score this mark.

A1: Achieves $y=-3x+25$

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2. For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0 \Rightarrow (x-a)^2+(y-b)^2=k \Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,-2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.
The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k. It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k. See General Principles.
The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

<u>Look carefully at the method used. It is possible to attempt this using gradients</u>

(b) Alt 1	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5}$, $y=\frac{-10\pm3\sqrt{5}}{5}$, $k=y-3x \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

	For $p = \text{awrt } 63100$ or $q = \text{awrt } 1.122$	A1	1.1b
	For correct equations in p and q $p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$= \text{awrt } (£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p = 10^{4.8}$ or $q = 10^{0.05}$ but may be $\log q = 0.05$ or $\log p = 4.8$

A1: For $p = \text{awrt } 63100$ or $q = \text{awrt } 1.122$

M1: For linking the two equations and forming correct equations in p and q. This is usually $p = 10^{4.8}$ and $q = 10^{0.05}$ but may be $\log q = 0.05$ and $\log p = 4.8$

A1: For $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$ Both these values implies M1 M1

……………………………………………………………………………………………

ALT I(a)

M1: Substitutes $t = 0$ and states that $\log p = 4.8$

A1: $p = \text{awrt } 63100$

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p = \text{awrt } 63100$ and $q = \text{awrt } 1.122$

……………………………………………………………………………………………

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " *p* is........... " and "*q* is ...."

(c)

M1: For substituting $t = 30$ into $V = pq^t$ using their values for p and q or substituting $t = 30$ into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = \ldots$	M1	1.1b

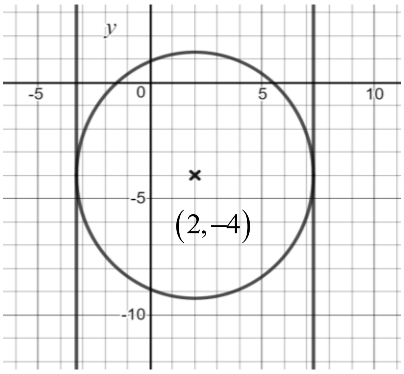
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $\left(1+k^2\right)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their *a*, *b* and *c* leading to values for *k* $"(10k-6)^2-36\left(1+k^2\right)...0" \to k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)
M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$
This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$
(i) A1: Centre $(3,-5)$
(ii) A1: Radius 5. Do not accept $\sqrt{25}$
Answers only (no working) scores all three marks

(b)
B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant0$ or even with greater than symbols
M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just *x* and *k* . It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.
A1: Correct 3TQ $\left(1+k^2\right)x^2+(10k-6)x+9=0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as
$x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of *a*, *b* and *c* are used in $b^2-4ac...0$.
FYI The equation in *y* and *k* is $\left(1+k^2\right)y^2+\left(10k^2-6k\right)y+9k^2=0$ oe

M1: Attempts to find two critical values for *k* using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.
dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both *a* and *b* must have been expressions in *k*.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.
$b^2-4ac=(k-6)^2-4\times\left(1+k^2\right)\times9>0 \Rightarrow -35k^2-12k>0 \Rightarrow k(35k+12)<0$

Question	Scheme	Marks	AOs
10(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract 'r' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	
			(5 marks)

Notes

(a)

M1: Attempts to complete the square. Look for $(x\pm2)^2+(y\pm4)^2...$

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm2,\pm4)$ Condone $a=\pm2, b=\pm4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds x/k by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root. E.g. $b^2-4ac=0 \Rightarrow 64-4\left(k^2-4k-8\right)=0 \Rightarrow k=..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm5.29 \rightarrow k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

Question	Scheme	Marks	AOs
11. (i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" = $\dfrac{7-1}{-5+9}=\left(\dfrac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\dfrac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

Notes

(i)

M1: Attempts $(x\pm9)^2....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\dfrac{7-\text{"}1\text{"}}{-5-\text{"}-9\text{"}}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\dfrac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in Z$

....

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\dfrac{dy}{dx}+18-2\dfrac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\dfrac{dy}{dx}+18-2\dfrac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\dfrac{dy}{dx}$

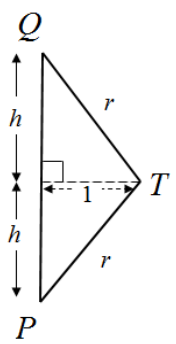
....

Question	Scheme		Marks	AOs
17 (a)	**Way 1:** Finds circle equation $(x \pm 2)^2 + (y \mp 6)^2 = (10 \pm (-2))^2 + (11 \mp 6)^2$	**Way 2:** Finds distance between $(-2, 6)$ and $(10, 11)$	M1	3.1a
	Checks whether (10, 1) satisfies their circle equation	Finds distance between $(-2, 6)$ and $(10, 1)$	M1	1.1b
	Obtains $(x+2)^2 + (y-6)^2 = 13^2$ and checks that $(10+2)^2 + (1-6)^2 = 13^2$ so states that (10, 1) lies on C *	Concludes that as distance is the same (10, 1) lies on the circle C *	A1*	2.1
			(3)	
(b)	Finds radius gradient $\dfrac{11-6}{10-(-2)}$ or $\dfrac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\dfrac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or –23		A1	1.1b
	Way 1: Deduces gradient of second tangent	**Way 2:** Deduces midpoint of PQ from symmetry (0, 6)	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ = 35 + 23= 58*		A1*	1.1b
			(7)	
			(10 marks)	

Question Number	Scheme	Marks
4. (a)	**Usually answered in radians: Uses** $BCD = 3.5\times(angle)$, $= 3.5\times 1.77 = 6.195$ (m) (accept awrt 6.20)	M1 A1 **(2)**
(b)	Area = $\frac{1}{2}(3.5)^2\times 1.77 = 10.84$ (m^2)	M1 A1 **(2)**
(c)	Area of triangle = $\frac{1}{2}\times 3.7\times 3.5\times\sin(\text{angle})$, $= \frac{1}{2}\times 3.7\times 3.5\times\sin(\frac{\pi}{2}-\frac{1.77}{2})$ (=awrt 4.1)	M1, A1
	Total area = $"10.84"+2\times"4.101"$	M1
	= 19.04	A1cao **(4)** **[8]**

	Notes
(a)	M1: uses $s = 3.5\times\theta$ with θ in radians or completely correct work in degrees
	A1: awrt 6.20 or just 6.2 (do not need to see units) Correct answer can imply the method.
(b)	M1 for attempt to use $A=\frac{1}{2}\times 3.5^2\times\theta$ (Accept θ in degrees.)
	A1 for awrt 10.84 (do not need to see units) isw if correct answer is followed by 10.8. Correct answer can imply the method.
(c)	M1: Uses area of triangle $\frac{1}{2}\times 3.7\times 3.5\times\sin(\text{angle})$ Must be correct method for area of triangle but may be less direct.
	A1: Correct expression using correct angle i.e. $\frac{\pi}{2}-\frac{1.77}{2}$ or awrt 0.69 or awrt 39 degrees (need at least 2 sf if no other work seen, but may be implied by correct final answer) If correct expression is given then isw (so e.g. isw an answer of 0.0775 implying angle set to degrees on calculator)
	M1: Adds **twice** their **second calculated area** (even if rectangle or segment) to their sector area (may have been slips or errors in one or both formulae – such as missing ½ or mixture of degrees and radians or weak attempt at triangle area) so M0A0M1A0 is a possible mark distribution
	A1: 19.04 cao (common answer through insufficient accuracy is 19.08 which loses final mark)
	Special Case. The mark profile M1A0M1A0M1A0M1A0 can be given if the angle is misunderstood as 1.77π or as *AFB* for example
	If $"10.84"+3.5\times 3.7\sin(\text{angle})$ is used then this can gain both M marks and the A marks if correct.
	But use of $3.5\times 3.7\sin(\text{angle})$ and later doubled and added to "10.84" is 1st M0, 2nd M1.

Question number	Scheme	Marks
5	$x^2+y^2-10x+6y+30=0$	
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x\pm 5)^2}$ $+\underline{\underline{(y\pm 3)^2}}$ $= \ldots$. Accept (±5,±3) as indication of this.	M1

	Centre is $(5, -3)$.	A1	**(2)**
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give $r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30 - 25 - 9$)	M1	
	$r = 2$	A1cao	**(2)**
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1	
	$r = 2$	A1	**(2)**
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1	
	e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10\times 4 + 6y + 30 = 0$		
	Solve their quadratic in y and obtain **two** solutions for y	dM1	
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1	**(3)**
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$,	M1	
	Find h and evaluate $"-3" \pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1	
	So $y = -3 \pm \sqrt{3}$	A1	**(3)** **[7]**

Question Number	Scheme		Marks
3.	$P(7, 8)$ and $Q(10, 13)$		
(a)	$\{PQ =\}\ \sqrt{(7-10)^2 + (8-13)^2}$ or $\sqrt{(10-7)^2 + (13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) **Way 1**	$(x-7)^2 + (y-8)^2 = 34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive value.	M1
		$(x-7)^2 + (y-8)^2 = 34$	A1 oe
			[2]
(b) **Way 2**	$x^2 + y^2 - 14x - 16y + 79 = 0$	$x^2 + y^2 \pm 14x \pm 16y + c = 0$, where c is any value < 113.	M1
		$x^2 + y^2 - 14x - 16y + 79 = 0$	A1 oe
			[2]
(c) **Way 1**	$\{\text{Gradient of radius}\} = \frac{13-8}{10-7}$ or $\frac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $= -\frac{1}{m}\left(= -\frac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\frac{1}{\text{gradient of radius}}$	M1
	$y - 13 = -\frac{3}{5}(x-10)$	$y - 13 = (\text{their changed gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 2**	$2(x-7) + 2(y-8)\frac{dy}{dx} = 0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7) + 2(13-8)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$	Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\frac{dy}{dx}$	M1
	$y - 13 = -\frac{3}{5}(x-10)$	$y - 13 = (\text{their gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 3**	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	B1
		$10x + 13y - 7(x+10) - 8(y+13) + c = 0$ where c is any value < 113	M2
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
			8

Question Number	Scheme	Marks
13 (a)(i)	$(3,-4)$	B1
(a)(ii)	$\sqrt{30}$	B1
		[2]
(b)	Attempts $(6-3)^2+(k+4)^2, < 30$	M1,M1
	$k^2+8k-5<0$	A1*
		[3]
(c)	Solves quadratic by formula or completion of square to give $k=$	M1
	$k=-4\pm\sqrt{21}$	A1
	Chooses region between two values and deduces $-4-\sqrt{21}<k<-4+\sqrt{21}$	M1
		A1**cao**
		[4]
		(9 marks

(a)(i)(ii)

B1 $(3,-4)$ Accept as $x=\ ,\ y=$ or even without the brackets

B1 $\sqrt{30}$ Do not accept decimals here but remember to isw

(b) This is scored M1 A1 A1 on e -pen. We are marking it M1 M1 A1

M1 Attempts to find the length or length^2 from P $(6,k)$, to the centre of C $(3,-4)$ following through on their C. Look for, using a correct C, either $(6-'3')^2+(k+'4')^2$ or $\sqrt{(6-'3')^2+(k+'4')^2}$

Another way is to substitute $(6,k)$ into $(x-3)^2+(y+4)^2=30$ but it is very difficult to score either of the other two marks using this method.

M1 Forms an inequality by using the length from P to the centre of C < the radius of C

$(6-3)^2+(k+4)^2<30$. In almost all cases I would expect to see < 30 before < 0

Using the alternative method, they would also need the line $(6-3)^2+(k+4)^2<30$. (As if the point lies on another circle, the radius/distance would need to be smaller than 30)

A1* $k^2+8k-5<0$

This is a given answer and you must check that all aspects are correct. **In most cases you should expect to see an intermediate line (with < 30) before the final answer appear with < 0.**

(c)

M1 Solves the equation $k^2+8k-5=0$ by formula or completing the square.

Factorisation to integer roots is not a suitable method in this case and scores M0.

The answers could just appear from a graphical calculator. Accept decimals for the M's only

A1 Accept $k=-4\pm\sqrt{21}$ or exact equivalent $k=\dfrac{-8\pm\sqrt{84}}{2}$

Do not accept decimal equivalents $k=-8.58,(+)0.58$ 2dp for this mark

M1 Chooses inside region from their two roots. The roots could just appear or have been derived by factorisation.

A1 cao $-4-\sqrt{21}<k<-4+\sqrt{21}$ Accept equivalents such as $\left(-4-\sqrt{21},-4+\sqrt{21}\right)$,

$k>-4-\sqrt{21}$ and $k<-4+\sqrt{21}$, even $k>-4-\sqrt{21}$, $k<-4+\sqrt{21}$

Accept for 3 out of 4 $\left[-4-\sqrt{21},-4+\sqrt{21}\right]$, $k>-4-\sqrt{21}$ or $k<-4+\sqrt{21}$, $-4-\sqrt{21}\leqslant k\leqslant -4+\sqrt{21}$

Do not accept $-4-\sqrt{21}<x<-4+\sqrt{21}$ for this final mark

Question Number	Scheme	Marks
14 (a)	$\left(\frac{1+7}{2},\frac{4+8}{2}\right)=(4,6)$	M1A1
		(2)
(b)	$\frac{\sqrt{(7-1)^2+(8-4)^2}}{2}$ Or $\sqrt{('4'-1)^2+('6'-4)^2}$ Or $\sqrt{(7-'4')^2+(8-'6')^2}$	M1
	(Radius of circle) = $\sqrt{13}$	A1
		(2)
(c)	Equation of C_2 is $x^2+y^2=r^2$	M1
	Attempts either value of r as $\left(\sqrt{'4'^2+'6'^2}\pm \text{their } r\right)$	M1
	When $r=\sqrt{52}-\sqrt{13}=\sqrt{13} \Rightarrow x^2+y^2=13$	A1
	When $r=\sqrt{52}+\sqrt{13}=3\sqrt{13} \Rightarrow x^2+y^2=117$	A1
		(4) **(8marks)**

(a)

M1 For an attempt at $\left(\frac{1+7}{2},\frac{4+8}{2}\right)$ May be implied by either correct coordinate

A1 $(4,6)$. No working is required, Correct answer scores both marks. Condone lack of brackets

(b)

M1 Scored for using Pythagoras' theorem to find the distance between their centre and a point. Look for an attempt at $\sqrt{('4'-1)^2+('6'-4)^2}$ or similar. If the original coordinates are used then there must be some attempt to halve.

A1 $=\sqrt{13}$ Correct answer scores both marks

(c)

M1 For stating the equation of C_2 is $x^2+y^2=r^2$ or $(x-0)^2+(y-0)^2=r^2$ for any 'r' including an algebraic 'r' Accept $x^2+y^2=k$ If a value of k is given then k must be positive

M1 Attempts either value of r Look for $\left(\sqrt{'4'^2+'6'^2}\pm \text{their } r\right)$ Accept $r=\frac{\sqrt{4^2+6^2}}{2}$

A1 Either of $x^2+y^2=13$ or $x^2+y^2=117$

Allow for this mark variations like $(x-0)^2+(y-0)^2=\sqrt{13}^2$

A1 Both of $x^2+y^2=13$ and $x^2+y^2=117$. Equations must be simplified as seen here

Any one correct equation will imply the first two M's.

……

Alt method to find equations using the intersections:

M1: As above

M1: Solves 'their' $y='\frac{3}{2}'x$ with their $(x-'4')^2+(y-'6')^2='13' \Rightarrow$ Intersections $(2,3)$ and $(6,9)$

So this time the method is scored for either $\sqrt{'2'^2+'3'^2}$ or $\sqrt{'6'^2+'9'^2}$

A1 A1 as before

Question Number	Scheme		Marks
2 (a)	**Mark (a) and (b) together**		
	$(x\pm4)...(y\pm2)$	Attempts to complete the square on x and y or sight of $(x\pm4)$ **and** $(y\pm2)$. May be implied by a centre of $(\pm4,\pm2)$. Or if considering $x^2+y^2+2gx+2fy+c=0$, centre is $(\pm g,\pm f)$.	M1
	Centre $C=(4,-2)$	Correct centre (allow $x = 4$, $y = -2$) But not $g = ..$, $f = …$ or $p = …$, $q = …$ etc.	A1
	Correct answer scores both marks		
			(2)
(b)	$r^2=12+(\pm4)^2+(\pm2)^2$	**Must reach:** $r^2=12+\text{their }(\pm4)^2+\text{their }(\pm2)^2$ or $r=\sqrt{12+\text{their }(\pm4)^2+\text{their }(\pm2)^2}$ or if considering $x^2+y^2+2gx+2fx+c=0$, $r^2=g^2+f^2-c$ or $r=\sqrt{g^2+f^2-c}$ Must clearly be identifying the radius or radius^2 May be implied by a correct exact radius or awrt 5.66	M1
	$r=\sqrt{32}$	$r=\sqrt{32}$. Accept exact equivalents such as $4\sqrt{2}$. $r = …$ not needed but must clearly be the radius. Do not allow $\pm\sqrt{32}$ unless minus is rejected	A1
	Correct answer scores both marks		
			(2)
(c)	$x=0\Rightarrow y^2+4y-12=0$	Correct quadratic. Allow $16+(y+2)^2=32$	B1
	$(y+6)(y-2)=0\Rightarrow y=...$	Attempts to solve a 3TQ that has come from substituting $x = 0$ or $y = 0$ into the given equation or their 'changed' equation. May be implied by correct answers for their quadratic.	M1
	$y=2,-6$ or $(0,2)$ and $(0,-6)$	Correct y values or correct coordinates. Accept sight of these for all 3 marks if no incorrect working seen but must clearly be y values or correct coordinates. This may be implied by the correct roots of a quadratic in y.	A1
			(3)
			(7 marks)

Question	Scheme		Marks
13 (a)	See $(x\pm1)^2+(y\pm3)^2=r^2$	Or see $x^2+y^2\pm2x\pm6y+c=0$	M1
	Attempt $\sqrt{(8-1)^2+(-2-(-3))^2}$ **or** $(8-1)^2+(-2-(-3))^2$	Substitute $(8,-2)$ into equation	M1
	$(x-1)^2+(y+3)^2,=50$	$x^2+y^2-2x+6y-40=0$	A1, A1 **[4]**
(b)	Gradient of $AP=\frac{1}{7}$		B1
	So gradient of tangent is -7		M1
	Equation of tangent is $(y+2)=-7(x-8)$		dM1
	$y=-7x+54$ or $m=-7$, $c=54$		A1 **[4]**
	Way 1	Way 2	
(c)	$y=x+6$ meets circle when $(x-1)^2+(x+9)^2=50$ or when $(y-7)^2+(y+3)^2=50$	As tangent has gradient 1 AQ has gradient -1 and $\frac{y-(-3)}{x-1}=-1$	M1
	i.e. $2x^2+16x+32=0$ **or** when $2y^2-8y+8=0$	$y+x=-2$	A1
	Solve to give x or $y=$	Solve $y+x=-2$ with $y=x+6$ or alternatively solve $y+x=-2$ with the equation of the circle to give x or $y=$	M1
	Substitute to give $y=$ (**or** $x=$)		dM1
	$(-4, 2)$ only		A1 **[5]**
			13 marks
	Notes		

(a)

M1 : Scored for centre at $(1,-3)\Rightarrow(x\pm1)^2+(y\pm3)^2=\ldots$ or $x^2+y^2\pm2x\pm6y+\ldots=0$

M1: Scored for an attempt at finding the radius or the radius ^{2} (see scheme).
It need not be in the equation It can be <u>implied</u> by $\sqrt{50}$ *or* $5\sqrt{2}$ or 50
If the form $x^2+y^2\pm2x\pm6y+c=0$ is used it is for substituting $(8,-2)$ into the equation

A1: LHS or RHS correct $(x-1)^2+(y+3)^2,=\ldots$ or $(x\pm a)^2+(y\pm b)^2,=50$ $x^2+y^2-2x+6y\ldots=0$

A1: Correct equation. Accept $(x-1)^2+(y+3)^2=50$ or $x^2+y^2-2x+6y-40=0$ or $x^2+y^2-2x+6y=40$

(b)

B1 : Obtain 1/7 . Implied by use of – 7 in their tangent

M1: Uses negative reciprocal

dM1: Linear equation through point (8, -2) with their negative reciprocal gradient

A1: cao

(c)

M1: Eliminates x or y from two relevant equations, that is whose intersection is Q.

A1: Correct quadratic in x or in y

M1: Solves (with usual rules) to give first variable. The first M must have been scored

dM1: Substitute in either (relevant) equation to give second coordinate, dependent upon both previous M's

A1: Correct answer accept $x=-4$, $y=2$. Withhold this if two answers given

Question Number	Scheme	Marks
12(a)	Writes C as $(x-a)^2+(y-0)^2=a^2$	M1A1 **(2)**
(b)	Subs $(4,-3)$ $\Rightarrow(4-a)^2+(-3-0)^2=a^2$	M1
	$\Rightarrow 16-8a+a^2+9=a^2$	
	$\Rightarrow 25=8a$	
	$\Rightarrow a=\dfrac{25}{8}$	dM1A1 **(3)**
		(5 marks)

Mark parts (a) and (b) together. Award marks in (a) from (b) and vice versa, but see note

(a)

M1 Attempts to find the equation of C centre $(a,0)$ radius a. Accept $(x\pm a)^2+y^2=a^2$ oe

If the alternative form of the circle is used accept $x^2+y^2\pm 2ax=a^2-a^2$

Allow for the M1 $(x\pm a)^2+(y\pm 0)^2=r^2$

A1 Writes C as $(x-a)^2+(y-0)^2=a^2$ or equivalent $x^2+y^2-2ax=0$.

(b)

M1 Subs $x=4$ and $y=-3$ into their circle equation for C which must be of the form $(x\pm a)^2+(y\pm 0)^2=a^2$

dM1 Proceeds to a linear equation in 'a' and reaches a=.... Condone numerical slips

A1 $a=\dfrac{25}{8}$ Accept exact alternatives

Note: There are some candidates who write the equation of the circle as $(x-a)^2+(y-0)^2=r^2$ in part (a)

This is M1 A0

However in part (b) they substitute $(4,-3)$ and write down $(4-a)^2+(-3)^2=a^2$

We will allow them to score all 3 marks in part (b).

Had they written $(x-a)^2+y^2=a^2$ in (b) we would allow them to score all 5 marks

Question Number	Scheme	Marks
15 (a)	Mid-point of AB = (2, - 3)	M1 A1
	$(r^2) = (12 - "2")^2 + (2 - "-3")^2$ or $(r^2) = (-8 - "2")^2 + (-8 - "-3")^2$ or $(d^2) = (-8-12)^2 + (-8-2)^2$	M1
	$r^2 = 125$	A1
	$"125" = (\boldsymbol{x} \pm "2")^2 + (\boldsymbol{y} \pm "-3")^2$	M1
	$125 = (x-2)^2 + (y+3)^2$	A1
		[6]
(b)	gradient from "(2, −3)" to (4, 8) $= \dfrac{8 - "-3"}{4 - "2"}, \left(= \dfrac{11}{2}\right)$	M1
	ZM has gradient $-\dfrac{1}{m}$ $\left(= -\dfrac{2}{11}\right)$	M1
	Either : $y - 8 = "-\frac{2}{11}"(x-4)$ **or**: $y = "-\frac{2}{11}"x + c$ and $8 = "-\frac{2}{11}"(4) + c \Rightarrow c = "8\frac{8}{11}"$	ddM1
	$2x + 11y - 96 = 0$	A1
		[4]
		(10marks)
	Notes	
(a)	**M1:** Uses midpoint formula, or implied by y coordinate of -3 or x coordinate of 2 **A1:** cao **M1:** Finds radius or radius^2, diameter or diameter^2 using any valid method – probably distance from centre to one of the points. **Need not state r =… so ignore lhs – you are just looking for correct use of Pythagoras with or without the square root so ignore how they reference it for this mark.** **A1:** for any equivalent $r^2 = 125$ or $r = \sqrt{125}$ (11.18...) etc . Their numeric answer must be identified here as either r or r^2 (may be implied by their equation). If they halve it or double it, this is M1 A0. **M1:** Attempt to use a true equation for circle with their centre and radius or the letter r, allow sign slips in brackets but do not allow use or r instead of r^2 in the equation. So must be using $r^2 = (\boldsymbol{x} \pm ...)^2 + (\boldsymbol{y} \pm ...)^2$ **A1:** correct answer only (Allow $\left(5\sqrt{5}\right)^2$ instead of 125 but not $5\sqrt{5}^{\,2}$)	
(b)	**M1:** States or uses gradient equation correctly **with their centre and (4, 8).** Must be using their centre and (4, 8). If no method is shown and gradient incorrect for their values score M0. **M1:** Finds negative reciprocal. Follow through their gradient **ddM1:** Correct straight line method with (4, 8) and perpendicular gradient. Dependent on **both** previous method marks having been scored. **A1:** cao – accept multiples of this equation **(Note integer coefficients not required)** A common error here is to use the diameter to find the gradient. This usually scores M0M1ddM0A0 i.e. just one mark for the perpendicular gradient rule.	
	(b) Alternative uses implicit differentiation: e.g. $125 = (x-2)^2 + (y+3)^2 \Rightarrow 2(x-2) + 2(y+3)\dfrac{dy}{dx} = 0$ M1(correct implicit differentiation) oe $\Rightarrow \dfrac{dy}{dx} = \dfrac{2-x}{y+3} = \dfrac{2-4}{8+3}$ M1(Substitution) Then follow the scheme.	

Question number	Scheme	Marks
15 (a)	$\sqrt{(12-5)^2+(7-6)^2}$,= $\sqrt{50}$ or $5\sqrt{2}$	M1, A1 [2]
(b)	See $(x \pm 5)^2 + (y \pm 6)^2 = (\text{their numerical } r)^2$	M1
	$(x-5)^2+(y-6)^2 =, 50$	B1, A1 [3]
(c)	Gradient of $AP = \frac{1}{7}$	B1
	So gradient of tangent is -7	M1
	Equation of tangent is $(y-7) = -7(x-12)$	dM1 A1 [4]
(d)	$AB = \sqrt{180} = \left(6\sqrt{5}\right)$, $BC = \sqrt{160} = \left(4\sqrt{10}\right)$, $AC = 10$	M1 A1 A1
	$\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{160+100-180}{20\sqrt{160}}$	M1
	So C = awrt 71.6	A1 [5] **14 marks**

Question Number	Scheme	Marks
15. (a)	gradient $=\frac{11-3}{6-0}$, $=\frac{4}{3}$	M1 A1 **[2]**
(b)	Mid-point of XY = (3, 7)	M1 A1
	ZM has gradient $-\frac{1}{m}$ $\quad\left(=-\frac{3}{4}\right)$	B1ft
	Either : $y-"7"="-\tfrac{3}{4}"\left(x-"3"\right)$ **or**: $y="-\tfrac{3}{4}"x+c$ and $"7"="-\tfrac{3}{4}"\left("3"\right)+c \Rightarrow c="9\tfrac{1}{4}"$	M1
	$4y+3x-37=0$ or $y-7=-\tfrac{3}{4}\left(x-3\right)$ Or $y=-\tfrac{3}{4}x+9\tfrac{1}{4}$	A1 **[5]**
(c)	Substitute y = 10 into their line equation to give x =	M1
	x = -1	A1 **[2]**
(d)	$(r^2)=(-1-0)^2+(10-3)^2$ or $(r^2)=(-1-6)^2+(10-11)^2$	M1
	$r^2=50$	A1
	$"50"=(x\pm"(-1)")^2+(y\pm"10")^2$	M1
	$"50"=(x-"(-1)")^2+(y-"10")^2$	A1ft
	$x^2+y^2+2x-20y+51=0$	A1 **[5]** **(14 marks)**
	Alternative methods to part (d) (i)Use equation $x^2+y^2+ax+by+c=0$ and substitute three points, usually (0,3), (6,11) and another point on the circle maybe (-2,17) or (-8,9) - **not** point Z Solves simultaneous equations a = 2, b = -20 and c = 51	M1 dM1 A1,A1,A1
	(ii) Uses centre to write a = and b = (doubles x coordinate and y coordinate respectively, $\pm"2"$ *and* $\pm"20"$) Obtains a= 2 and b = -20 (or just writes these values down so these answers imply M1A1) Completes method to find c, (could substitute one of the points on the circle) or could find r Accurate work e.g. $r^2=50$ or e.g. $x^2+y^2+2x-20y=(-8)^2+9^2+2\times-8-20\times9=$ $c=51$	M1 A1 dM1 A1 A1